VOLUME 108 PART A NUMBER 41 OCTOBER 1961

The Proceedings

OF

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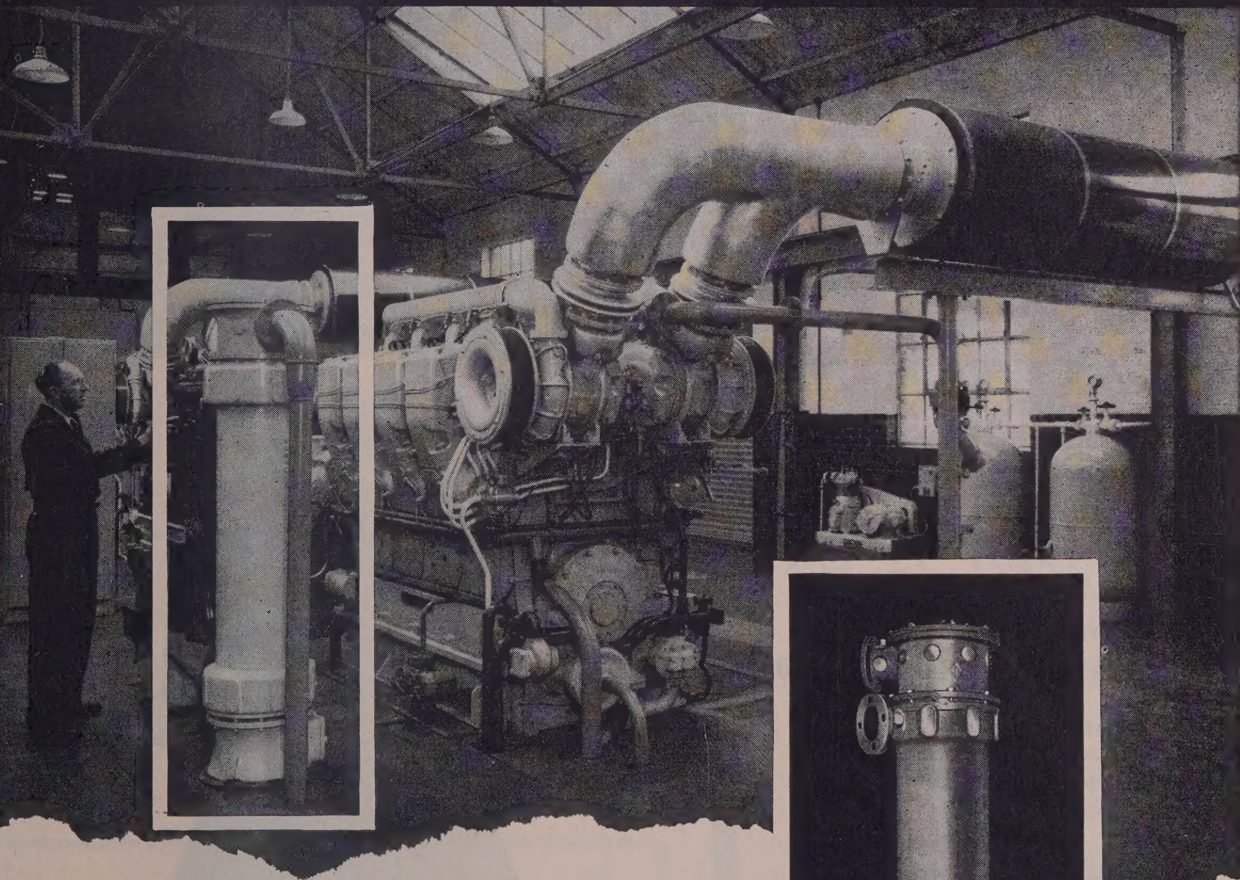

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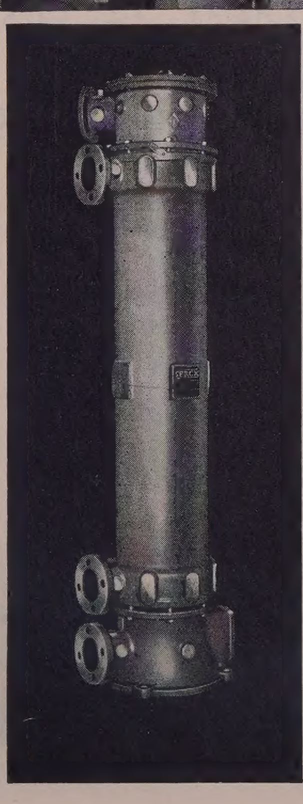

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HARACTERISTICS

ectrical

	BT. 109	BT. 113
thode type	Directly heated.	Directly heated.
lament voltage	2.5 volts	2.5 volts
lament current (max.)	23 amps	5.3 amps
lament current (min.)	19 amps.	4.5 amps.
ltage drop (max.)	15 volts.	15 volts.
thode heating time	60 secs.	30 secs.
isation time (approx.)	4 micro secs.	10 micro secs.
ionisation time (approx.)	1000 micro secs.	1000 micro secs.
ode-control capacitance	4 $\mu\mu$ farads.	2 $\mu\mu$ farads.
ntrol grid—cathode capacitance	18 $\mu\mu$ farads.	7 $\mu\mu$ farads.

echanical

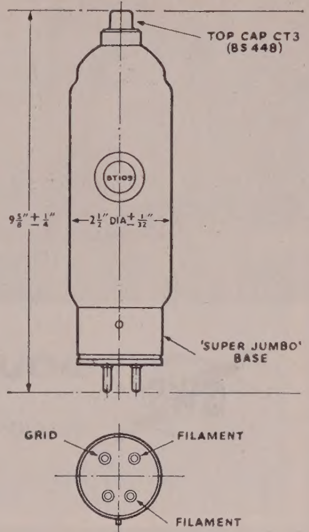
pe of cooling	Convection.	Convection.
ounting position	Vertical, base down.	Vertical, base down.
eight (max.)	(11½ oz). (328 gm).	(2¾ oz.) (80 gm).
uilibrium condensed mercury mperature rise above nbient (full load)	32°C.	(approx.) 24°C.
(no load)	29°C.	(approx.) 23°C.

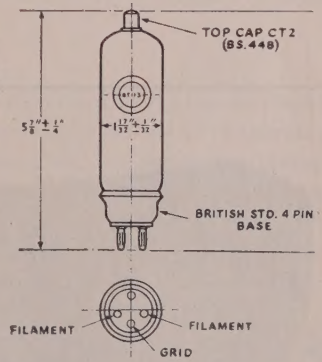
tings

x. peak anode voltage: forward	1500 volts.	1500 volts.
inverse	1500 volts.	1500 volts.
x. cathode current: peak	77 amps.	2 amps.
average	6.4 amps.	0.5 amps.
x. averaging time	15 secs.	15 secs.
nbient temperature range*	—40°C. to +40°C.	—40°C. to +40°C.

till air temperature near the base of valves.

though valves will operate satisfactorily at ambient temperatures of —40°C. to +10°C., e will be reduced at these low temperatures. For maximum life the valve should be operated ambient temperatures within the range +15°C. to +40°C.





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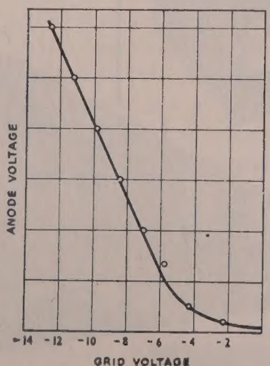
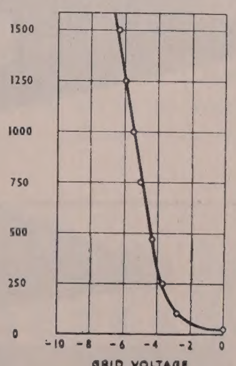
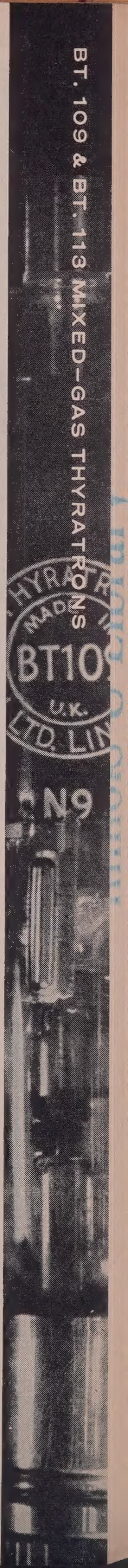


BT 113 Thyratron control characteristic



BT 109 Thyratron control characteristic



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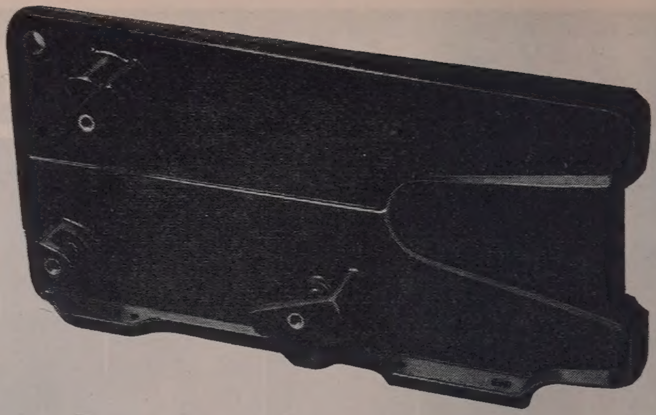

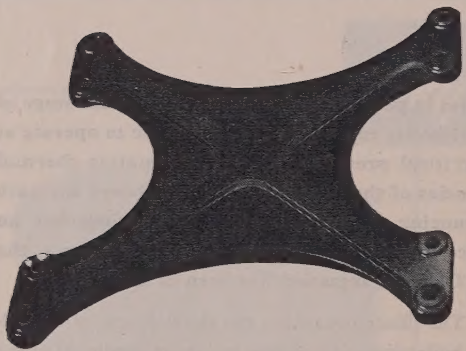

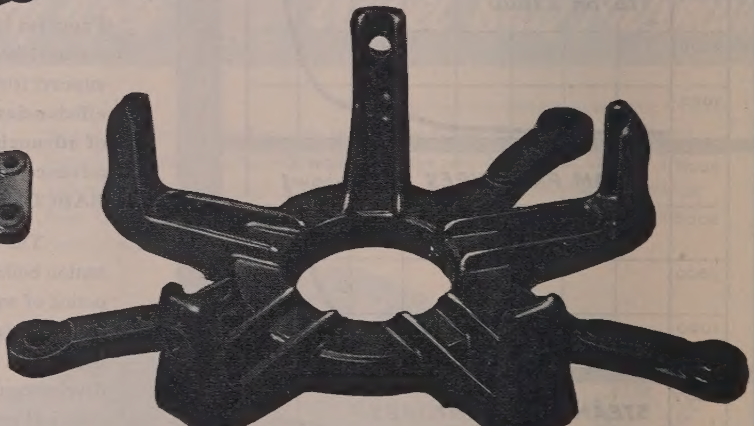

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Summary — An essential part of a Paper is the **Summary**, which should not exceed 200 words.

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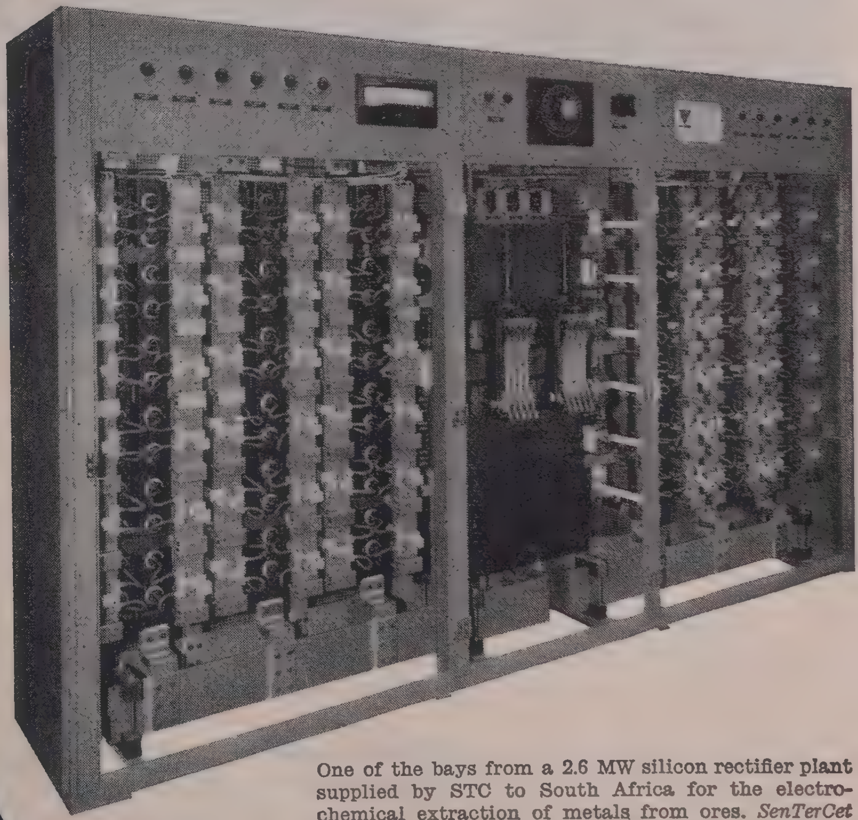

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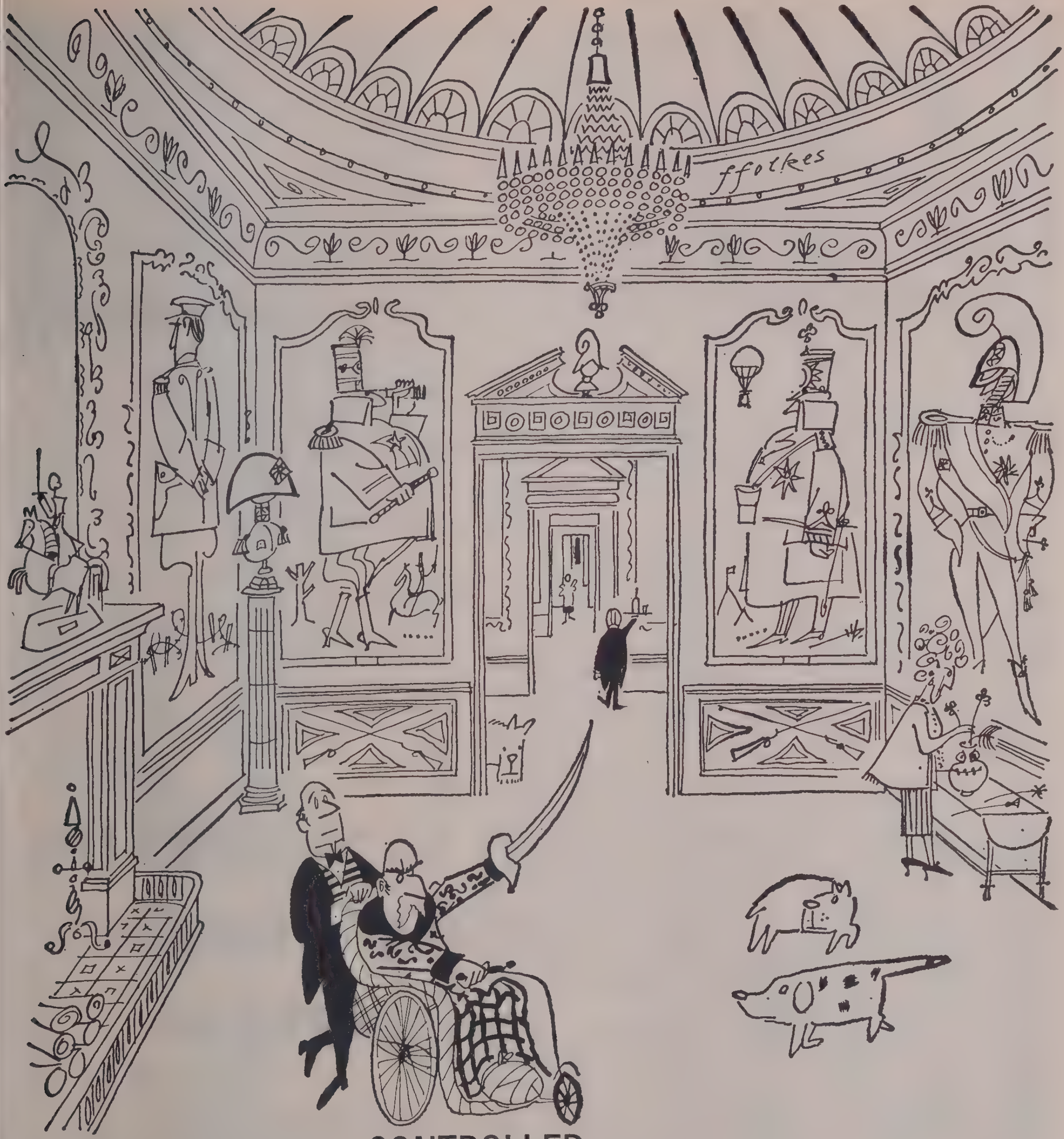
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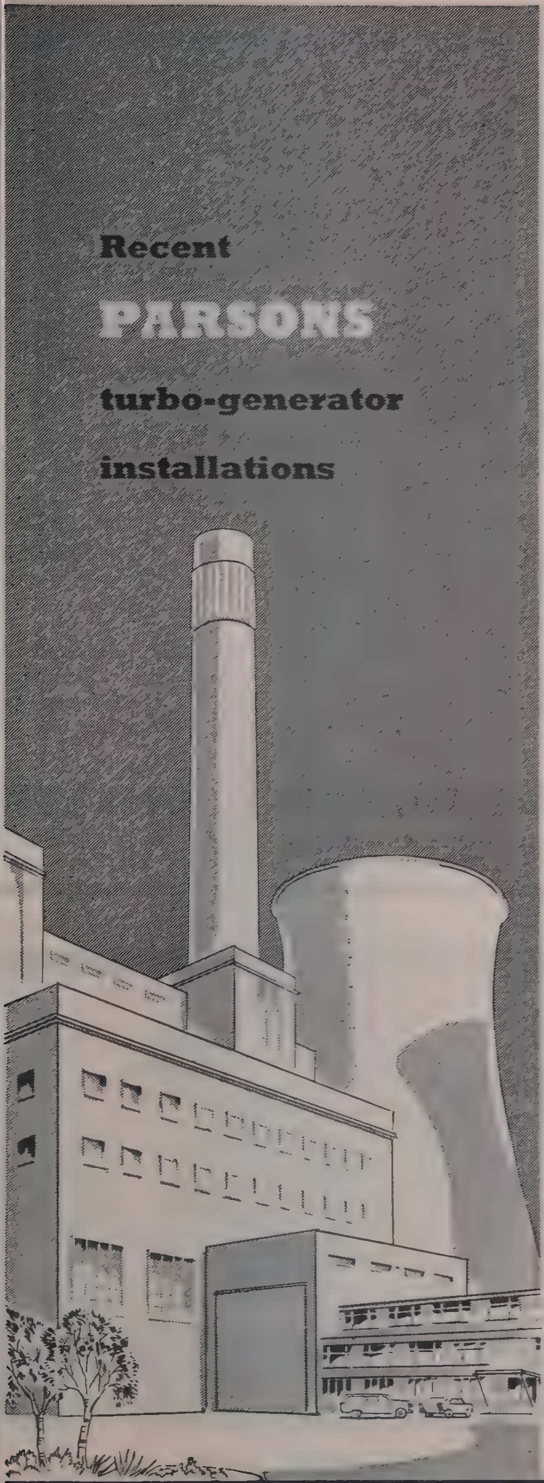
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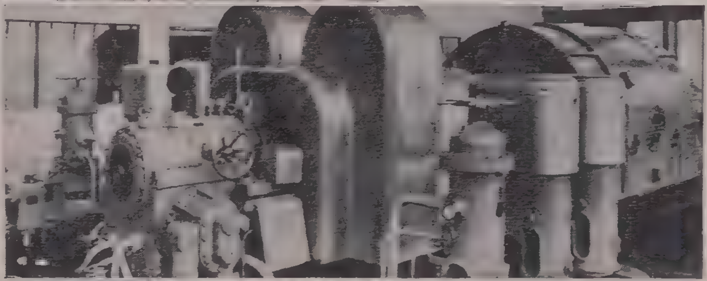

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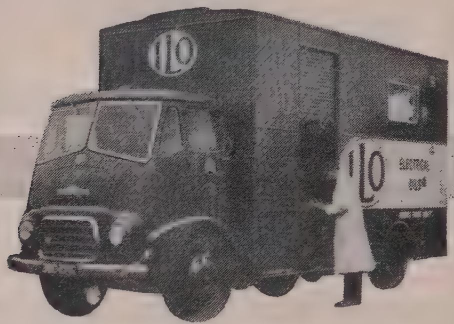

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Nov. 1960
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A GENERAL THEORY OF DEPRECIATION OF ENGINEERING PLANT

By D. RUDD, B.Sc.(Eng.), A.M.I.Mech.E., Associate Member.

(The paper was first received 5th May, and in revised form 17th August, 1960. It was published in November, 1960, and was read before the SUPPLY SECTION *15th February, the* NORTH-EASTERN CENTRE *27th February, and the* SOUTH-EAST SCOTLAND SUB-CENTRE *4th April, 1961.)*

SUMMARY

The conventional methods of providing for the depreciation of gineering plant are criticized on the grounds that they contain bitrary features, and a general theory is formulated which is not bject to such criticism. The general theory is applied first to dividual projects, showing how the factors which affect depreciation erate, and secondly to an integrated industry, showing how the ogrammes for writing off the investments in the industry can be -ordinated and the economic assessment of new projects facilitated consequence. The effects of variations in the value of money are so considered.

The propositions are illustrated by numerical examples drawn, in e case of the application to an integrated industry, from the field of blic electricity supply.

(1) INTRODUCTION

Most engineering projects require fairly heavy capital invest-ent, and it is the normal practice to charge interest on the pital and to provide annually for depreciation of the investment. he classical method of providing for depreciation is to set up a nking fund into which equal annual payments are made, the yments being calculated so that, at the end of an appointed riod, the fund, with accumulated compound interest, amounts the original capital cost. If the interest on the fund is equal that on the capital this method is exactly equivalent to the ethod of redeeming mortgages on house property whereby, it of a constant annual charge, interest is first paid on the nount of the loan outstanding and the remainder of the charge en goes to redeeming a portion of the loan, the constant annual arge being calculated so as to redeem the whole of the loan by e end of the period.

An alternative method, which is known as the 'straight-line' ethod and is used for existing investments but not as a le for the economic assessment of future projects, is to ake the annual provision for depreciation constant instead of e total annual charge, so that the interest payments decrease iformly to zero at the end of the period. The classical method and the straight-line method will be referred to as the conventional methods of providing for depreciation.

In many concerns, notably those which invest at frequent intervals, the process of borrowing money may be separate from the process of investment. The borrowing periods may then bear little or no relation to the periods over which the investments are written off, and the provision for depreciation of one investment may be used to defray part of the cost of a later investment, or for the purchase of securities, or as working capital, instead of to repay an external debt. It is therefore necessary to distinguish between provision for depreciation, which may be a matter of purely internal accountancy but which nevertheless determines the annual cost attributed to an investment, and the arrangements for incurring and discharging external debts, the details of which are relevant in the present context only in so far as they affect the rate of interest.

In the classical method of providing for depreciation, the total annual charge is made constant without taking account of how the factors which affect depreciation operate, and this arbitrary feature has the effect of casting doubt on assessments of the costs and economic lives of engineering projects and on comparisons of alternative schemes. The straight-line method is perhaps a little more realistic in this respect, but the constant provision for depreciation is arbitrary. Accordingly, the present purpose is to formulate a general theory of depreciation (Section 2), which takes account of how the factors affecting depreciation operate, and to apply it first to individual projects (Section 3) and then to an integrated industry having a more or less continuous programme of investment (Section 4). In the latter application, it is found to be possible to co-ordinate the programmes for writing off the individual investments in a manner which the arbitrary features inherent in the conventional methods have hitherto prevented.

One of the phenomena to be taken into account in any programme for writing off investments is that of variations in the value of money, i.e. monetary inflation and deflation. The nature of this phenomenon is such, however, that it is easier to consider its effect on the fully developed theory than to introduce it at the outset. Accordingly, no mention is made of variations in the value of money until Section 5.

Mr. Rudd is with Humphreys and Glasgow Ltd.

(2) GENERAL THEORY

The term 'engineering project' is now widely used to connote any plant, aggregation of plant, station, works or other piece of engineering which is under consideration or in course of construction. In the absence of a recognized generic term to describe what an engineering project becomes when its construction is complete and it passes into service, the term 'engineering plant' will be used with this meaning.

The total annual cost of an engineering plant in this sense can be resolved into two components:

(*a*) *The capital charge*, which is defined as the sum of the interest payable in respect of the plant and the provision for depreciation.
(*b*) *The current cost*, which is defined as all other expenditure in respect of the plant.

The general theory of depreciation is that the capital charge should be such as to make the total annual cost in every year no greater than the cost of the most economic alternative method of providing the same service. It is implied that the cost of the most economic alternative is itself assessed in accordance with the theory, but it will become apparent that this does not lead to an infinite regression.

In terms of this theory, the *economic life* of an engineering plant is defined as the period after which it is more economical to replace the plant than to continue to use it, or the period after which either the plant is no longer physically capable of providing the service required or the demand for the service ceases, whichever is the shortest. In either of the latter circumstances, the determination of the life of the plant does not involve economic considerations and is not considered further here. In the former case, it is more economical to replace a plant than to continue to use it only if the total annual cost (including an appropriate capital charge on the cost of replacement) after the replacement is less than the current cost (excluding any capital charge which may still be payable on the old plant) before the replacement.

(3) APPLICATION TO INDIVIDUAL PROJECTS

There are two primary factors which limit the economic life of an engineering plant:

(*a*) *Physical deterioration*, i.e. wear and tear resulting from use and the action of the weather etc., whereby the plant becomes progressively less suitable for the purpose for which it was provided and consequently progressively more expensive to use.
(*b*) *Technological advance*, whereby a cheaper or more efficient plant, or one requiring fewer operators or less maintenance, becomes available during the life of the plant installed.

The following secondary factors, as will be shown, affect the economic life as limited by the primary factors:

(*c*) *The capital cost of the plant.*
(*d*) *The rate of interest.*
(*e*) *The load factor.*

These factors are considered individually in the following Sections, the necessary mathematical formulae being derived in Appendix 8.1.

(3.1) Physical Deterioration

Assuming for initial simplicity that there is no technological advance and that the secondary factors remain constant, physical deterioration must bring about an increase in the current cost from year to year. In these circumstances the theory requires the total annual cost to be constant, so that it is never greater than that of a new plant constructed to provide the same service. The capital charge should therefore decrease at the same rate as the current cost increases, and the initial charge should be calculated so as to redeem the whole capital cost by the time the current cost equals the total annual cost, i.e. when the capital

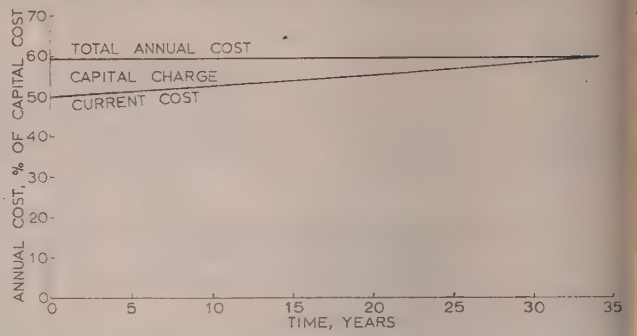


Fig. 1.—Variation of current cost, capital charge and total annual cost of a project in the absence of technological advance and at constant load factor.

Initial current cost = 50% of capital cost per annum.
Rate of increase of current cost = ½% per annum.
Rate of interest = 5% per annum.

charge decreases to zero. If the trend of current cost has been correctly estimated, this event then marks the end of the economic life of the plant. Such a process is illustrated graphically in Fig. 1.

(3.2) Technological Advance

Technological advance reduces the total annual cost of a new plant capable of providing the same service as the plant installed. The theory requires, therefore, that, if such advance is expected, the capital charge on the plant installed shall decrease year by year so as to make its total annual cost no greater than the decreasing total annual cost of a new plant constructed at any time, as illustrated in Fig. 2. Thus, the effect of technological

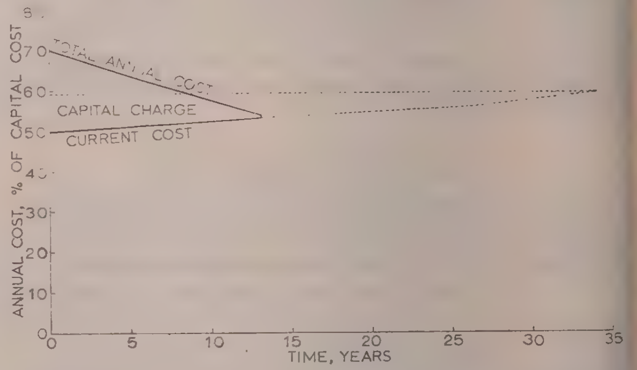


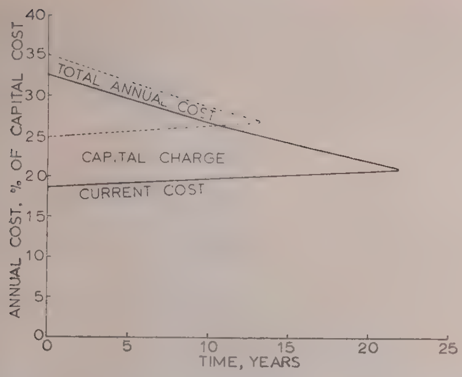
Fig. 2.—Effect of technological advance on the economic life, capital charge and total annual cost of a project (constant load factor).

Rate of decrease of total annual cost = 2% per annum.
All other data the same as in Fig. 1.
------ Fig. 1 repeated for comparison.

advance is to increase the total annual cost of the plant in the early years and to shorten its economic life: in the example considered, the total annual cost in the first year is increased by 16% and the economic life is shortened from 34 to 13 years by a rate of decrease of the total annual cost of 2% per annum.

(3.3) Capital Cost

The effect of the capital cost on the economic life of a plant can be illustrated by comparing alternative schemes for providing the service required. If, for example, as an alternative to the scheme illustrated in Fig. 2, the current cost could be reduced by 25% by doubling the capital cost, the economic life would be



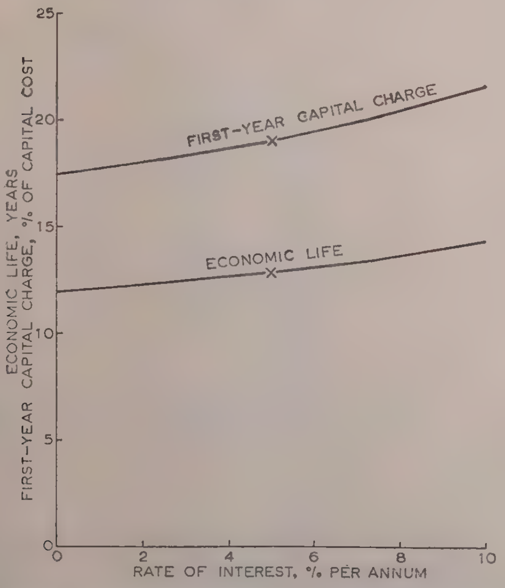
ig. 3.—Effect of capital cost on the economic life, capital charge and total annual cost of a project (constant load factor).

Capital cost = 200% of capital cost in Fig. 2.
Initial current cost = 18·75% of capital cost per annum, i.e. 75% of initial current cost in Fig. 2.
All other data the same as in Fig. 2.
------ Fig. 2 repeated for comparison, with appropriate change of scale.

ncreased to 22 years and the total annual cost reduced by 7% hroughout that period, as shown in Fig. 3. It is, of course, ecessary that a higher capital cost should result in a lower urrent cost; otherwise the alternative would not need to be nvestigated. But with this proviso, it is clear from Fig. 3 that, vhen alternative schemes are compared, the scheme involving he higher capital cost will have the longer economic life.

(3.4) Rate of Interest

The effect of the rate of interest on the economic life of a lant and the capital charge in the first year are illustrated in ig. 4. The economic life increases slightly with the rate of nterest, and this increase partly offsets the effect which the igher rate of interest would otherwise have on the capital harge in the first year. Hence the capital charge is less affected



ig. 4.—Effect of rate of interest on the economic life and first-year charge on a project (constant load factor).

Data the same as in Fig. 2 except that the rate of interest is variable.
× × Points represented by Fig. 2.

by the rate of interest than it would be if either conventional method of providing for depreciation were used.

(3.5) Load Factor

The use which is made of a plant, i.e. its load factor, directly affects its current cost. Reverting to Fig. 2, if the load factor were such that the current cost were only half as great as in that Figure (all the other data being unaltered), the economic life would be 19 years instead of 13 years, as illustrated in Fig. 5.

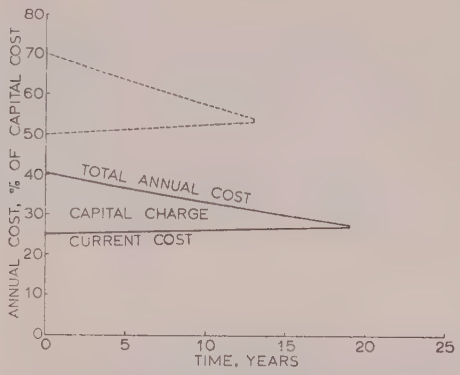


Fig. 5.—Effect of load factor on the economic life, capital charge and total annual cost of a project.

Initial current cost = 25% of capital cost per annum, i.e. 50% of initial current cost in Fig. 2.
All other data the same as in Fig. 2.
---- Fig. 2 repeated for comparison.

Moreover, if the plant were operated at the higher load factor for 13 years and circumstances then permitted the load factor to be reduced, the economic life would be correspondingly extended. In general, the economic life of a plant is sensibly determined by the final load factor at which it is operated.

(3.6) Capital Value

At any time after an engineering plant has been constructed (or even while it is still being constructed), it may become evident that some of the factors affecting depreciation have been imperfectly predicted, and that in consequence the total annual cost, including the capital charge laid down in the original programme, is appreciably more or less than the cost of the best alternative method of providing the same service. It is therefore necessary to distinguish between the unredeemed capital cost of the plant at any instant and its capital value. The *unredeemed capital cost* is defined as the sum which remains after deducting the successive provisions for depreciation from the original capital cost. The *capital value*, on the other hand, is defined as the greatest capital sum which it is possible to write off within the remainder of the economic life of the plant in accordance with the general theory, using the best available data at that instant. Whether and how often it is worth undertaking the valuation or revaluation of a plant depends on whether and to what extent its subsequent use is affected thereby.

(4) APPLICATION TO AN INTEGRATED INDUSTRY

In Section 3, attention was focused on calculating the temporal series of annual capital charges on a single engineering plant, by comparison with alternative methods of providing the same service. But, within an integrated industry having a more or less continuous investment programme for the purpose of providing a service, the general theory can be developed to enable the total annual costs of—and hence the capital charges

on—the plants providing that service to be compared with one another, year by year, without reference to the temporal series of charges or to costs obtained outside the industry. The method of application and the consequent development of the theory must depend on the factors from which the economy of each individual industry derives: it is, therefore, only feasible in the present context to apply the theory to one industry, namely public electricity supply, and within that industry to the provision of generating capacity.

Electricity generation for public supply in Great Britain takes place in a number of generating stations, almost all of them thermal stations, interconnected by the Grid and Supergrid. (The existence of a small number of hydro-electric stations, depending on water storage, necessitates a modification of the development of the general theory, but in the present treatment this modification is omitted in the interest of simplicity.) In the natural course of events the stations in use at any one time differ widely in age, capacity, capital cost, thermal efficiency and works cost, but for any one thermal station, the total annual cost can be related to the output from the station by the approximate formula:

$$\delta c = S\delta p + R\delta w + Q\delta v \qquad . \quad . \quad . \qquad (1)$$

where

δc = Total annual cost of station, £.
δp = Its output capacity, kW or MW.
δw = Its annual output, kWh or GWh.
δv = Its potential annual output, kWh or GWh, i.e. the annual output which would be produced if the generators in operation at any time were on full load.
S, R and Q = Constants which do not vary with the output or potential output and will be termed 'cost parameters'.

The term $S\delta p$ is commonly referred to as the 'standing cost' of the station and is written here as a product purely for convenience. The sum $R\delta w + Q\delta v$ is commonly referred to as the 'running cost' of the station, and it is a matter of experience that the running cost is related to the annual and potential annual outputs approximately as stated. (The running cost is frequently regarded as proportional to the annual output alone, but the binomial expression is a better approximation.)

In order eventually to compare stations of unequal capacity, it is convenient at this juncture to eliminate δp from eqn. (1) by simple division, obtaining

$$C = S + RF + QE \qquad . \quad . \quad . \quad . \quad . \qquad (2)$$

where $\quad C = \delta c/\delta p, \quad F = \delta w/\delta p$ and $E = \delta v/\delta p$

F is then proportional to the annual load factor of the station, whilst F/E is equal to the 'plant load factor' in the sense in which that term is normally used.

Now the total capacity of all the stations is determined by the peak demand on the supply system, but the demand fluctuates daily and annually over a wide range and one of the principal advantages of public electricity supply, compared with private generation, is that the supply undertaking can co-ordinate the outputs of the generating stations in order to minimize the total running cost: thus, instead of allowing the output of every station to fall proportionately when the demand falls, the outputs of stations with low running-cost parameters can be maintained preferentially compared with stations with high running-cost parameters. Neglecting as a first approximation the electrical losses in the interconnecting system, the required co-ordination is achieved by listing all the stations in order of increasing running-cost parameters, and bringing them into operation in that order as the demand increases and shutting them down in the reverse order as the demand decreases.

On the basis of this list, the total capacity can be divided into sections (as many as may be convenient) of equal capacity, arranged in order of decreasing annual output. In Table 1* the total capacity in Great Britain in the year 1948–49 is divided into twelve sections, and columns (ii)–(vi) give the sectional data from which the factors, F and E, and running cost parameters, R and Q, can be calculated, as described in Appendix 8.2. Figs. 6 and 7 show how these quantities vary from section to

Table 1

OUTPUTS AND COSTS

Section number	Output capacity	Annual output		Running cost component proportional to		Current component of standing cost	Unredeemed capital cost brought forward	Interest	Standing cost		Unredeemed capital cost carried forward	Total annual cost
		Actual	Potential	Annual output	Potential annual output				Total	Capital charge		
(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)	(xii)	(xiii)
	MW	GWh	GWh	£ × 10³	£ × 10³	£ × 10³	£ × 10³	£ × 10³	£ × 10³	£ × 10³	£ × 10³	£ × 10³
1	857·5	5793	6852	7497	298	527	41677	1873	5846	5319	38231	13641
2	857·5	5775	6851	7925	580	573	30554	1373	5102	4529	27398	13607
3	857·5	5421	6850	8140	921	617	19058	856	4039	3422	16492	13100
4	857·5	4971	6511	7507	1123	670	11991	539	3737	3067	9463	12367
5	857·5	4389	5934	6809	1208	700	7222	324	3355	2655	4891	11372
6	857·5	4093	5532	6635	1296	739	4187	188	2885	2164	2229	10816
7	857·5	3798	5191	6539	1359	782	2215	100	2340	1558	757	10238
8	857·5	3172	4351	5607	1399	838	861	39	1909	1071	17*	8915
9	857·5	2219	3393	4537	1269	896	419	19	1033	137	113†	6839
10	857·5	1921	2818	4268	1254	931	—	—	931	—	—	6453
11	857·5	1315	2217	3444	1153	1057	—	—	1057	—	—	5654
12	857·5	710	1432	2697	932	1222	—	—	1222	—	—	4851
Totals	10290	43582	57932	71605	12792	9552	118184	5311	33456	23904	33456	117853

* £188 000 of surplus assets deducted.
† £188 000 of dead liabilities written off.

* All the data in columns (ii)–(ix) of Table 1 are obtainable from the station output records, plant performance records and station accounts, but the figures in the Table, though realistic, are not authentic. The costs of hydro-electric stations are replaced by the costs of equivalent thermal stations, i.e. thermal stations having cost parameters which would result in their occupying the respective places in the list.

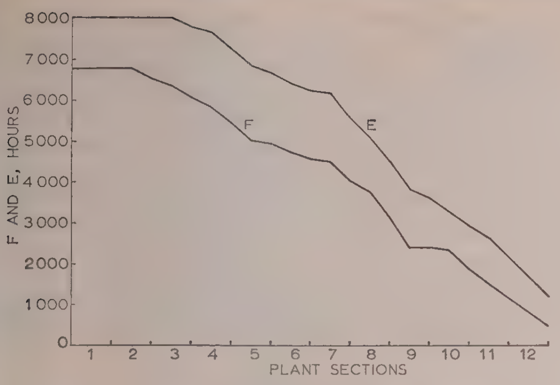


Fig. 6.—Variation of F and E over plant sections.

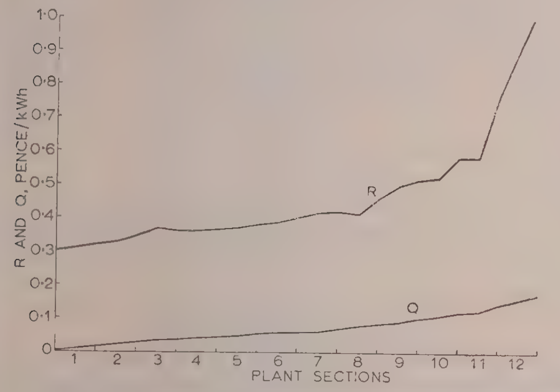


Fig. 7.—Variation of R and Q over plant sections.

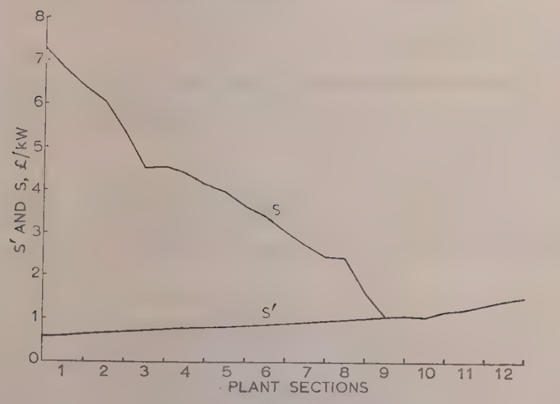


Fig. 8.—Variation of S and S' over plant sections.

section and within the sections. Column (vii) gives the current component of the standing cost, i.e. that part of the standing cost which is classified as current cost, for each section, and the corresponding cost parameter, S' (the current component of S) appears in Fig. 8. Finally column (viii) of the Table shows the unredeemed capital cost of each section brought forward from the previous annual account, and column (ix) the interest payable thereon in the current year.

By means of these data and eqn. (2), the total annual cost of any station can be compared with the cost of obtaining the output of that station by the alternative method of generating it in another station, e.g. the next station in the list. Neglecting as a first approximation the cost of interconnection, the general theory requires the former cost to be no greater than the latter, i.e.

$$S_1 + R_1F_1 + Q_1E_1 \leqslant S_2 + R_2F_1 + Q_2E_1 \quad . \quad . \quad (3)$$

where the suffices apply to the station first considered and the adjacent station respectively. Similarly, comparing the total annual cost of the second station with the cost of obtaining its output from the first station, the requirement is that

$$S_2 + R_2F_2 + Q_2E_2 \leqslant S_1 + R_1F_2 + Q_1E_2 \quad . \quad . \quad (4)$$

But the capacity of any station is very small compared with the total capacity and hence F and E closely approximate to continuous functions, as illustrated in Fig. 6. Furthermore, in the year 1948–49, virtually all the thermal stations burned fossil fuel, used steam as the working medium and were listed for the most part in order of increasing age for operation purposes. In such circumstances, R and Q also approximate to continuous functions, as illustrated in Fig. 7. Hence eqns. (3) and (4) can be combined into the differential equation

$$dS + dR\,F + dQ\,E = 0 \quad . \quad . \quad . \quad . \quad (5)$$

The advent of nuclear power and the increasing construction of new peak-load stations, which are specially designed for the purpose and do not necessarily use steam as the working medium, make it necessary now or in the near future to recognize discontinuities in the graphs of R and Q. At any such discontinuity, eqn. (5) is replaced by the difference equation

$$\Delta S + \Delta R\,F + \Delta Q\,E = 0 \quad . \quad . \quad . \quad . \quad (6)$$

where ΔS, ΔR and ΔQ denote the discontinuities in S, R and Q respectively.

In the present context, however, it is only feasible to give a numerical illustration of the general theory in respect of the single year 1948–49, and for that year eqn. (5) provides a sufficiently accurate approximation for the purpose.

The range of validity of eqn. (5) is limited by the requirement that the capital charge on a station shall not be negative, i.e. the equation ceases to be valid where it leads to the standing cost parameter, S, being less than its current component, S'. Starting from the point at which $S = S'$, which has to be determined by iteration, the equation can be solved numerically, as described in Appendix 8.3, and the solution is presented in columns (x)–(xiii) of Table 1 and in Fig. 8. Column (x) of the Table shows the standing cost of each section, as calculated from the equation, and column (xi), the capital charge. The variations within the sections of the standing cost parameter, S, and of its current and capital charge components, are shown in Fig. 8, from which the capital charge on any particular station can be ascertained by interpolation. Finally, column (xii) shows the unredeemed capital cost of each section, carried forward to the next annual account, and column (xiii), the total annual cost in the current year.

In the foregoing solution, the point at which $S = S'$ is in the ninth section and there is no capital charge against any of the stations which lie beyond this point in the list. These stations have reached the ends of their economic lives and should be classified as obsolete, although some of them have not been completely written off in the previous annual account. Conversely, the capital charges against some of the stations which are not yet obsolete are greater than necessary to write off completely their unredeemed capital costs. In the former case the unredeemed capital cost of an obsolete station constitutes a liability for which there is no corresponding asset and which may therefore be termed a 'dead' liability, whilst in the

latter case the excess capital charge on a station, after the capital cost has been completely written off, constitutes an asset for which there is no corresponding liability, and which may therefore be termed a 'surplus' asset. Accordingly, the point at which $S = S'$ has been determined so as to equate the total dead liabilities and the total surplus assets, as described in Appendix 8.4 and illustrated in Fig. 9. (For the purpose of illustrating the

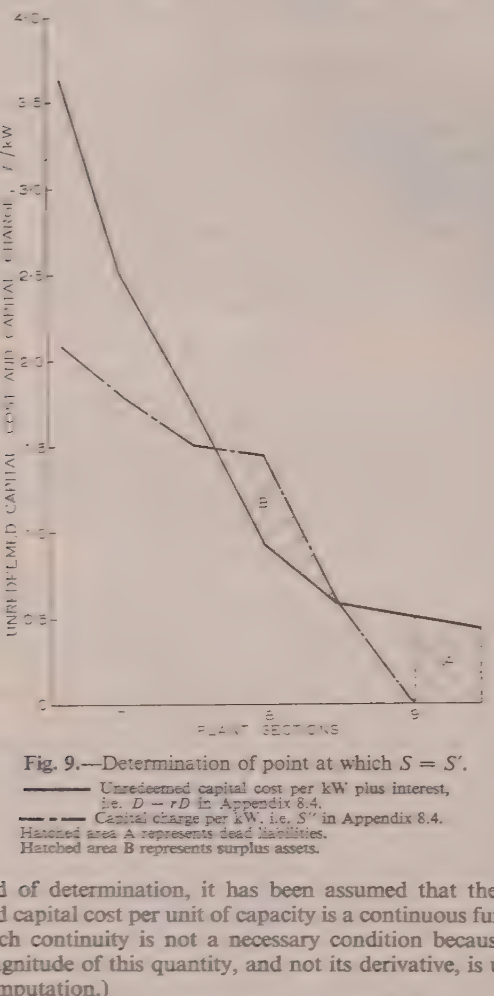


Fig. 9.—Determination of point at which $S = S'$.
——— Unredeemed capital cost per kW plus interest, i.e. $D - rD$ in Appendix 8.4.
— - — Capital charge per kW, i.e. S'' in Appendix 8.4.
Hatched area A represents dead liabilities.
Hatched area B represents surplus assets.

method of determination, it has been assumed that the unredeemed capital cost per unit of capacity is a continuous function, but such continuity is not a necessary condition because only the magnitude of this quantity, and not its derivative, is used in the computation.)

In the first year of application of the general theory, the unredeemed capital costs brought forward from the previous account must necessarily have been obtained by a conventional method of providing for depreciation. If the method has resulted in the total capital charge in previous years being smaller than would have been required by the general theory, the effect of making the foregoing totals precisely equal in the first and every year would be to bring about a transient increase in the total charge beyond what is ultimately necessary, or a transient reduction if the total charge was previously too large. Moreover, the inevitable imperfections of load forecasting and station design, which lead to the accumulation of dead liabilities, would probably produce erratic fluctuations in the total charge, even after the initial transient fluctuation had decayed. It may therefore be necessary to permit temporary inequalities between the total dead liabilities and the total surplus assets in order to even out such fluctuations and prevent them from being reflected in the trading account. Nevertheless, the general theory enables the total capital charge and the charge on every station to be determined year by year, without the necessity of long-term estimates of the economic lives of the stations or the future trends of the factors which affect depreciation.

One of the purposes of computing the data in Table 1 is to enable the estimated costs of new projects to be assessed against the background of the whole industry. For this purpose it is necessary to extrapolate the data provided by successive annual analyses to cover the prospective economic lives of the projects to be assessed, and the accuracy of the extrapolation will diminish with distance into the future. But the conjectural element in such extrapolation is greatly reduced if the programmes for writing off the individual stations are co-ordinated. Thus, it is apparent from Fig. 10 that, if obsolete plant is

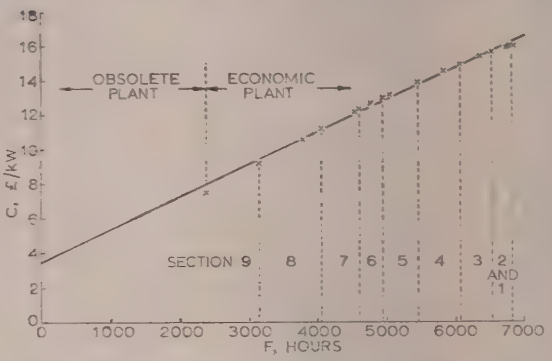


Fig. 10.—Relation between total annual cost and output.
× × × Points calculated from Table 1.
——— Empirical relation.

excluded, the total annual cost of any station is related to its annual output by the approximate empirical formula

$$C = H + KF \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where $H =$ £3·5 per kW,

$K =$ £0·001 86 (0·447d.) per kWh,

and that greater accuracy could be obtained, if it were desired, by introducing a term in F^2.

Now, from eqns. (2) and (5),

$$dC = RdF + QdE \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and, from eqns. (2) and (6), at any discontinuity in R or Q,

$$\Delta C = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

C is therefore a continuous function of F and E, and it is reasonable to suppose that some simple relationship, such as that exemplified by eqn. (7), will be found to persist in successive annual analyses and that the coefficients will vary only gradually from year to year. If this supposition is correct, the rate of variation of the coefficients is a valid measure of the rate of technological advance in the supply industry, and the coefficients themselves can be forecast in much the same manner as the peak demand and total annual output are forecast at present.

(5) VARIATIONS IN THE VALUE OF MONEY

Monetary inflation is one of the oldest of economic phenomena, but it is only in recent history that serious attempts have been made to measure variations in the value of money. Such attempts presuppose that money is no longer to be regarded as the sole standard of value, and that it is necessary to set up a new standard, values which are related to the new stan-

dard being then described as 'real' to distinguish them from monetary values. Perhaps the most acceptable standard of real value at the present time is that afforded by the Retail Price Index, in as much as it appears to be less affected than the other price indices by variations in individual factors, such as the prices of particular materials and the terms of foreign trade, when such variations occur independently of the value of money; however, the propositions in this Section are not affected by which index is adopted and the foregoing suggestion is no more than tentative.

The capital charge, x (£), on an engineering plant (or collection of plants) in a given year comprises the interest payable on the unredeemed capital cost brought forward from the previous annual account plus the provision for depreciation: i.e.

$$x = rA + (A - B) \quad . \quad . \quad . \quad . \quad (10)^*$$

where $100r$ denotes the rate of interest (% p.a.) compounded annually and A and B denote the unredeemed capital costs (£) brought forward from the previous annual account and carried forward to the next annual account respectively.

If x, A and B are real quantities, the corresponding monetary quantities, x', A' and B', are given by

$$x' = xJ, \qquad A' = AI, \qquad B' = BJ \quad . \quad . \quad (11)$$

where $100I$ and $100J$ denote the price index (base 100) at the beginning and end of the year respectively.

The corresponding equation in terms of monetary values is then

$$x' = r'A' + (A' - B') \quad . \quad . \quad . \quad . \quad (12)$$

where $100r'$ denotes the monetary rate of interest.

From eqns. (11), if the price index increases by $100i$% during the year, i.e. $J = I(1 + i)$, the monetary provision for depreciation is related to the real provision by the equation

$$A' - B' = (A - B)J - AIi \quad . \quad . \quad . \quad (13)$$

and, from eqns. (10), (11) and (12), the monetary rate of interest is related to the real rate by the equation

$$r' = r + i + ri \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Hence, applying the general theory to the real quantities, i.e. to the real capital charge and the real total annual cost, it can be shown that the effect of variations in the price index depends on:

(*a*) Whether the financial arrangements by which the capital is made available are expressed in terms of real or monetary values, i.e. whether a real or monetary rate of interest is specified.

(*b*) Whether the variations in the price index can be predicted.

From eqns. (13) and (14), if the real rate of interest were specified, the effect of inflation would be to reduce the successive monetary provisions for depreciation and to increase the monetary rate of interest. For example, for the project considered in Fig. 2, if the price index were to vary from 100 to 104 in the first year of operation, the calculations would be as follows:

	Real account	*Monetary account*
Capital charge (from Fig. 2)	18·8	19·6
Subtract interest payment	5·0	9·2
Provision for depreciation	13·8	10·4

* Cf. eqn. (16) in Appendix 8.1.

	Real account	*Monetary account*
Initial capital cost brought forward	100·0	100·0
Subtract provision for depreciation	13·8	10·4
Unredeemed capital cost carried forward	86·2	89·6

If this rate of inflation (4% per annum) were to persist throughout the life of the plant, it would be written off as illustrated in Fig. 11.

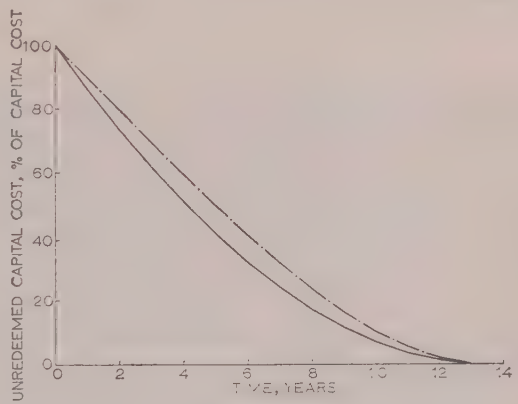


Fig. 11.—Programme for writing off an engineering plant.

Rate of monetary inflation = 4% per annum.
All other data the same as in Fig. 2.
———— Real value.
— — — Monetary value.

Deflation would have the opposite effect, i.e. it would increase the monetary provisions for depreciation and reduce the monetary rate of interest. But the economic life of the plant, being calculated in terms of real values, would be unaffected by variations of the index in either direction. It would, therefore, make no difference whether the variations could be predicted or not.

In practice, however, capital for engineering projects is normally made available by means of loans at specified monetary rates of interest. (In the privately owned sector of industry, capital is raised by issuing equity shares, but an individual project is normally required to pay interest or to produce a return on its capital cost at a specified monetary rate.) With such an arrangement, the real rate of interest, r, equivalent to the specified monetary rate, r', can be derived from eqn. (14), obtaining

$$r = \frac{r' - i}{1 - i} \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

If, therefore, the movements of the price index can be predicted, the equivalent real rate of interest can be incorporated in the programme for writing off the plant. Inflation then reduces the real rate of interest and hence reduces both the economic life of the project and the real capital charge, as illustrated in Fig. 4. For example, if the monetary rate of interest were 5% per annum and the predicted rate of inflation were 4% per annum, the real rate of interest would be 1·0% per annum. The economic life of the project considered in Fig. 2 would then be 12 years (instead of 13 years) and the initial real capital charge would be 17·7% (instead of 18·8%)

of the original capital cost. The calculations for the first year would be as follows:

	Real account	Monetary account
Capital charge (from Fig. 4) ..	17·7	18·4
Subtract interest payment	1·0	5·0
Provision for depreciation	16·7	13·4
Initial capital cost brought forward	100·0	100·0
Subtract provision for depreciation	16·7	13·4
Unredeemed capital cost carried forward	83·3	86·6

It may be objected that a reduction in the real rate of interest which results from inflation is obtained at the expense of those who have lent the capital at a specified monetary rate of interest, and hence that it is not in the national interest to take advantage of the reduction in drawing up the programme of capital charges. To overcome this objection, however, it is only necessary to stipulate that the predicted trend of the price index shall be available to both parties to the loan: it is then a fair assumption that both parties have discounted the trend and accept the equivalent real rate of interest. With this stipulation there is no reason why the equivalent real rate should not be used in calculating the capital charges.

The process of predicting the trend of the price index is a separate study which will not be pursued here. It is evident, however, that the index is subject to erratic variations which cannot be reliably predicted. Such variations can be dealt with only by correcting the capital charge as and when they occur. Now the real capital value of a plant at any instant is determined by the prospective real rate of interest, as calculated from the monetary rate of interest and the predicted trend of the price index, during the remainder of the economic life of the plant. The real capital value is therefore unaffected by unpredicted variations in the index and the only correction to be made is to the interest payment, in accordance with eqn. (15). Assuming for simplicity that the predicted variation in the index was zero, but that the index actually increased from 100 to 104 in the first year, the calculations for the example considered in Fig. 2 would be

	Real account	Monetary account
Initial capital cost	100·0	100·0
Subtract capital value, with a prospective real rate of interest of 5% p.a.	86·2	89·6
Depreciation	13·8	10·4
Add interest at the monetary rate of 5% p.a.	1·0	5·0
Capital charge	14·8	15·4
Predicted capital charge	18·8	

There is thus a discrepancy between the predicted real capital charge and the charge actually made. The magnitude of the discrepancy depends on the capital value of the plant, and the discrepancies in successive years will therefore decrease as the plant depreciates, but they may be cumulative over the life of the plant and can then be regarded only as a windfall gain (or loss). But whereas a windfall gain or loss which comes about as the result of predicting incorrectly the rate of technological advance, for example, is the inevitable consequence of the inherent unpredictability of the course of events, the discrepancies arising from monetary inflation or deflation could be obviated by paying interest at a specified real rate instead of at a specified monetary rate. The variations in the price index, whether predicted or not, would then have no effect on the real capital charge.

Whatever the arrangements by which the capital is made available, the real and monetary capital values of the plant must both eventually decrease to zero, as illustrated in Fig. 11. Hence, apart from differences arising out of imperfect prediction of the factors affecting the economic life of the plant, the sum of the real provisions for depreciation over the whole economic life must equal the original real capital cost and the sum of the monetary provisions for depreciation must equal the original monetary capital cost, i.e. the 'historic' cost in both cases, not the cost of replacement. Thus if, for example, an undertaking which does not expand or contract or advance technically has a more or less continuous programme of investment to replace its plants as they reach the ends of their economic lives, the total real capital value of the plants will remain approximately constant. In the event of inflation the monetary replacement costs of the plants will continually increase, necessitating a continuous increase in the total monetary liability of the undertaking, but the total monetary capital value of the plants will increase at exactly the same rate, so maintaining the necessary balance between the total assets and total liabilities of the undertaking. Any attempt to provide for the replacement costs of the plants would produce an imbalance between these two totals.

(6) CONCLUSIONS

The general theory of depreciation of engineering plant, which has been derived from acceptable economic principle, is applicable to individual projects and to integrated industries having more or less continuous investment programmes.

The factors which affect the economic life of an individual engineering project are primarily physical deterioration and technological advance and secondarily the capital cost of the project, the rate of interest and the load factor. The general theory shows that the economic life varies inversely with the rate of physical deterioration, the rate of technological advance and the load factor, and directly with the capital cost and the rate of interest.

From a knowledge of the factors from which the economy of an integrated industry derives, the general theory can be developed to provide a virtually automatic method of determining year by year the total capital charge for the industry and the charge on every plant, without the necessity of long-term estimates of the economic lives of the individual plants or the future trends of the factors which affect depreciation. As a result of this development, the conjectural element in economic assessments of new projects, which rely on the extrapolation of cost data for the whole industry, is greatly reduced, and the way is pointed to correlating the data provided by successive annual analyses and extracting cost coefficients which are a valid measure of technological advance throughout the industry.

With the normal method of borrowing capital, at a specified monetary rate of interest, predicted variations in the value of money can be incorporated in the programme of capital charges on a project or group of projects by a simple computation of the real rate of interest in terms of the monetary rate and the rate of inflation or deflation. But unpredictable variations cannot be so incorporated and may lead to cumulative discrepancies

between the predicted capital charges and the charges actually made. These discrepancies would be obviated if the real rate of interest were specified: variations in the value of money, whether predicted or not, would then have no effect on the real capital charge.

Whatever the method of raising capital, apart from inexact predictions, the sum of the provisions for depreciation of a plant over its whole economic life must equal the 'historic' cost of the plant, not its replacement cost.

(7) ACKNOWLEDGMENT

I acknowledge with thanks the valuable comments which I have received from Prof. W. G. Bickley of the Imperial College of Science and Technology, and also his help with some of the mathematics in the very early stages.

(8) APPENDICES

(8.1) Formulae for the Capital Charge on an Individual Project

Let A_0 = Capital cost of project, £.
A_m = Unredeemed capital cost after m years, £.
n = Economic life, years.
$100r$ = Rate of interest (% p.a.) compounded annually.
x_m = Capital charge in mth year, £.

Then
$$x_m = rA_{m-1} + (A_{m-1} - A_m) \quad . \quad . \quad . \quad (16)$$
whence
$$\begin{aligned} A_m &= A_{m-1}(1+r) - x_m \\ &= A_{m-2}(1+r)^2 - x_{m-1}(1+r) - x_m \\ &= A_0(1+r)^m - x_1(1+r)^{m-1} - x_2(1+r)^{m-2} \ldots \\ &\qquad - x_{m-1}(1+r) - x_m \quad . \quad . \quad (17) \end{aligned}$$
and it is a necessary condition that $A_n = 0$ . . . . . (18)

(8.1.1) Constant Capital Charge (Classical Method).

$$x_m = x_1 = \text{constant}$$
Therefore, from eqns. (17) and (18),
$$A_n = A_0(1+r)^n - \frac{x_1}{r}[(1+r)^n - 1] = 0$$
whence
$$\frac{x_1}{A_0} = \frac{r}{1-(1+r)^{-n}} \quad . \quad . \quad . \quad . \quad . \quad (19)$$

(8.1.2) Exponentially Decreasing Total Annual Cost with Exponentially Increasing Current Cost (Figs. 1–5 and 11).

Let y_m = Current cost in mth year, £.
$100a$ = Rate of increase (% p.a.) in current cost.
$100b$ = Rate of decrease (% p.a.) in total annual cost.

Then
$$y_m = y_1(1+a)^{m-1} \quad . \quad . \quad . \quad . \quad . \quad (20)$$
and
$$x_m + y_m = (x_1 + y_1)(1-b)^{m-1} \quad . \quad . \quad . \quad (21)$$
Hence, from eqns. (17) and (18),
$$A_0(1+r)^n - (x_1+y_1)\left[\frac{(1+r)^n - (1-b)^n}{r+b}\right] + y_1\left[\frac{(1+r)^n - (1+a)^n}{r-a}\right] = 0$$
Now
$$(x_1+y_1)(1-b)^n = y_1(1+a)^n \quad . \quad . \quad . \quad (22)$$
Therefore
$$A_0(r+b)(r-a) = x_1(r-a) - y_1(a+b)[1-(1+r)^{-n}(1+a)^n]$$

Substituting $s = \dfrac{r-a}{1+a}$,
$$x_1 = A_0(r+b) + y_1\frac{(a+b)[1-(1+s)^{-n}]}{(1+a)s}$$
Therefore, substituting $u = \dfrac{s}{1-(1+s)^{-n}}$
$$x_1 = A_0(r+b) + \frac{y_1(a+b)}{u(1+a)} \quad . \quad . \quad . \quad . \quad (23)$$
where u is obtainable from orthodox annuity Tables [cf. eqn. (19)].

And from eqn. (22),
$$n = \frac{\log(1 + x_1/y_1)}{\log[(1+a)/(1-b)]} \quad . \quad . \quad . \quad . \quad (24)$$

Thus, x_1 and n can be computed from eqns. (23) and (24) by iteration.

(8.2) Construction of Figs. 6 and 7 and S' in Fig. 8

The Figures are constructed from the data in Table 1 by means of the following approximations (which can be replaced by more elaborate approximations if greater accuracy of computation is desired):

(a) All the quantities represented, i.e. F, E, S', R and Q, vary continuously with respect to p, the total capacity of all the stations preceding a given station in the list; i.e. each quantity has a single magnitude corresponding to any magnitude of p.

(b) The quantities vary rectilinearly over each half-section of plant, and over the whole of the first and twelfth sections.

(c) The magnitude of a quantity at the boundary between two sections is equal to the average of the mean magnitudes of the quantity within the sections.

Now let P = Capacity of every section (kW or MW),
$F_0, F_{\frac{1}{2}}, F_1$, etc. = Magnitude of F when $p = 0, \frac{1}{2}P, P$, etc.
$\bar{F}_{01}, \bar{F}_{12}, \bar{F}_{23}$, etc. = Mean of F between the limits $0 < p < P$, $P < p < 2P$, $2P < p < 3P$, etc.

Then, in accordance with the foregoing approximations, the magnitudes of F at the boundaries and centres of the sections are given by
$$\left.\begin{aligned} F_0 &= \bar{F}_{01} + \tfrac{1}{2}(\bar{F}_{01} - \bar{F}_{12}) \\ F_{\frac{1}{2}} &= \bar{F}_{01} \\ F_1 &= \tfrac{1}{2}(\bar{F}_{01} + \bar{F}_{12}) \\ F_{1\frac{1}{2}} &= \bar{F}_{12} + \tfrac{1}{4}[(\bar{F}_{12} - \bar{F}_{23}) - (\bar{F}_{01} - \bar{F}_{12})] \\ F_2 &= \tfrac{1}{2}(\bar{F}_{12} + \bar{F}_{23}) \\ F_{2\frac{1}{2}} &= \bar{F}_{23} + \tfrac{1}{4}[(\bar{F}_{23} - \bar{F}_{34}) - (\bar{F}_{12} - \bar{F}_{23})] \\ &\text{etc.} \end{aligned}\right\} \quad . \quad (25)$$
and the equations for E, S', R and Q are exactly similar.

(8.3) Numerical Solution of Eqn. (5)

From eqn. (5), using the approximations in Appendix 8.2, the difference between the magnitudes of S across any half-section, within the range of validity of the equation, can be obtained by integration. For example, over the first half of the third section,
$$S_2 - S_{2\frac{1}{2}} = \tfrac{1}{2}(R_{2\frac{1}{2}} - R_2)(F_2 + F_{2\frac{1}{2}}) + \tfrac{1}{2}(Q_{2\frac{1}{2}} - Q_2)(E_2 + E_{2\frac{1}{2}}) \quad . \quad . \quad . \quad . \quad (26)$$

Thus, if the magnitude of S is known at any point, the magnitudes at the boundaries and centre of every section can be computed.

Also, the magnitude of $\bar{S}$ for any section can be obtained by a second integration. Thus, for the first half of the third section,

$$\int_{2}^{2\frac{1}{2}} Sd(p/P) = \tfrac{1}{4}(S_2 + S_{2\frac{1}{2}}) + \tfrac{1}{24}(R_{2\frac{1}{2}} - R_2)(F_2 - F_{2\frac{1}{2}}) + \tfrac{1}{24}(Q_{2\frac{1}{2}} - Q_2)(E_2 - E_{2\frac{1}{2}})$$

in which it is found in practice that the second and third terms on the right-hand side are negligibly small in comparison with the first term.

Similarly, $$\int_{2\frac{1}{2}}^{3} Sd(p/P) = \tfrac{1}{4}(S_{2\frac{1}{2}} + S_3)$$

whence $$\bar{S}_{23} = \tfrac{1}{4}(S_2 + 2S_{2\frac{1}{2}} + S_3) \quad . \quad . \quad . \quad . \quad (27)$$

(8.4) Determination of Point at which $S = S'$

Let S'' = Capital charge component of S, i.e. $S = S' + S''$, and when $S = S'$, $S'' = 0$.

D = Unredeemed capital cost per kilowatt of output capacity brought forward from previous annual account, £/kW.

rD = Interest payable on D, £/kW.

Then the unredeemed capital cost per kilowatt, carried forward to the next annual account, is equal to $D + rD - S''$.

Using the same approximations for D as were used in Appendix 8.2 for F, etc., graphs can be plotted of $D + rD$ and S'' over, say, the seventh, eighth and ninth sections, for any trial point at which $S'' = 0$, as illustrated in Fig. 9.

Then the area under the graph of $D + rD$ beyond the point at which $S'' = 0$ represents the total dead liabilities, and the area bounded by the graph of S'' and the graph of $D + rD$, over the range in which $S'' > D + rD$, represents the total surplus assets.

Now if the graph of S'' were lifted bodily by a small increment, so maintaining the differences calculated from eqn. (26), it can be seen that the point at which $S'' = 0$ would move to the right, the total dead liabilities would be reduced and the total surplus assets increased. Lowering the graph of S'' would have the opposite effects. Hence the point at which $S'' = 0$ can be chosen so as to equate the two totals, and has been so chosen in Fig. 9.

DISCUSSION BEFORE THE SUPPLY SECTION, 15TH FEBRUARY, 1961

Mr. J. M. Drummond: The Central Electricity Generating Board use the straight-line method of depreciation. The conventional accounting period is one year, and we must ensure that in that period we do not overstate our profits and that the accounts include a proper charge for depreciation, as laid down in the Electricity Act, which also requires us to keep our accounts in accordance with the best commercial standards. Depreciation provision must allow for wear and tear and obsolescence. In addition, treatment should be uniform and consistent.

In my view there is no such thing as a precise measurement of depreciation over one year; it is an arbitrary assessment. For generating plant—incidentally, I should be interested to know how the author would apply his general theory to the Board's transmission assets rather than the generating plant—past experience shows that an economic life of some 30 years can be expected. In present circumstances it is economic to keep old plant available for peak purposes although it may be fully written-off.

It does not follow that the future will show exactly the same course of events and there is another element to be considered, that of financial prudence. Allowing for developments (who, 30 years ago, would have foreseen present nuclear developments?) and obsolescence, it is not financially prudent to assume that generating plant will last so long, and these considerations have led the Board to decide to write off conventional generating plant over 25 years. The basis adopted is simple and facilitates an accurate forecast of depreciation provision for many years ahead. Accurate forecasts are essential because we have to submit capital investment estimates to the Treasury and obtain capital allocations.

In the author's general theory I assume that by 'providing the same service' he means producing electricity in some other way. For the current year, that is a straightforward question of incremental fuel costs and depreciation does not enter into it. I am forced to the conclusion that the author is looking several years ahead and assuming that plant will be provided as required by the theory. His paper bears this out, because he mentions the necessity to make assumptions about factors such as the rate of technological advance, load factor, rates of interest and capital costs. We need a period of five years for planning and constructing power stations. Therefore, if we are to compare what the cost may be in five years time with the plant in the lowest merit order at that time, it is necessary to get a reasonable prediction of all those factors.

All the factors introduce the element of possible error into the forecast, not only of the depreciation provision itself, but of the quantity of plant programmed. In Section 3.6 the author writes: '. . . it may become evident that some of the factors affecting depreciation have been imperfectly predicted, and that in consequence the total annual cost, including the capital charge laid down in the original programme, is appreciably more or less than the cost of the best alternative method of providing the same service.' I suggest that all these factors are likely to be imperfectly predicted.

The author has gone back to the 1948–49 generating plant position for his illustration and points out that the extremely small hydro-electric plant then existing would necessitate modification of his theory. How then would he cope with the forecasting of nuclear plant development and the economics of pumped-storage schemes?

I ask myself whether the author's theory is not more related to the economic assessment of projects than to depreciation. I see no harm in drawing a distinction between the basis for an economic assessment and that used for depreciation calculations. There is an element of financial prudence which must be considered in the statement of profits.

Professor W. G. Bickley: I have watched the development of the author's ideas and their expression in mathematical language with interest. When I first started trying to teach engineers mathematics many years ago, mathematics among engineers was hardly respectable, and, indeed, many lecturers in engineering were loath to use it. However, nowadays there are very few mathematical techniques which some engineer has has not used to help solve his problems.

When one reflects upon the material and philosophical changes which men such as Newton, Maxwell, Kelvin and Einstein have brought about in human progress, I think one must admit that not only in physical science but in other regions as well the impact of mathematics produces remarkable effects. In par-

icular, the time has come when mathematical thinking should be applied much more consistently in the regions of economics and sociology. The author has in fact tried to do something of this, admittedly at the moment in only a small region.

Mathematics, of course, is not only a set of formulae: it is also a way of thinking.

I have sometimes been impressed by the faith which my engineering students, and in particular postgraduates, seem to have in the ability of mathematics to work miracles. It will not do that, of course, but what it can do is to enable one to express rather more accurately one's ideas about things, and then to deduce consequences which can show whether the ideas have some pretence to be sound or not.

I am absolutely sure that the way of thinking about this problem which the author has exemplified is bound, if one follows it patiently and honestly, to lead to results which are going to be of practical importance. It is in contrast to the complacency with which the accountants retain their old ideas and to the way in which they appear to resist, and profess themselves unable to understand, the new ones. The marks which they make on paper do not always correspond with tangible realities.

Ultimately, however, it is upon the soundness of the basic ideas that the truth of the conclusion rests. Even mathematics cannot produce truth out of falsehood.

Mr. D. J. Bolton: In his introductory remarks the author referred to the difference in practice between the project engineer, who has to make an economic assessment, and the accountant, who has to deal with the plant when it is bought. The project engineer uses the sinking-fund method, or some economically equivalent device, because it gives equal capital charges for each year of the asset's life.

The comparison drawn between the effects of the two methods is misleading. It is true that if you have a single asset the difference between the straight-line and sinking-fund methods is substantial, but that is not the position with an electricity undertaking which may have several thousand assets with some hundreds of different times of installation and replacement. The difference between the two methods in respect of annual depreciation contribution is then slight and may be either way round. (There is, of course, a substantial difference in respect of the accumulated reserve and it is always the same way round.*)

A classical definition of depreciation would be 'expired capital outlay' or 'the fall in value of wasting assets arising out of their use or tenure'. The definition in Section 2 has nothing to do with the assets but relates to depreciation provision, and the annual spread of such provision is necessarily arbitrary. I agree with Mr. Drummond that you cannot define annual depreciation in any exact manner. You buy at a certain price and you scrap at a certain price so many years later. Total depreciation is factual, but how you apportion it between the years must be arbitrary, and the author's method is as arbitrary as any of the others. It must stand or fall on the basis of convenience and on that basis I suggest it falls. The uses of a depreciation formula are twofold, economic assessment and accounting. The author's formula is unsuitable for the first and is less convenient than either of the standard methods for the second.

The effects of his proposals are to load the earlier years more heavily and the later years less heavily, but one of the reasons given for this, namely technological advance, is not a valid one. What happens in the outside world does not affect the distribution of depreciation. It affects the question of when to scrap but not what proportion is depreciated each year.

Dr. S. Eilon (*read by Mr. J. R. King*): Of the two conventional methods mentioned for evaluating depreciation, the sinking fund provides for an imaginary fund into which equal annual payments are made, so that with compound interest the annual depreciation figures gradually increase and the 'book value' curve of the present equipment is convex upwards, as shown in Fig. A*; in the straight-line method the annual depre-

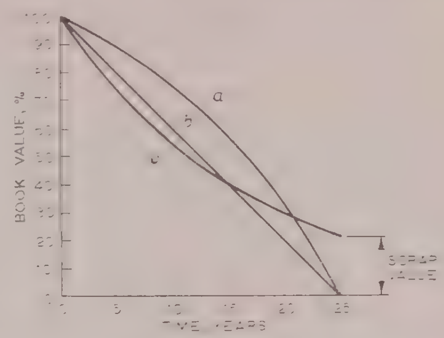


Fig. A.—Three depreciation methods over 25 years.

(*a*) Sinking fund (6%).
(*b*) Straight line (4%).
(*c*) Declining balance (6%).

ciation figures are constant and the 'book value' is a linearly decreasing function. A third method which deserves to be mentioned here is that of declining balance, where depreciation is computed as a percentage of the current book value, which now becomes convex and is in some cases a more realistic representation of the actual value of the equipment.† Conventional theories of economic equipment replacement are based on an analysis of the total annual cost function (the economic life corresponding to the minimum on this curve) and on comparison with similar cost functions of challenging equipment. While such an analysis is usually demonstrated for the straight-line method, it can be adapted and extended for other depreciation methods, accounts of which have been published.‡ Will the author comment on these methods and indicate the difficulties, if any, in applying them in practice?

The criticism that all these conventional depreciation methods are arbitrary is justified, and a cursory study of the depreciation percentages by the straight-line method allowed by the Inland Revenue, for example, is enough to convince one of this argument. But in expounding the general theory of depreciation the author still left the door wide open for arbitrary decisions. First, he suggests that 'the capital charge should be such as to make the total annual cost in every year no greater than the cost of the most economic method of providing the same service', and secondly that 'the cost of the most economic alternative is itself assessed in accordance with the theory'. This argument implies that the total cost function is arbitrarily fixed (by appropriate adjustments of the depreciation values) at some level between the current cost and the total cost of the alternative equipment (the challenger), which is in itself arbitrary as it must be defined between the current cost and that of another challenger. Surely, the dominant factor here would be the economic life of the equipment, during which the acquisition costs must be written off. The excellent treatment of

* Bolton, D. J.: 'Electrical Engineering Economics' Vol. 1 (Chapman and Hall, 1936).

* See also Grant, E. L.: 'Principles of Engineering Economy' (Ronal Press, 1950), Chapter 10.

† See, for example, Terborgh, G.: 'Dynamic Equipment Policy' (McGraw-Hill, 1949), p. 21.

‡ Terborgh, G.: *loc. cit.*
Maynard, H. B. (ed.): 'Industrial Engineering Handbook' (McGraw-Hill, 1956), Section 7, Chapter 3.
Griffiths, C. W.: 'The Economics of Plant Renewal and Replacement', *Proceedings of The Institution of Mechanical Engineers*, 1957, 171, p. 469.

annual capital charges in Section 8.1 suggests that, if depreciation is not confined to any of the conventional methods, there is enough room for manipulating suitable values of x_m for a wide variety of total cost functions, but that such an analysis cannot be separated from considerations of economic equipment replacement. Will the author comment on how he would make comparisons with challengers without reverting to a lengthy regression, when apparently all the 'standards', so to speak, that we want to compare with are in themselves arbitrary and interdependent?

Returning to the author's first contention that the total annual cost in every year should not exceed that of a challenger's, I suggest it would be both sound and realistic to allow the total cost to exceed that of a challenger during some years, provided that the average cost during the equipment's lifetime was lower. Take as a simple illustration the case where the current cost y_m in the mth year is a linear function $y_m = y_0 + am$, where y_0, a are constants, and the total cost T'_m of a challenger in the mth year is $T'_m = T'_o - b'm$, where T'_o and b' are constants. If the total annual cost, T_m, of the present equipment ($T_m = T_o - bm$) is set in such a way that initially it is below, but after some time above, the total annual cost of the challenger (Fig. B), then, assuming no scrap value at the end, the life of

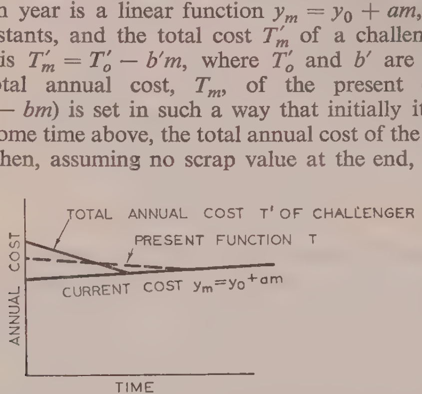


Fig. B.—Alternative total annual cost functions.

the present equipment is obtained when $T_o - bn = y_o + an$, or $n = T_o - y_o/(a + b)$, the average annual cost being $\bar{T} = \frac{1}{2}(T_o + T_n) = T_o - \frac{1}{2}b(T_o - y_o)/(a + b)$. Similarly for the challenger the average annual cost is $\bar{T}'_m = T'_o - \frac{1}{2}b'(T'_o - y_o)/(a + b')$.

Such a method is worth while when $\bar{T}' > \bar{T}$, i.e. if

$$\frac{T'_o - y_o}{T_o - y_o} > \frac{(2a + b)(a + b')}{(2a + b')(a + b)}$$

where $T'_o > T_o$ and $b' > b$; this condition is quite feasible. To minimize the total costs over the equipment's lifetime, several cost functions relating to different depreciation methods have to be constructed and compared. In this way the search for an optimal depreciation method becomes an integral part of equipment replacement analysis. Whether this is a practical method is a different question. A plant having many types of machines will involve lengthy studies and computations for each type, and different optimal depreciation methods may emerge for different machines, while a standard method to be applied to all machines may be preferable for the sake of simplicity.

The author's treatment of the variations in the value of money is worthy of attention, as this facet is often glossed over. To make a depreciation method realistic, it is sometimes essential to take into account changes in the acquisition costs of new equipment, and in this connection attention is drawn to Griffiths's paper (*loc. cit.*).

Mr. T. E. Norris: For accountancy purposes, provision for depreciation is affected by many factors, e.g. the tax laws, the method of loan repayment, the revenue or profit-earning capacity of the asset, the use to which depreciation appropriations are put and the financial policy. The exercise of judgment on all these factors results in a wide range of possible depreciation methods, and to this extent depreciation is inherently arbitrary in both method and accountancy life. These arbitrary features form a very useful instrument of financial policy, and no business man would willingly forgo them in order to be bound by a general method such as the author's.

For economic studies it is necessary to consider the financial yardstick by which alternatives are compared. The annual savings of one scheme relative to another, after allowing for interest and depreciation, are often expressed as a percentage of any additional capital and variously referred to as the marginal or net return or profit. The minimum acceptable profit is a matter of judgment which will depend on the method of depreciation. Ideally, all methods when coupled with an appropriate profit will result in the same scheme being adopted. Other things being equal, therefore, the obvious method to choose is the simplest, i.e. the sinking fund.

In Section 4, the author effectively says that all plant fully depreciated is uneconomic and this applies to 30% of the total plant in the example (see Fig. 8). I think this alarming conclusion is incorrect for two reasons. First, the economic life should be determined by comparison with the most economic alternative and not by comparison with the next best group of stations in the order-of-merit list. Second, the system considered had only experienced one year of the author's method, and it is not correct to say that plant is uneconomic because it is fully depreciated.

Mr. C. J. O. Garrard: Mr. Drummond said 'one has depreciation around one's neck and one can do nothing about it'. In the sense that plant wears out or inasmuch as the Government or one's banker insists that some particular method must be used, this is true, but in the context of the paper the opposite is the truth and depreciation is, in fact, the only factor about which one can do exactly as one likes.

It is important that more or less arbitrary economic factors should not be allowed to weigh too heavily against arguments based on engineering common sense. To take one example, for almost the whole of my lifetime the electrification of our railways has been held up mainly by decisions based upon current rates of interest and current ideas about depreciation. I doubt whether today it would be possible to prove that it would not have paid us handsomely to have begun electrification at any time in the last 50 years.

The main point of the author's argument is that the choice of a method of depreciation is largely arbitrary and that an undertaking should choose that method which enables it to produce a continuing service in the cheapest possible way. I was slightly shocked by the idea that as straight-line depreciation is the simplest method it therefore must be the best. If accountants are innocent of mathematics, it does not follow that a non-mathematical approach is the right one.

I hope that those who have sufficient time, energy and knowledge of mathematics will continue to worry their accountant colleagues into a more open frame of mind.

Dr. H. D. Einhorn: The author's definition of depreciation is interesting because it makes use of a general economic concept, that of opportunity cost.

Another general economic distinction merits explicit mention, that between commitments and avoidable costs. As engineers we have to make decisions, and for economic decisions avoidable costs only must be considered, while commitments must be ignored. For this reason the author correctly allows for a capital charge on the cost of replacement plant but excludes any capital charge payable on the old plant when considering its replacement.

Some of the comments during the discussion indicated the absurd conclusions reached if this important principle is ignored, and the accountant's need of writing off debts incurred in the past (which are commitments) is confused with a cost element for the purpose of engineering decisions.

Whether these unredeemed capital costs should or should not be allowed for under the heading of depreciation costs when making a different decision, namely that of assessing the price (e.g. electricity tariff), is a thorny problem in the case of a public utility.

THE AUTHOR'S REPLY TO THE ABOVE DISCUSSION

Mr. D. Rudd (*in reply*): Mr. Drummond has stated that the C.E.G.B. use the straight-line method of depreciation and that the Electricity Act requires the Board to keep their accounts in accordance with the best commercial standards. The implication is that the straight-line method is widely accepted as representing the best commercial standards at the present time. But standards can and do change in all fields of human activity, and in presenting the paper I assume that my proposals will eventually be judged on their merits. If they involved no modification of existing practice, they would be of no interest.

On the question of financial prudence, there are two ways of providing for losses which may arise from imperfect forecasting. One way is to set up a reserve fund, and this has the advantage that it shows clearly in the accounts how much provision has been made. The other way is to be conservative in deciding what provision to make for depreciation. If the straight-line method is to be used, this involves using a shorter economic life than past experience has indicated. If the general theory is to be used, the equivalent adjustment is to use a higher rate of technological advance than has been previously experienced. But if the effect of the general theory is to ascribe a longer life to a particular project than would have been used with the straight-line method, it does not follow that it is less prudent to adopt the general theory. On the contrary, in the example of the coal-fired generating station which I cited in my oral presentation, the general theory required 33% greater provision for depreciation in the first year than the straight-line method and parity of provision was not reached until the 23rd year, as shown in Fig. C, by which time over 90% of the capital cost of

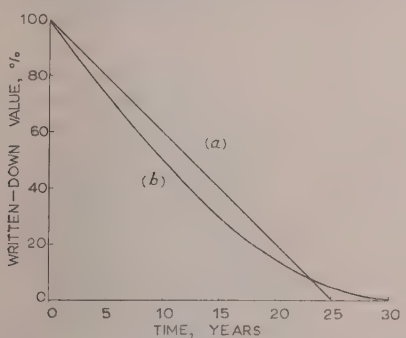


Fig. C.—Comparison of methods of depreciation provision for coal-fired generating station.

(*a*) Straight-line method (25 years).
(*b*) General theory ($a = 0\cdot05$ p.a., $b = 0\cdot08$ p.a., $r = 0\cdot04$ p.a., $y_1/A_0 = 0\cdot20$ p.a.)

the station would have been written off by either method. Thereafter, if the original forecasts were still valid, the general theory would make less provision for depreciation than the straight-line method, but it is one of the incidental advantages of the general theory that it facilitates periodic reappraisals of the programme in the light of experience, and it would be reasonable to expect some evidence of the magnitude of any inaccuracies in the original forecast to appear within 23 years of the commissioning date. Whilst, therefore, I do not advocate the general theory on the grounds of financial prudence, it so happens that, in the only example of its application to a real project which has so far been presented, it would have resulted in a greater degree of prudence than the straight-line method now in use.

I agree that in making provision for depreciation it is necessary to look ahead, and that all the five factors I have mentioned as affecting the depreciation of individual projects are likely to be imperfectly predicted. But the capital estimates which the C.E.G.B. have to submit to the Treasury relate to an integrated industry, and I have shown in Section 4 that the total capital charge and the charge on every station can be determined year by year without the necessity for long-term estimates of the future trends of those factors. If the relevant data were made available, I would undertake to show that the necessary forecasts of depreciation provision can be obtained by the general theory with no less accuracy than by the straight-line method. Admittedly the arithmetic involved in using the general theory is more elaborate (although with modern computing machines this is not a serious bar to its use), but the arithmetical simplicity of the straight-line method is only obtained by relinquishing the aim of representing the true financial state of a firm or undertaking in its accounts. For some purposes this may not matter, but one of the chief purposes of keeping accounts at all is to provide the information on which decisions regarding future operations and investments can be based. In many firms I believe the accounts are well used for just such purposes, but in large undertakings such as the C.E.G.B., engineering decisions affecting the whole national economy are evidently taken without much assistance from the accounts on the important question of providing for depreciation. The harm in this situation is that, in making shift without all the information which scientific accounting is inherently capable of providing, engineers may adopt schemes which do not achieve the minimum possible annual cost. A good example of an economic assessment which was critically dependent on the method of providing for depreciation was the assessment in 1955 of the economic possibilities of nuclear generation stations in Great Britain.*

A separate treatise would be required to deal with the Board's transmission assets and the slight modification which is necessary to cater for hydro-electric and pumped-storage schemes (which still account for only a very small fraction of the total generating capacity in Great Britain). The advent of nuclear power is dealt with briefly in eqn. (6) and the preceding paragraph of the paper. Although nuclear power was foreshadowed by a world-shaking technological revolution, the important point in the present context is that it is having and is expected to have only a very gradual effect on the cost of electricity and hence on the economic lives of conventional stations.

In reply to Mr. Bolton, although it is true that in a static undertaking the total annual provision for depreciation—being approximately equal to the total annual investment—would be practically unaffected by the method of provision, in an expanding undertaking the straight-line method requires more total provision than the annuity method, and the general theory more than either.

Mr. Bolton's main criticism is that the general theory is as

* PASK, V. A., and DUCKWORTH, J. C.: 'The Place of Nuclear Energy in United Kingdom Power Development', *Journal of the British Nuclear Energy Conference*, 1956, 1, p. 13.

arbitrary as any of the other methods of providing for depreciation. But, whereas I have specified the arbitrary features of the conventional methods, i.e. the constant capital charge of the annuity method and the constant provision for depreciation of the straight-line method, it is evident that the general theory has neither of these features, and Mr. Bolton has not specified in which other respects the theory is arbitrary. It is therefore impossible to reply succinctly, but in general terms I deny the allegation of arbitrariness on the ground that the theory has been derived from a principle which is already generally accepted as the basis for the design and operation of engineering plants, namely that the total annual cost should not exceed the cost of the most economic alternative method of providing the same service. The new claim of the general theory is that this principle can be used, not only to design and operate plants, but also to provide for depreciation. In applying the theory, I have introduced approximations, e.g. that technological advance brings about an exponential reduction in the total annual cost, but an approximation is not arbitrary if its correspondence with the phenomenon it represents can be tested. I have therefore shown how, in the supply industry, cost coefficients can be calculated whose rate of reduction would be a valid measure of the rate of technological advance in the industry. If it is ultimately found that the reduction can be more accurately represented by some other approximation, the mathematical formulae will change but the theory will be unaffected.

With regard to Mr. Bolton's other criticisms, the simplest definition of 'depreciate' is 'fall in value'. I have used the word in this sense and I think it is clear from Section 3 that I distinguish between the effects of use or tenure and the effects of external events, but that I include both in the term 'depreciation'. If technological advance (which is gradual in its economic effect in spite of the erratic publicity which is given to its more dramatic manifestations) can reduce the value of a plant to its scrap value, it is reasonable to suppose that the reduction occurs gradually and therefore that the rate of advance affects the distribution of depreciation over the economic life.

Replying to Dr. Eilon, I understand the reducing (or declining) balance method is used almost exclusively for calculating income tax liability (the C.E.G.B. use it for this purpose), but the percentage rate of reduction is not, in fact, determined by the scrap value at the end of the economic life but by the class of plant, standard rates having been laid down for various classes of plant. The principal difficulty of using the scrap value to determine the percentage rate of reduction appears to be that the scrap value is frequently a very small (sometimes a negligibly small) fraction of the capital cost: hence the percentage rate would be unrealistically high. A more fundamental objection to the method itself is that it has no theoretical foundation.

I cannot agree that, in expounding the general theory, I have 'still left the door wide open for arbitrary decisions', but I freely admit that every engineering project involves an element of speculation and that the practical difficulty of applying the general theory is directly related to the degree of speculation in the project. Thus, at one end of the scale is, for example, the problem of designing and providing for the depreciation of a generating station in an undeveloped region where there is almost no demand initially, but probably a large potential demand for electricity at a reasonable price, and where the political situation is unstable. In such a case the general theory would be ludicrously elaborate. At the other end of the scale is the supply industry in Great Britain, for which—as I have shown in Section 4—the calculation of the provision for depreciation of every station could be virtually automatic. In the latter case, the adoption of the general theory would enable cost coefficients to be computed every year [see eqn. (7)] and a trend established from which the total annual cost of generating electricity at any load factor could be computed for any year in prospect. This would then provide a background against which any new scheme for generating electricity could be compared. The comparison would consist in subtracting the prospective current cost of the scheme from the background total annual cost in every successive year, so obtaining a diminishing series of capital charges. The scheme would be economic if the capital cost could be served and paid off by these charges before they diminished to zero. Without describing the whole process in minute detail, I hope it is clear that such an assessment and the resulting programme of provision for depreciation would be in no way arbitrary. I agree that planning the provision for depreciation would be inseparable from deciding which scheme to adopt, although once the plant was in operation the provision for depreciation would be determined by the overall programme for the industry.

In a less integrated industry some degree of judgment is required, principally in estimating the rate of technological advance. At first sight a wide variety of total cost functions might be employed, but there is no point in introducing complication for its own sake, and between an exponential and a straight-line function (which are of equal simplicity in application), the former is to be preferred because it avoids the absurdity of negative cost in the foreseeable future. It is by introducing this concept of a rate of technological advance, which is common to all the schemes under consideration, that the lengthy regression referred to by Dr. Eilon is avoided. The process of comparison with a challenger is illustrated in Fig. 3.

Referring to Fig. B, I do not think it would be regarded as 'sound' in the business sense to allow the total cost to be greater than that of a challenger during some years. Bearing in mind that the situation would arise only in a non-integrated industry in which there would be competition, the product would probably have to be sold below cost during those years, and for this reason it would be preferred to arrange the provision for depreciation so that the product could compete against the challenger throughout the life of the plant without a subsidy.

Turning to Dr. Eilon's last comment that 'it is sometimes essential to take into account changes (due to variations in the value of money) in the acquisition costs of new equipment', I would argue from the statement in the paper referred to (p. 478, line 38) that 'the object of depreciation policy is to ensure that sufficient funds are recovered in selling prices and set aside year by year to make good the diminishing value of the plant, while at the same time there must be no question of overcharging'. This statement is unexceptionable, but I contend that the author has disregarded it in his subsequent treatment, when he advocates 'Depreciation Based on Replacement Values' and sets out a numerical example in which £1 450 is to be set aside for the depreciation of a machine costing £1 000. Clearly, the value of the machine has not diminished by more than its first cost. The effect of the advocated policy must therefore be to prevent the total liability of the concern, which remains constant, from keeping in step with the value of its assets, which continually increase; in other words to finance part of its investment out of current revenue. Whether it is in a firm's or the national interest to finance investment out of current revenue is a matter for debate, but it is not the same thing as providing for depreciation.

Replying to Mr. Norris, I do not think a trained accountant would agree that for accountancy purposes the method of depreciation should be affected by any of the six factors he mentions, except perhaps the last. I certainly do not. His sweeping assertion, that no business man would willingly forgo

the arbitrary features in the conventional methods because they form a useful instrument of financial policy, is presumably not meant to be taken literally. If he means that business men value the wide range of present depreciation methods because they give them room to manoeuvre, I would venture the contrary opinion that those in charge of successful large businesses are often inclined to believe their success stems from the use of modern scientific methods and are rather relieved when decisions, which used to be the prerogative of the head of the firm, can now be safely taken outside the board room.

I agree that, when two schemes are being compared and one has a higher capital cost than the other, it is a common practice to require the one scheme to show a minimum annual saving after charging interest in order to justify its higher capital cost. There is no difficulty in employing this technique in conjunction with the general theory (although it would be simpler and more rational to use a higher interest rate representing the total return required on the capital, after providing for depreciation), but it is not true, ideally or otherwise, that all methods of providing for depreciation will result in the same scheme being adopted. In some cases comparisons based on the general theory will justify greater expenditure of capital than equivalent comparisons based on the sinking-fund method: in other cases the reverse will be true.

I have made it clear that the data used in Section 4 are not authentic, but I should expect authentic data to show that a substantial proportion of the power stations in Great Britain was obsolete in 1949. When one remembers that the supply industry was struggling to eliminate electricity cuts and was keeping in service any plant which could produce electricity, almost regardless of cost, this conclusion is not surprising. Mr. Norris's first reason for doubting its correctness is mistaken because eqn. (5) ensures that the most economic alternative to any non-obsolete station is, in fact, the next station in the list. His second reason has more weight, and I have mentioned in Section 4 that some adjustments might be necessary in the light of data from several successive years. However, I did not say or imply that 'plant is uneconomic because it is fully depreciated'. What I did imply was that the general theory shows which stations are obsolete and refrains from making any capital charge on such stations. It would be economic to replace them, but whether it is practicable to do so depends, of course, on the availability of capital and the physical resources of industry.

Replying to Mr. Garrard, I am in sympathy with much of what he says, although my intention is to remove the element of free choice from the field of depreciation provision by showing that, in any given situation, only one choice can be made without distorting the economic picture presented by the accounts. I do not believe the conclusions which stem from the general theory will, when carefully examined, be found to offend against engineering common sense.

Replying to Dr. Einhorn, the formulation of electricity tariffs appears to be beset with difficulties, but I think the difficulty he mentions is successfully disposed of by arranging that the unredeemed capital costs of obsolete stations, which constitute dead liabilities, are always balanced by a fund into which is paid the capital charges on stations which, though not yet obsolete, are completely written off. Hence in formulating the tariffs there is no cost to be recovered under the heading of unredeemed capital cost.

DISCUSSION ON 'PROPERTIES AND THEORIES OF THE ELECTRIC ARC'*

Dr. A. E. Guile, Dr. T. J. Lewis and Dr. P. E. Secker (*communicated*): The limitation of space imposed on Dr. Edels in reviewing the vast and diverse literature on the electric arc has resulted in over-simplification of his remarks on the behaviour of the arc in a magnetic field (Section 2). This might mislead engineers, who, although dealing with arcs, have little time for detailed study of the physics of the processes.

In the case of a metallic conductor in a magnetic field, the latter acts upon the conduction electrons, but because of the interactions between the electrons and the lattice the electron momentum is transferred to the solid conductor as a whole. In the arc column the magnetic field acts on the electrons, and to a certain extent on the positive ions. Part of the momentum acquired in the direction mutually perpendicular to the arc-current axis and the lines of magnetic flux is lost in collisions with neutral gas molecules, and since these, unlike the lattice of the solid, are in random motion, this directed motion is mainly lost. Thus there is little mass movement of the neutral gas in the neighbourhood of the column,[A] and it is unreasonable to assume that the normal laws relating to metallic conductors will apply either for the driving or for the retarding forces. There is some evidence for the fact that the speed of arc-column movement is independent of current (at least in the range 30–700 A)[B]; this behaviour would not be expected for a solid conductor.

Because of the complex relation between movement of the ions and electrons and the mass of neutral gas, the effect of a transverse gas flow on the column cannot easily be calculated. The high drift velocity of the electrons and ions along the column will reduce considerably the deflection of the plasma in the direction of gas flow.

We would stress the great importance of the cathode fall in determining arc motion in a transverse magnetic field. In many cases it is conditions within this region rather than in the column that determine the motion both in the Amperian and retrograde modes. As recent experiments have shown, arc movement in an opposing wind can be the same as in still air, because of the controlling influence of the cathode spot,[C] and cathode material and surface conditions can alter the arc velocity by several orders.[B, D, E]

It has been shown recently[F, G] that Amperian arc movement can be explained in terms of the electron and ion paths in the cathode fall region, and this now provides a unified theory with that of retrograde motion.[H]

* EDELS, H.: Paper No. 3498, February, 1961 (see 108 A, p. 55).

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DISCUSSION ON 'SPEED-CHANGING INDUCTION MOTORS'*

WESTERN CENTRE, AT BRISTOL, 10TH APRIL, 1961

Mr. P. Collins: The paper mentions that the possible pole combinations to which this method of speed changing can be applied are limited to those in which neither pole number is a multiple of 3. This 3 presumably stems from the fact that a 3-phase machine is being considered. The essence of this pole-changing technique is contained in the expression† for a resultant flux density

$$B_\theta = \frac{C}{2}\left[\cos\left(\frac{P}{2} - k\right)\theta - \cos\left(\frac{P}{2} + k\right)\theta\right]$$

which is essentially that for a single-phase field. Could the authors state whether the method would be applicable to single-phase motors, and if so, would there be similar limitations on the possible pole combinations?

Mr. A. J. Parsons: It would appear that, although a variety of stator slot numbers has been tried, there is a distinct preference for 72 slots, where diameters permit this number to be used. Is the choice of the number of slots equally as wide, over a range of outputs, as with orthodox windings, and what is the lowest number that could be used successfully, as for example on fractional-horsepower motors?

The authors also state that the motor efficiencies when working with the modulated connection were not quite as good as the conventional single-speed motors of the same output. Has a detailed analysis of the losses been made? Greater core losses could be expected as a result of harmonics in the flux distribution.

Mr. C. W. H. Minchin: Induction motors have been made now for some 70 years and pole changing with a tapped winding has been employed since the beginning of the century, but this has almost invariably been for a 2 : 1 speed range. These windings employ 120° phase belts on the low-speed connection which may give even harmonics and high magnetizing currents, and it is often difficult to obtain the best flux density for both speeds and almost impossible to have the same flux density for both. Most often, it is found that on low speed the machine is greatly over-fluxed with consequent high magnetizing current and low power factor.

The authors must be congratulated for extending change pole connections with only six leads to other speed ratios, and for obtaining practically the same flux density for both speeds. We must not, however, consider flux density alone to be the criterion of performance, for, as shown by the circle diagram, it is the leakage reactance which chiefly counts. If the leakage reactance can be reduced with the same flux and resistance we have an all-round improvement. Some windings, such as single layer with $1\frac{1}{2}$ slots per pole per phase, have a very high zigzag leakage. This is not evident by the overlapping permeance method of calculation as shown by Adams, but is by the method suggested by Alger.†

I wonder whether the authors' windings have been checked for differential leakage factors and whether these are high, as is the case with some other irregular windings. This would be more likely to show up on the modulated winding and could decrease the torque more than the flux density would indicate.

I can foresee some design trouble on larger machines, where it is generally necessary to use windings with poles connected in parallel. I think that this would not always be possible, and, together with the use of partly filled slots, might limit the output of some frames.

Prof. G. H. Rawcliffe and **Dr. W. Fong** (*in reply*): When the paper was written, pole numbers which were multiples of 3 were excluded. We are now able to obtain a 2-speed winding for any pole combination. There is every prospect, arising from recent developments, of applying this whole system to single-phase operation.

A very wide choice of possible slottings is available, and the lowest slot number regularly used is 36. It is accepted that there will be some extra core losses due to harmonic fluxes, but tests have shown that these are trivial in amount. Further, a 2-speed motor with a single winding will be much smaller, for a given output, than a double-wound machine. The main core losses will therefore be substantially less, which will more than compensate for any harmonic losses.

The leakage reactance of these machines, as tested, differs only in the second order from that of corresponding double-layer single-speed machines. We have never used single-layer windings. It is just possible that difficulties may arise in large machines from the necessity to use parallel paths, but difficulties exist to be overcome. We have already done so in many respects in the production of the 2-speed single-winding induction motor.

* RAWCLIFFE, G. H., and FONG, W.: Paper No. 3306, December, 1960 (see **107** A, p. 513).

† See Reference 1 of the paper.

† ADAMS, C. A.: 'The Design of Induction Motors', *Transactions of the American I.E.E.*, 1905, **24**, p. 649.

ALGER, P. L.: 'The Calculation of the Armature Reactance of Synchronous Machines', *ibid.*, 1928, **47**, p. 493.

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SPEED-CHANGING INDUCTION MOTORS

Reduction of Pole Number by Sinusoidal Pole-Amplitude Modulation

By Professor G. H. RAWCLIFFE, M.A., D.Sc., Member, and W. FONG, Ph.D., Associate Member.

(*The paper was first received* 13*th July, and in revised form* 22*nd October*, 1960.)

SUMMARY

Previous papers on pole-amplitude modulation have described methods of increasing the pole number and reducing the speed, by modulation. The reciprocal possibility has already been pointed out; and it has now been found that, under some conditions, it is better to use modulation to reduce the pole number and increase the speed. The particular form of modulation used in the examples described here gives exceptionally good performance. By way of example, test results are recorded for a small laboratory machine, and for a very large machine made by an industrial company, both for 10/8 poles, and both being very successful. The general conclusions are drawn that, for very many purposes, a 10/8-pole motor is to be preferred to an 8/10-pole one; and that modulation to reduce the pole number of an electrical machine has important industrial possibilities.

(1) INTRODUCTION. REDUCTION OF POLE NUMBERS

The first paper[1] published relating to this new type of squirrel-cage induction motor was implicitly based on the assumption of a load torque rising considerably with rise of speed: commonly proportional to the square of the speed. Different considerations apply for a constant-torque—or constant-horsepower—drive, and these have led to the use of an alternative type of pole-amplitude modulation,[2] which gives something approaching a constant-torque output.

In Section 4 of Reference 1 it was explained that there was a reciprocal principle in the theory of pole-amplitude modulation, which always gave a choice between starting with $(6n + 2)$ poles and changing to $(6n + 4)$ poles, or starting with $(6n + 4)$ poles and changing to $(6n + 2)$ poles. Since the performance after modulation is normally a little inferior to that before modulation, and since the load on a motor normally falls off with reduction in speed, it was therefore stated that modulation to increase the pole number was likely to prove more important than modulation to reduce it.

However, somewhat contrary to expectations, it has proved possible in many cases to apply the principle of pole-amplitude modulation more efficiently, and to obtain a higher average winding factor, a lower harmonic content and a better air-gap flux-density ratio when the winding is originally wound for the higher number of poles. If the optimum performance at the higher speed is desired, regardless of any deterioration in performance at the lower speed, it is often still true, in general, that the motor should be initially wound for the lower pole number and that the speed should be reduced by modulation. If, however, a small reduction in rating at the higher speed can be tolerated, a striking improvement in rating at the lower speed may be achieved by winding initially for the higher pole number and increasing the speed by modulation. This inversion of the previous procedures is especially applicable to irregular fractional-slot windings of the type discussed in an earlier paper[2] in which modulation is obtained by coil reversal, without omission of any part of the winding. The reasons for these slightly surprising conclusions are as follows.

(1.1) Orders of Resultant Harmonics for Odd and Even Pole Pairs

If modulation is used to change an even number of pole pairs into an odd number, the possible numbers of pole pairs in the unmodulated winding are given by $m = 2, 4, 8, 10, 14$, etc.; and in the modulated winding by $m = 1, 5, 7, 11, 13$, etc. (This was so, for example, in the change from 4 to 5 pole pairs considered in previous papers.[1,2]) If, however, modulation is used to change an odd number of pole pairs to an even number, these two series are interchanged. Now potentially the most objectionable harmonics, which cannot be removed by chording, are those of lowest order, and an unwanted 4-pole field ($m = 2$) will tend to be smaller, and will usually be less objectionable in its effects than an unwanted 2-pole field ($m = 1$). In general, therefore, there is a *prima facie* argument for modulating from an odd number of pole pairs to an even number, rather than the reverse, so that the harmonics after modulation are of the higher, even orders given in the first series above. This favours modulation from 5 pole pairs to 4 pole pairs, rather than from 4 to 5, as previously treated fully.[1,2]

It follows that, where modulation from an odd number of pole pairs to an even number (e.g. 7 to 8) also increases the number of pole pairs, the arguments in favour of reducing speed by modulation act in the same direction as those for modulating from an odd number of pole pairs to an even number; but where such modulation decreases the number of pole pairs (e.g. 5 to 4) one consideration acts in opposition to the other, and the arguments are nearly balanced. For this reason, therefore, there was a *prima facie* case for investigating in detail the effect of 10/8 pole-amplitude modulation.

(1.2) Harmonic Modulation Products

In addition to the fundamental modulation product there are two kinds of harmonic modulation products, as discussed in an earlier paper.[2] These result from harmonic modulations of the fundamental m.m.f., and from fundamental modulation of the harmonics m.m.f.s, the former being the more important cause of harmonic resultants. It is to be noted that a third harmonic can and does exist in the m.m.f.s of the separate phase-windings, and this harmonic—the first of the series—is normally the only one which is of any significance. The orders of harmonic involved can readily be seen from Table 1.

It will be seen at once that the orders of the resultant harmonics which arise from an initial pole number of $(6m + 4)$ are considerably higher, in general, than the orders of those which arise from an initial pole number of $(6m + 2)$, and the former are thus likely to be of lower magnitude, on the average. This, of course, does not mean that some particular harmonic, which happens to have a higher harmonic winding factor, cannot be of larger magnitude than the other harmonics of high order. The

Written contributions on papers published without being read at meetings are invited for consideration with a view to publication.

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Table 1

RESULTANT HARMONIC POLE NUMBERS FOR POLE-AMPLITUDE MODULATION OF WINDINGS OF $(6m + 2)$ OR $(6m + 4)$ POLES

Initial fundamental pole number	Harmonic modulating pole numbers	Resultant harmonic pole numbers
$6m + 4$	6	$6m + 10$ $6m - 2$
	10	$6m + 14$ $6m - 6$ (vanishes)
	14	$6m + 18$ (vanishes) $6m - 10$
$6m + 2$	6	$6m + 8$ $6m - 4$
	10	$6m + 12$ (vanishes) $6m - 8$
	14	$6m + 16$ $6m - 12$ (vanishes)
Initial harmonic pole numbers	**Fundamental modulating pole number**	**Resultant harmonic pole numbers**
$18m + 12$	2	$18m + 14$ $18m + 10$
$18m + 6$	2	$18m + 8$ $18m + 4$

argument of this Subsection relates to the trend of the harmonic magnitudes, and not to the exact value of every harmonic.

(1.3) Quasi-Sinusoidal Distribution of Coil Groups

It has been shown in an earlier paper[2] that a considerable improvement in performance in the modulated connection can be obtained by quasi-sinusoidal initial distribution of the coil groups in each phase-winding, these irregularities in the unmodulated winding normally being quite acceptable. Quasi-sinusoidal coil-group distribution can in principle be achieved with equal ease for any initial pole number; and this type of winding can therefore be employed for modulation from a larger to a smaller pole number (10/8 poles are here considered) in the same way as it was previously used for modulation from a smaller to a larger pole number (8/10 poles). In the detailed analysis 10/8 poles proved to be a specially favourable pole combination for quasi-sinusoidal coil-group distribution; and, as Table 2 and Section 3 both show, windings of this type were found both by theory and in practice to be exceptionally successful.

(1.4) Undesired Modulation in Normal Connection

When windings in which the coil groups are originally distributed sinusoidally are used, there is one further argument, related to the unmodulated condition, in favour of starting with $(6m + 4)$ poles, rather than with $(6m + 2)$ poles ($m = 1, 2, 3$, etc.). Sinusoidal coil-group distribution, without reversal of one-half of each phase-winding, is equivalent to a 4-pole modulation of the original winding, the resultant pairs of pole numbers, respectively, being $(6m + 8)$ and $6m$ for a winding basically of $(6m + 4)$ poles, and $(6m + 6)$ and $(6m - 2)$ for a winding basically of $(6m + 2)$ poles. The pole numbers $6m$ and $(6m + 6)$ are, respectively, eliminated in the two cases, when the three phase-windings are combined; and the unwanted product of modulation is an m.m.f. of $(6m + 8)$ poles in the former case and $(6m - 2)$ poles in the latter case. The former is of much higher order, and thus likely to be both less objectionable and more readily eliminated by chording. Again, this argument favours a winding for the higher number of poles initially, in this case 10 poles rather than 8 poles, for obtaining modulation between two pole numbers, m having here the particular value of unity in the general expressions just quoted.

(1.5) Effect of Chording Factors

Another reason why it may sometimes be better to use modulation to reduce the pole number, rather than to increase it, arises from a consideration of the chording factors. The coil pitch of windings designed for pole-amplitude modulation is apt to be critical, being largely determined by the need to suppress undesired m.m.f. harmonics, as explained in earlier papers;[1,2] and the air-gap flux-density ratio is also greatly influenced by the degree of chording. The coil pitch having been settled on these grounds, the consequent chording factors have to be accepted. For 8/10-pole modulation it has been shown that a coil pitch of two-thirds full pitch for 8 poles, which is equivalent to five-sixths full pitch for 10 poles, is practically essential. This gives chording factors of 0·866 and 0·966, respectively, or an average of 0·916. For 10/8-pole modulation a coil pitch of 1·25 × full pitch for 10 poles, which is equivalent to 1·0 × full pitch for 8 poles, is completely satisfactory. This gives chording factors of 0·924 and 1·000, respectively, or an average of 0·962. All other things being equal, a high average chording factor is a good thing, and its effect must certainly be considered in relation to the two alternative ways of using modulation.

(1.6) Effect of Phase Spread

When the pole number of a winding is reduced, the electrical angle of spread is also reduced, and the spread factor is therefore numerically increased. Whilst spread factor is only discernible as a separate quantity for windings which initially are integral-slot windings, and in which no coil group is divided on modulation, the effect of phase spread is always implicitly present in the layer factor of any winding. Consequently, in relation to phase spread, modulation 'down' to a lower pole number tends to increase the layer and winding factors, whereas modulation 'up' tends to reduce them.

(1.7) Effect of Winding Factor

For most 2-speed windings, the value of the winding factor after modulation tends to be somewhat lower than the winding factor before modulation. As a broad average, the reduction of winding factor is about 8% to 10% as will be seen in Table 2. The same general trend will be observed in Table 6 of an earlier paper,[2] which related to increase of pole number by pole-amplitude modulation.

Now, parallel-star connection of a winding gives a greater total flux per pole than is given by series-delta connection; and it is therefore customary to use parallel-star connection for the lower pole number. The increase, thus obtained, in total polar flux for the lower pole-number is in the ratio $2/\sqrt{3} = 1{\cdot}15$. For pole ratios, such as 10/8, which are greater than 1·15, this phase reconnection is therefore not sufficient, in itself, to give the same air-gap flux density at both speeds; and it is desirable to obtain a lower value of winding factor for the lower pole number than for the higher pole number. This consideration therefore favours modulation to reduce pole number, for pole ratios greater than 1·15.

(1.8) Flux-Density Ratio

In general, the air-gap flux-densities for the two methods of connection of a 2-speed winding, with a given voltage, are not equal. The maximum rated voltage is settled by reference to the connection giving the higher flux density, and the flux density in the other connection will be less than the permissible value.

Table 2

WINDINGS FOR POLE-AMPLITUDE MODULATION: TEN ALTERNATIVE TYPES FOR 10/8 POLES. RELATIVE M.M.F. AMPLITUDES, WINDING FACTORS AND FLUX DENSITIES

Types of winding		(a)	(b)	(c)	(d)*	(e)*	(f)*	(g)	(h)	(i)*	(j)
m	Pole numbers of modulated winding	60 slots(ii) coil pitch: 8 slots: 1·33 (full-pitch) Modulated 2–2–2–2–0 2–2–2–2–0(vi)	60 slots(ii) coil pitch: 8 slots: 1·33 (full pitch) Modulated 10–2–2–2–01 10–2–2–2–01(iii)	90 slots(ii) coil pitch: 12 slots: 1·33 (full-pitch) Modulated 100–3–3–3–001 100–3–3–3–001(iii)	54 slots(iv) coil pitch: 7 slots: 1·30 (full-pitch) Modulated 1–2–3–2–1 1–2–3–2–1	126 slots(iv) coil pitch: 16 slots: 1·27 (full-pitch) Modulated 2–5–7–5–2 2–5–7–5–2	72 slots(iv) coil pitch: 9 slots: 1·25 (full-pitch) Modulated 1–3–4–3–1 1–3–4–3–1	162 slots(iv) coil pitch: 20 slots: 1·24 (full-pitch) Modulated 2–7–9–7–2 2–7–9–7–2	90 slots(iv) coil pitch: 11 slots: 1·22 (full-pitch) Modulated 1–4–5–4–1 1–4–5–4–1	120 slots(iv) coil pitch: 15 slots: 1·25 (full pitch) Modulated 2–5–6–5–2 2–5–6–5–2(vii)	132 slots(iv) coil pitch: 16 slots: 1·21 (full-pitch) Modulated 2–5–8–5–2 2–5–8–5–2(vii)
2	4-pole	35·9	23·8	10·7	2·9	7·9	11·3	14·1	16·0	1·0	11·5
4	8-pole	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0
8	16-pole	2·3	0·5	0·3	0·5	0·3	0·0	0·3	0·5	0·0	0·1
10	20-pole	0·0	0·0	4·0	2·0	0·5	0·3	0·8	1·2	1·0	1·9
14	28-pole	8·9	10·7	10·0	10·4	10·4	11·3	10·9	11·1	11·5	16·9
16	32-pole	7·0	4·2	4·1	1·6	0·6	0·0	0·4	0·7	0·0	0·6
	Pole numbers of unmodulated winding										
1	2-pole	—	—	—	9·3	2·1	3·1	6·9	10·1	7·8	8·7
5	10-pole	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0
7	14-pole	—	—	—	3·9	4·9	5·7	6·2	6·7	4·5	6·9
11	22-pole	—	—	—	3·7	1·8	0·6	0·3	0·9	0·3	7·9
13	26-pole	—	—	—	5·2	6·2	7·6	8·3	9·2	6·7	13·6
17	34-pole	—	—	—	3·4	2·4	2·0	1·4	1·1	1·5	0·7
Winding factor 8-pole		0·599(v)	0·609(v)	0·580(v)	0·758	0·766	0·781	0·788	0·796	0·757	0·812
Winding factor 10-pole		0·827	0·827	0·833	0·844	0·845	0·855	0·855	0·858	0·862	0·875
Average winding factor		0·713	0·718	0·711	0·801	0·806	0·818	0·822	0·827	0·810	0·844
B_{10}/B_8 (air gap)		1·04	1·06	1·00	0·972	0·983	0·991	1·00	1·01	0·951	1·00
Connections (10-pole/8-pole)		Parallel-star/delta	Parallel-star/delta	Parallel-star/delta	Delta/parallel-star	Delta/parallel-star	Delta/parallel-star	Delta/parallel-star	Delta/parallel-star	Delta/parallel-star	Delta/parallel-star

* Preferred designs.

(i) All coil pitches in terms of 10 poles, for which the winding is originally wound.
(ii) For windings types (a), (b) and (c), the unmodulated winding is a standard 10-pole winding, and thus contains no harmonics lower than the fifth harmonic of 10 poles, i.e. 50 poles ($m = 25$).
(iii) The inner coils in the outer coil groups must be omitted; not the outer coils as in 8/10-pole modulation.
(iv) For windings types (d)–(j) the whole winding is used at both speeds and all coils are identical. The unmodulated winding is a 10-pole fractional-slot winding of irregular coil-group distribution, and its m.m.f. waveform contains the usual odd harmonics, especially a sub-harmonic 2-pole field.
(v) The 8-pole winding factors are calculated with reference to the total number of conductors. For windings (a)–(c), the winding factors of the conductors actually in circuit are correspondingly higher.
(vi) Winding type (a) could be wound in 90 slots with three coils per group and modulated 3–3–3–3–0 (etc.).
(vii) Winding type (i) is intermediate between windings types (d) and (e); winding type (j) is intermediate between windings types (e) and (f).

For a single-speed winding it is accepted that the winding factor is a measure of its effectiveness; but for a 2-speed winding the effectiveness of a winding is truly given by the winding factor only for that connection which gives full flux density when the rated voltage is applied. The present authors suggest that the 'effectiveness' in the other connection could reasonably be measured by

$$\text{Winding factor} \times \left[\frac{\text{Actual flux-density}}{\text{Full flux-density}}\right]^2$$

The squared term arises from the fact that induction-motor torques are proportional to the square of the flux density.

It is therefore clear that there is a strong advantage in using a connection which gives a ratio of nearly unity between the flux densities at the two speeds. Now it has been shown[2] that, for the preferred 8/10-pole winding, the winding factors are 0·820 and 0·760, respectively, and the flux-density ratio, B_8/B_{10}, is 0·855. This gives an 'effectiveness' in the two connections which is 0·600 and 0·760 for 8 and 10 poles, respectively. On the other hand, for the 10/8-pole winding which was used in the large test machine (discussed in Sections 2.4 and 3.2) the winding factors were 0·855 and 0·781, respectively, and the flux-density ratio, B_{10}/B_8, was 0·991. The 'effectiveness' in the two connections is therefore 0·838 and 0·781 for 10 and 8 poles, respectively; and it will be observed that the 'effectiveness' for both pole numbers is substantially higher for the 10/8-pole winding than for the 8/10-pole winding. The superiority of the 10/8-pole winding in this respect is substantial.

In order to decide whether to use modulation to increase the pole number or to reduce it, it is therefore desirable always to examine the 'effectiveness' of the winding at both speeds, in this way, for the two alternative types of winding.

(1.9) Ampere-Conductor Modulation

The authors proposed the term 'pole-amplitude modulation', to describe the novel processes involved in this method of speed

changing, in the earlier stages of an investigation which has taken several years. They now incline to the opinion that scientifically a more correct term would have been 'ampere-conductor modulation', though it might have been less acceptable to the general reader. However that may be, it is certain that the m.m.f., and hence the pole amplitude, is, in fact, the integral of the ampere-conductor distribution. It thus follows that, when a field of p poles is modulated to give a mixture of $(p+2)$ poles and $(p-2)$ poles, the amplitudes of the two components tend to be in inverse proportion to their orders. If one discards the higher resultant pole number and retains the lower, one will therefore have a larger resultant pole amplitude than that which is obtained by discarding the lower pole number and retaining the higher.

This point is now regarded by the authors as one of considerable cogency, although it was not fully apparent in the earlier stages of these investigations. It is one more reason why modulation to reduce pole number is likely to be more favoured than modulation to increase it.

(1.10) Cooling for the Two Speeds

Modulation to reduce the pole number rather than to increase it may also be preferred, particularly when the number of poles is small, because the cooling of a machine is substantially better at the higher of two speeds, especially when the speeds differ considerably. For a given type of modulation, the simplest being single modulation, the speed difference is greatest when the actual pole numbers are small. Improved cooling at the higher speed may go far to compensate for a reduction in winding factor, by making possible a higher current density. For single modulation with a considerable number of poles, for example 16/14-pole modulation, the cooling does not differ greatly at the two speeds and this argument ceases to have much force.

(1.11) Accelerating Characteristics

It has proved possible, in general, to reduce the harmonic content of the m.m.f. waveform after modulation to an acceptably low value, and at the same time to obtain a good value of starting torque. In so far as there is any minor residual inferiority in accelerating properties in the modulated connection, however, it is better for this inferiority to occur at the higher speed and the lower pole number. It then becomes possible to start the motor unmodulated for the lower speed and higher pole number, and to switch to the higher speed and lower pole number whilst running. In this way, it is possible to sidestep any slight inferiority in starting and accelerating properties, over a large part of the speed range through which acceleration must take place.

As an engineering proposal, a requirement to start in the lower speed connection, and to switch to the higher speed whilst running, is much more acceptable than a requirement to start in the higher speed connection, and subsequently to reduce speed.

(2) OPTIMUM DESIGNS FOR 10/8 POLES

The general arguments in Section 1 can be made clearer by considering a particular pole combination which has been both analysed theoretically and tested experimentally.

(2.1) Designs using Fractional-Slot Windings

It has been found in this case that very good results are obtained by using fractional-slot windings, and these will therefore be first considered.

The principle of sinusoidal distribution of coil groups was originally developed[2] for an even number (4) of pole pairs; and in extending this principle from an even number (4) to an odd number (5), certain points became obvious from an inspection of Fig. 1. The coil-group sequence for each half phase-winding is taken as 1–x–$(x+1)$–x–1. Obviously the least number of coils is required in groups Nos. 1 and 5; and, in the first instance, this number is taken to be unity. The corresponding number of coils in groups Nos. 2 and 4 is taken to be x, and it is then reasonable to take the number of coils in group No. 3 as $(x+1)$. By inspection alone, it is clear that x and $(x+1)$ will have values of about 3·0 and 4·0, respectively. For convenience, x will be described as the coil-group factor.

If the theoretically desirable value of x proved to be substantially different from an integer, it would be necessary to

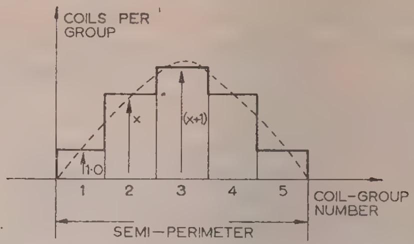


Fig. 1.—Quasi-sinusoidal coil-group distribution in one-half of a winding with an odd number (5) of pole pairs.

Coil-group sequence for complete phase-winding:
1–x–$(x+1)$–x–1—1–x–$(x+1)$–x–1
The signs of the coil groups are ignored.

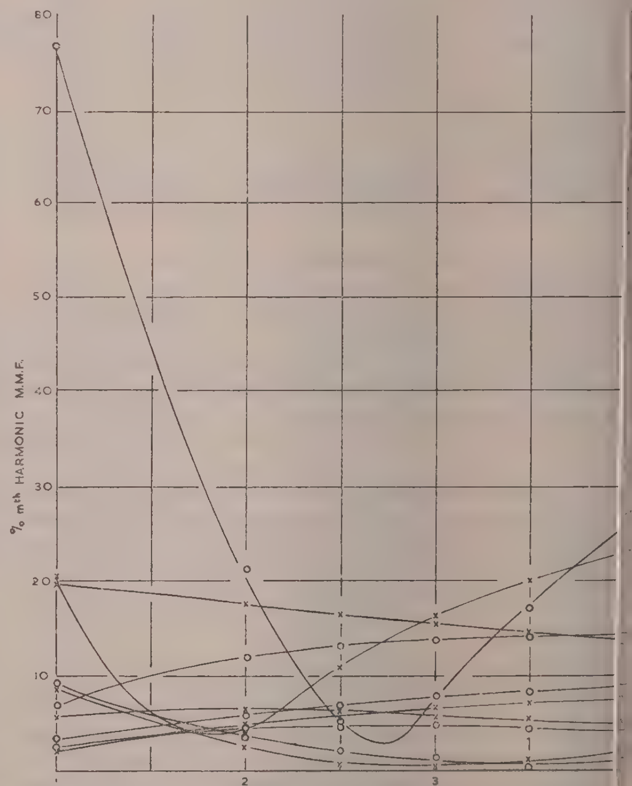


Fig. 2.—Harmonic m.m.f.s of one layer of irregular fractional-slot winding for 10/8 pole-amplitude modulation.

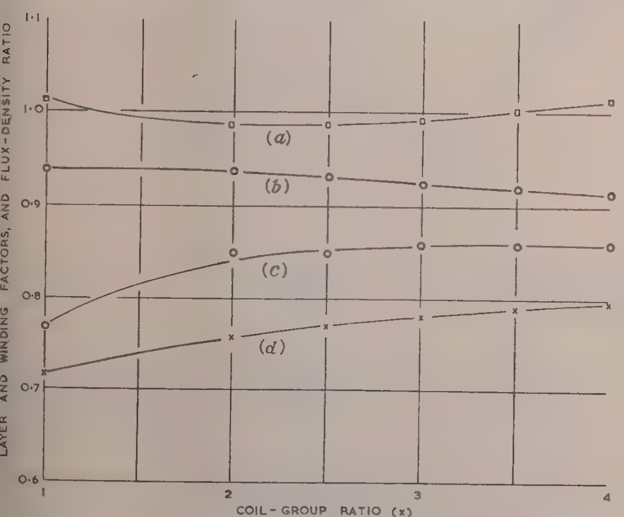
Coil grouping: 1–x–$(x+1)$–x–1—1–x–$(x+1)$–x–1
○ M.M.F.s before modulation, in terms of 10-pole m.m.f. ($m=5$).
× M.M.F.s after modulation, in terms of 8-pole m.m.f. ($m=4$).

ultiply the number of coils in all the coil groups by a suitable teger, in order to make integral the number of coils in every roup. For example, the sequence 1–2½–3½–2½–1 would be arried into effect as 2–5–7–5–2; and it will be shown that, by small margin, this latter is, in fact, the ideal coil-group equence. Further analysis has actually been carried out for ther types of sequence; e.g. 1–x–(x + 2)–x–1. This has, owever, merely served to confirm rigorously the intuitive hoice of 1–x–(x + 1)–x–1 as the prototype coil-group sequence.

The m.m.f. harmonic contents of one layer of a winding of his type, for a series of values of x, are plotted against x in ig. 2. The effect of chording was thus deliberately excluded. he reason for doing this is that the degree of chording used aries slightly with the type of winding, and the chording factors or the higher harmonics thus vary substantially from winding to inding.

The curves for which $m = 1$, 7, 11, 13 and 17 show the alues of harmonic before modulation, and those for which $n = 2$, 8, 10, 14 and 16 give the values of harmonic after nodulation; the basic m.m.f. being given by $m = 5$ or $m = 4$ n the two cases, respectively. It will clearly be seen from hese curves that $x = 2$ and $x = 3$ are the lower and upper imits, in practice, for the acceptable range of x, within which he overall harmonic content is relatively small. For $x = 2$, he coil-group sequence is 1–2–3–2–1 and can be embodied n 54 slots; for $x = 3$ the sequence is 1–3–4–3–1 and requires 2 slots; and for $x = 2\frac{1}{2}$ the sequence is 2–5–7–5–2 and needs 26 slots. As already stated, this is the ideal for minimum armonic content, but the number of slots is too large to use or a small machine.

The essential data for these and various other possible forms of quasi-sinusoidally grouped windings, for pole-amplitude nodulation, starting with a 10-pole winding initially, are shown n Table 2 columns (d)–(h), all these windings being capable of derivation ultimately from a distribution wave of the general orm shown in Fig. 1. In Fig. 3 are shown the layer factor



ig. 3.—Variations of layer and winding factors, and of flux-density ratio, in irregular 10/8-pole fractional-slot windings, for pole-amplitude modulation.

(a) Flux-density ratio.
(b) Layer factor: 10 poles.
(c) Winding factor: 10 poles.
(d) Layer and winding factors: 8 poles.

For 8 poles, all the windings are virtually full pitch and the chording factor is almost onstant and equal to unity. Hence the layer and winding factors are equal.
Coil-grouping: 1–x–(x + 1)–x–1—1–x–(x + 1)–x–1

and the winding factor, for both 8-pole and 10-pole connections, with respect to the fundamental component of flux, plotted as a function of the coil-group factor x, referred to above. For 8-pole connection the coil pitch is so nearly constant and equal to unity that the layer and winding factors are virtually equal. From these factors can be calculated the air-gap flux-density ratios, B_{10}/B_8, using delta/parallel-star connection throughout; and it will be seen that this is agreeably near to unity for all values of x.

A winding for the coil-group sequence 2–5–7–5–2 needs 126 slots; and two alternative designs could be brought into the range of possibility by adding or subtracting one coil to or from the centre coil group. The sequence 2–5–6–5–2 could be used in an armature with 120 slots, or the sequence 2–5–8–5–2 in an armature with 132 slots; and these would manifestly give results only differing marginally from their prototype of 2–5–7–5–2 in 126 slots. The m.m.f. analyses and other data for windings of these last types have accordingly been added, as designs (i) and (j), to those in Table 2. The sequence 2–5–6–5–2 (i.e. 1–2½–3–2½–1, doubled) could, in some senses, be regarded as intermediate between designs (d) and (e); and the sequence 2–5–8–5–2 (i.e. 1–2½–4–2½–1, doubled) as intermediate between designs (e) and (f); although neither of them is a member of the prototype 1–x–(x + 1)–x–1.

Clearly, both these designs (i) and (j) are very practicable forms of 10/8-pole winding, and it would be possible to make a useful direct assessment of design (i) by comparing the output of a machine frame wound for 10/8 poles with a single winding, of this sequence 2–5–6–5–2, with the output of the same frame wound with two separate integral-slot windings, respectively, for 10 and 8 poles and with 4 and 5 slots per pole per phase.

(2.2) Designs using Integral-Slot Windings

In Table 2 are also shown, in columns (a)–(c), the details and analysis of three designs of integral-slot winding, wound initially for 10 poles and modulated with omission of coils to 8 poles. The basic design in column (a) corresponds as nearly as possible to the basic design for 8/10-pole modulation, but it gives a higher harmonic content and would scarcely be satisfactory in practice. The modified design in column (b) is somewhat improved relatively, though it is again less good than its counterpart for 8/10-pole modulation. The reason for these facts is fairly clear. Leaving out one coil group—or two separate coils—from an original total of five coil groups does not give as good an approximation to sinusoidal coil-group distribution as when there are four coil groups originally from which one is omitted.

In column (c), there is shown the form of 10/8-pole modulation equivalent to that found most successful, when using an integral-slot winding initially, for 8/10-pole modulation. This is certainly the best form of modulation amongst those using a normal winding when unmodulated. It is of interest that the best coil-group arrangement for 10/8-pole modulation is 100–3–3–3–001, whereas the best coil-group arrangement for 8/10-pole modulation is 001–3–3–100. A little consideration shows that sinusoidal coil-group distribution in each case is most nearly reached with the coil-group distributions stated, but the difference should be remarked.

Table 2 can be regarded as the basic data sheet for 10/8 pole-amplitude modulation, and it forms a companion to Table 6 in an earlier paper,[2] which discussed 8/10 pole-amplitude modulation.

(2.3) Design of Small Test Machine

The design given in column (d) of Table 2 was the one selected for small-scale laboratory tests, because of the relatively small

Table 3

General Expressions for Layer Factor for 10/8 Pole-Amplitude Modulation

Modulation method according to Table 6	Slot numbers	Layer factor
Column (*a*)	60	$\sin\frac{m\pi}{10}\cos\frac{m\pi}{5}\cos\frac{m\pi}{60}$
Column (*b*)	60	$\sin\frac{m\pi}{10}\cos\frac{m\pi}{5}\sin\frac{7m\pi}{60}$
Column (*c*)	90	$\frac{1}{15}\left[1-2\cos\frac{m\pi}{5}\left(1+4\sin\frac{11m\pi}{90}\sin\frac{m\pi}{10}\right)\right]$
Column (*d*)	54	$\frac{1}{9}\left(2\cos\frac{5m\pi}{27}-1\right)\left(2\cos\frac{2m\pi}{9}-1\right)$
Column (*e*)	126	$\frac{1}{21}\left[1+4\sin\frac{5m\pi}{42}\sin\frac{11m\pi}{126}\left(1+2\cos\frac{m\pi}{63}\right)+\cos\frac{17m\pi}{42}\cos\frac{m\pi}{126}-\cos\frac{13m\pi}{63}\cos\frac{2m\pi}{63}\right]$
Column (*f*)	72	$\frac{1}{6}\left[\cos\frac{m\pi}{24}-\cos\frac{5m\pi}{24}\left(4\sin\frac{m\pi}{9}\sin\frac{m\pi}{12}+1\right)\right]$
Column (*g*)	162	$\frac{2}{27}\left\{\left(1-2\cos\frac{17m\pi}{81}\right)\left[\frac{1}{2}+\cos\frac{2m\pi}{81}\left(1+2\cos\frac{m\pi}{81}\right)\right]+\cos\frac{4m\pi}{81}+2\cos\frac{65m\pi}{162}\cos\frac{m\pi}{162}\right\}$
Column (*h*)	90	$\frac{1}{15}\left\{1+2\left[\cos\frac{m\pi}{45}\left(1-2\cos\frac{m\pi}{5}\right)-4\cos\frac{2m\pi}{9}\sin\frac{m\pi}{10}\sin\frac{7m\pi}{90}\right]\right\}$
Column (*i*)	120	$\frac{1}{10}\left[\cos\frac{5m\pi}{24}\left(1+8\sin\frac{m\pi}{12}\sin\frac{13m\pi}{120}\cos\frac{m\pi}{120}\right)-\cos\frac{m\pi}{24}\right]$
Column (*j*)	132	$\frac{1}{11}\left[2\sin\frac{7m\pi}{66}\left(\sin\frac{3m\pi}{44}-4\sin\frac{7m\pi}{66}\cos\frac{m\pi}{132}\cos\frac{7m\pi}{33}\right)+\cos\frac{7m\pi}{132}\right]$

number of slots (54) which this design requires, compared with the other designs in Table 2. The layer factor was determined by the methods described in an earlier paper,[2] the general expression in this case being

$$\frac{1}{9}\left(2\cos\frac{5m\pi}{27}-1\right)\left(2\cos\frac{2m\pi}{9}-1\right)$$

This expression is one of those contained in Table 3, which records general expressions for the layer factor, for all the windings covered by Table 2. By putting $m = 4$ and 5, successively, the layer factors for 8 and 10 poles are given as 0·758 and 0·941; the corresponding chord factors are 0·998 and 0·895; and the resultant winding factors are 0·756 and 0·844, the average being 0·80, which is a high value.

If the 10/8-pole connection is arranged series-delta/parallel-star, the ratio of the polar fluxes is given by

$$\frac{\Phi_{10}}{\Phi_8} = \frac{V}{n \times 0{\cdot}844} \times \frac{\frac{n}{2}0{\cdot}756}{V/\sqrt{3}}$$
$$= 0{\cdot}778$$

The flux-density ratio is

$$\frac{B_{10}}{B_8} = 1{\cdot}25 \times 0{\cdot}778 = 0{\cdot}972$$

Within a very near tolerance, therefore, the machine is fully fluxed at both speeds, as well as having a high average winding

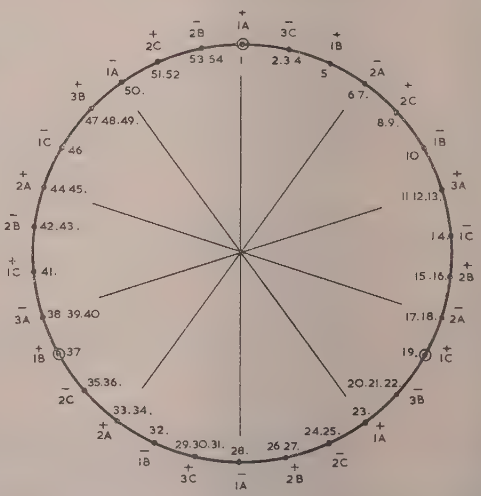


Fig. 4.—Fractional-slot winding in 54 slots, for pole-amplitude modulation: 10/8 poles.

Winding phase-sequence: ABC.
Phase-origin sequence: ACB.
○ Ideal phase origins.
10 poles: 54 slots: 9/5 slots per pole per phase.
Coil pitch: 7 slots = 1·30 (full pitch) for 10 poles.
Coil-group sequence per phase: 1–2–3–2–1—1–2–3–2–1.
Overall coil-group sequence: 1–3–1–2–2, etc. (six times).
Modulation by reversal of half of each phase-winding.

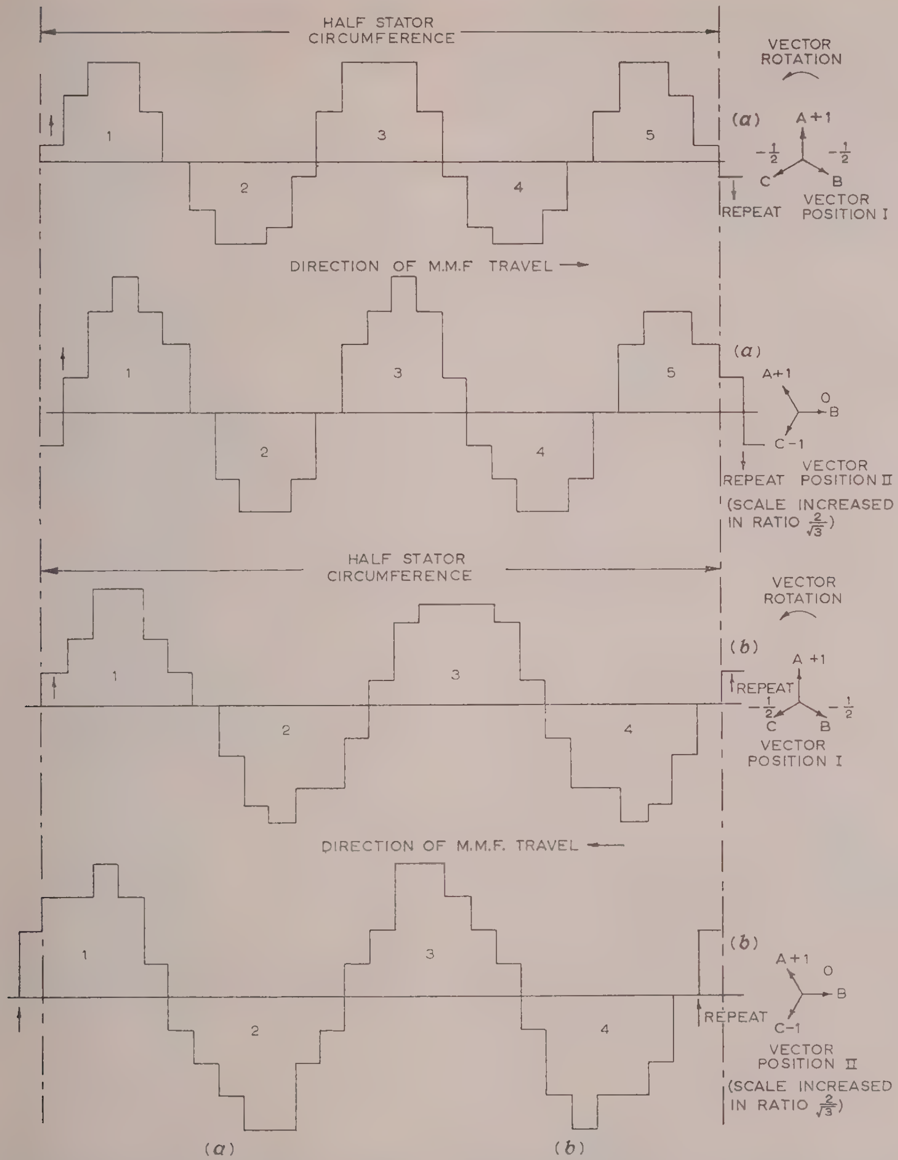


Fig. 5.—M.M.F. waveforms for the two typical vector positions, for 10/8-pole winding in 54 slots, grouped 1–2–3–2–1—1–2–3–2–1.
Coil-pitch: 7 slots = 1·30 (full pitch) for 10 poles.
(*a*) 10-pole unmodulated. (*b*) 8-pole modulated.

factor. The m.m.f. analyses are given in Table 2, from which it will be seen that the only harmonics which are not obviously negligible are a 2-pole sub-harmonic in the 10-pole connection, and a 28-pole harmonic in the 8-pole connection, which, fortunately, rotates in the opposite sense to the main 8-pole field.

The actual coil-group layout for the test machine is shown in the clock diagram of Fig. 4, in which the geometrical symmetry has been emphasized by marking the ideal phase origins. It will be observed that the main winding sequence, ABC, is opposite to the phase-origin sequence, ACB, and that modulation of the winding will therefore reduce the number of poles by 2.

In order that the excellent m.m.f. waveforms obtained from the designs in Table 2 may be fully appreciated, the m.m.f. waveforms for the small test machine have been drawn for both pole numbers, and for both the usual vector positions. They are shown in Fig. 5, the waveform being given in all cases for half the winding, the second half being identical with the first. Similar m.m.f. waveform diagrams can be readily constructed for any of the other designs in Table 2, and the visual impression gained from such diagrams would lead any experienced designer to expect satisfactory performance from machines using these windings.

(2.4) Design of Large Test Machine

Following the successful testing of the small laboratory machine wound to the authors' design, a large 6·6 kV test machine was built by an industrial company using design (*f*) of Table 2, with 144 stator slots, each stator phase-winding thus being grouped 2–6–8–6–2—2–6–8–6–2. This design can, of course, be derived directly from Table 2 by doubling each slot

of design (*f*), which is there shown as having 72 slots. The rating of the test machine was 450/800 hp for 10/8 poles, respectively. There were 100 rotor slots, although this number has no special significance. The winding factors and flux-density ratio are all shown in Table 2, the winding factors being high and the flux-density ratio being almost equal to unity.

The winding was placed in a stator core and frame identical in overall dimensions with that in which another 10/8-pole machine of the same rating was constructed, using two separate windings in 120 slots. The layout of the test winding is shown

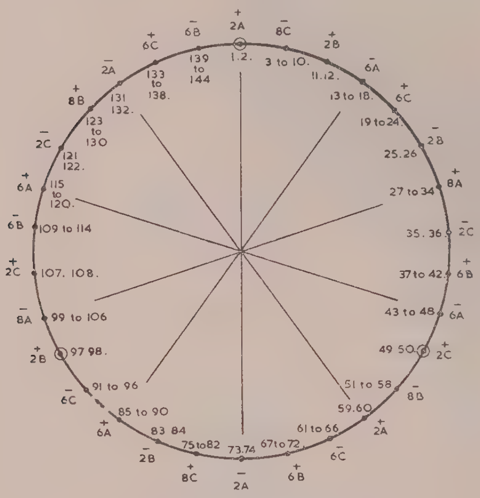


Fig. 6.—Fractional-slot winding in 144 slots, for pole-amplitude modulation: 10/8 poles.

Winding phase-sequence: ABC.
Phase-origin sequence: ACB.
○ Ideal phase origins.
10 poles: 144 slots: 24/5 slots per pole per phase.
Coil pitch: 18 slots = 1·25 (full pitch) for 10 poles.
Coil-group sequence per phase: 2–6–8–6–2—2–6–8–6–2.
Overall coil-group sequence: 2–8–2–6–6, etc. (six times).
Modulation by reversal of half of each phase-winding.

in the clock diagram of Fig. 6. The slots used for the single winding machine were, of course, very much shallower, and a considerably smaller frame and core could, in fact, have been used.

(3) TEST RESULTS

(3.1) Test Results for Small Test Machine

Before putting the machine on load, the usual no-load tests were performed; and curves showing the variation of magnetizing current and no-load power with variation of applied voltage are given in Figs. 7 and 8, respectively.

It can readily be shown that, when a winding is connected so as to be switched parallel-star/delta for p_1/p_2 poles, respectively, the ratio of the magnetizing currents per line, ignoring saturation, is given in general by

$$\frac{I_1}{I_2} = \frac{4}{3} \times \left(\frac{p_1}{p_2}\right)^2 \times \left(\frac{k_2}{k_1}\right)^2$$

where k_1 and k_2 are the corresponding winding factors. In this example, $p_1/p_2 = 0 \cdot 8$ and $k_2/k_1 = 1 \cdot 11$, and the numerical value of the ratio is 1·06; whereas the measured ratio of the initial slopes of the magnetizing-current curves in Fig. 7 is 1·12. This comparison is within 5%, and is quite as close as can be expected for such a calculation on a saturable circuit.

The curves of no-load power shown in Fig. 8 are very nearly coincident for both speeds, the power loss after modulation,

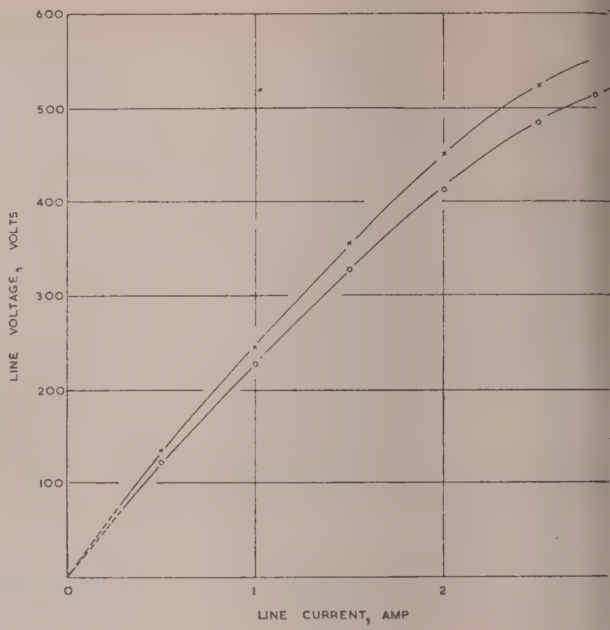


Fig. 7.—Magnetizing characteristics for 54-slot 10/8-pole motor.

Operating voltage = 500 volts.
○○○ 8-pole parallel-star connection.
××× 10-pole series-delta connection.
Ratio of initial slopes $I_8/I_{10} = 1 \cdot 12$, for a given applied voltage.

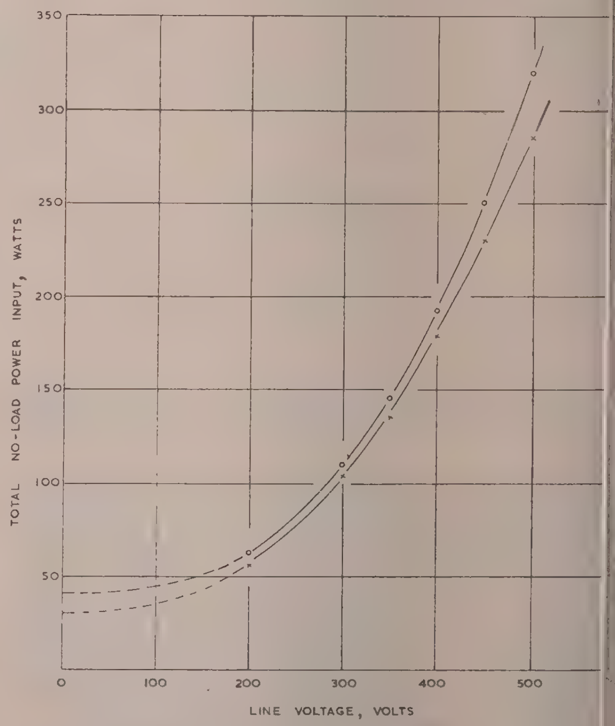


Fig. 8.—Variation of no-load power input with line voltage.

54-slot 10/8-pole motor.
Operating voltage = 500 volts.
○○○ 8-pole parallel-star connection.
××× 10-pole series-delta connection.

...en running on 8 poles, being only slightly higher. Merely on ...ount of increased speed, the no-load power would be expected ...ncrease more than in fact it does, and there is no question of ...dulation leading to excessive no-load losses.

(3.2) Test Results for Large Test Machine

...n every respect, the performance of the motor with a single ...e-amplitude-modulated winding was superior or equal to ...t of the double-wound machine, at all loads. At the lower ...ed, the power factor for the single-winding machine was ...preciably better. After a test of this thoroughness and on ...s scale, following earlier tests on many 8/10-pole motors,[2] ...an be said with confidence that, for this speed ratio at least, ... double-wound machine is now obsolete.

(3.3) General Note on Testing

Neither the small machine, which was tested under completely ...iet laboratory conditions, nor the large machine, which was ...ted on an industrial test bed, showed any signs whatever of ...normal noise at either speed, on full load or on no load.

The small machine was reverse-'plugged' at various voltages ...wn to about a quarter of the nominal voltage, for both speeds, ...t there was no trace of any tendency to crawl, in particular ... the 28-pole harmonic, which, as mentioned in Section 2.3, ...the only sensible harmonic in the 8-pole connection. In the ...-pole connection, all harmonics are trivial except the sub-...rmonic, and this cannot cause crawling. The large machine ...as of too great a rating to be reverse-plugged, and it was in ...ct started on reduced voltage. Compared with other standard ...ngle-speed industrial motors, its accelerating properties were ...garded as wholly satisfactory by the engineers of the industrial ...mpany who built the motor to the authors' specification.

The authors would, of course, have liked to carry out dynamic ...eed/torque tests, as was done for the medium-sized 8/10-pole ...achines discussed in an earlier paper,[2] but they do not yet ...ossess the necessary apparatus themselves, and the large ...achine was too large to be treated in this fashion. In the light ... the earlier exhaustive tests and of the extensive tests on these ...achines, it is, however, now possible virtually to discount any ...ggestion that these machines differ in any material respect from ...andard machines in relation to noise, vibration or accelerating ...roperties.

(4) DESIGN OF PRE-MODULATED WINDINGS

(4.1) Introduction and General Theory

An earlier paper[2] has shown, for a particular case, that ...odulation may be carried out by simple reversal of one-half ...f each phase-winding with respect to the other half and without ...mission of coils on modulation, provided that the original coil-...roup sequence is arranged according to a special non-uniform ...attern. The appropriate coil-group sequence was originally ...btained semi-intuitively; and the analytical treatment to deter-...ine the best coil grouping, both in the former paper and in ...is paper, has so far been directed to an analysis, *seriatim*, of ...e various possibilities. It is, however, desirable to find a ...mple general method of determining the best possible winding, ...r any particular slot number and pole combination, which does ...ot involve successive and full analysis of the various possible ...esigns. The following Sections deal with the development of ...ch a method of determination.

(4.2) Pre-Modulation of Polyphase Windings

The essence of pre-modulation of polyphase windings is that the magnitude, but not the sign, of the modulating wave is permanently impressed on the windings. The process of modulation then consists solely of adding the effect of sign, by appropriate reversals of coil groups. In effect, the amplitude of the m.m.f. due to the windings is permanently pre-modulated. Since the process of modulation essentially consists in multiplying the original m.m.f. waveform by a sine wave, it follows at once that the most acceptable form of pre-modulation is coil grouping according to a sinusoidal law, at least as far as the resultant m.m.f. waveform after modulation is concerned. Having grouped the coils in this way, simple reversal of half of them necessarily results in sinusoidal pole-amplitude modulation. It only remains, therefore, to decide whether this form of pre-modulation is equally acceptable in relation to the m.m.f. waveform before modulation.

Any non-uniform distribution of coil groups is, in itself, undesirable, because it inevitably introduces sub-harmonics into the m.m.f. waveform. Non-uniform distribution is ordinarily associated with fractional-slot windings, in which sub-harmonics are inherent, although there is a counterbalancing improvement of waveform in relation to high-order harmonics. The effective and actual numbers of slots per pole per phase in fractional-slot windings are not equal, and the higher harmonics are often much reduced in magnitude, in a manner corresponding to an increased number of slots per pole per phase.

Before considering further the question of winding distribution, some re-definition of certain terms is necessary. For example, a standard integral-slot 3-phase 8-pole winding in 72 slots will have a coil-group sequence 3–3–3–3—3–3–3–3 per phase, whereas the corresponding pre-modulated coil-group sequence is 2–4–4–2—2–4–4–2, which formed the basis of a successful earlier machine.[2] The *average* number of slots per pole per phase in the latter case is still 3, and the winding is therefore still an integral-slot winding, *taking an average*. It is, however, very doubtful whether it is proper to call such a winding an integral-slot winding, since this term has always been associated with uniform distribution. The present authors propose the use of the term *quasi-fractional-slot windings* for those in which the average number of slots per pole per phase is integral, but in which the distribution of coils is not uniform.

The term quasi-fractional-slot can also be applied to windings in which the average number of slots per pole per phase is fractional, but in which the coil-group distribution is more irregular than is required simply by the number of slots. For example, a 3-phase 8-pole winding in 36 slots will necessarily be a fractional-slot winding, with $1\frac{1}{2}$ slots per pole per phase, and in the accepted form of winding the coil-group sequence per phase will be 1–2–1–2—1–2–1–2, which is as near as possible to uniform distribution. For a quasi-fractional-slot 8-pole winding, intended for modulation to 10 poles, the coil-group sequence will be 1–2–2–1—1–2–2–1; and this is exactly comparable to the distribution 2–4–4–2—2–4–4–2, which was used in one large test machine.[2]

(4.3) Effect of Pre-Modulation

The effect of pre-modulation can be seen from the following general theorem:

The m.m.f. of the three phase-windings of a polyphase winding of $2p$ poles, where $2p = (6m + 2)$ or $(6m + 4)$, and m is any integer, can be expressed as

$$A \sin p\theta;\ A \sin p\left(\theta - \frac{2\pi}{3}\right);\ A \sin p\left(\theta - \frac{4\pi}{3}\right)$$

If the amplitude A is pre-modulated, so as to go through one cycle of variation in a half-perimeter, as is necessary for the

basic form of pole-amplitude modulation, these expressions may be rewritten:

$$A[1 - \Sigma M_k \cos k\theta] \sin p\theta$$

$$A\left[1 - \Sigma M_k \cos k\left(\theta - \frac{2\pi}{3}\right)\right] \sin p\left(\theta - \frac{2\pi}{3}\right)$$

$$A\left[1 - \Sigma M_k \cos k\left(\theta - \frac{4\pi}{3}\right)\right] \sin p\left(\theta - \frac{4\pi}{3}\right)$$

where $k = 2(1, 3, 5,$ etc.) for symmetrical pre-modulation, i.e. $k = 2, 6, 10,$ etc., and M_k is the depth of pre-modulation for each component of the modulation.

The direct term of each product gives the normal rotating field of p poles, and need not be considered any further. The additional terms, ignoring the coefficients AM_k, are:

$$\cos k\theta \sin p\theta = \frac{1}{2}[\sin (p+k)\theta + \sin (p-k\theta)]$$

$$\cos k\left(\theta - \frac{2\pi}{3}\right) \sin p\left(\theta - \frac{2\pi}{3}\right) = \frac{1}{2}\left[\sin (p+k)\left(\theta - \frac{2\pi}{3}\right) + \sin (p-k)\left(\theta - \frac{2\pi}{3}\right)\right]$$

$$\cos k\left(\theta - \frac{4\pi}{3}\right) \sin p\left(\theta - \frac{4\pi}{3}\right) = \frac{1}{2}\left[\sin (p+k)\left(\theta - \frac{4\pi}{3}\right) + \sin (p-k)\left(\theta - \frac{4\pi}{3}\right)\right]$$

This gives two rotating fields of $(p \pm k)$ pole-pairs; except that all values of $(p \pm k)$ which are multiples of 3 will disappear when the three phase-windings are combined.

In relation to the number of harmonics likely to be effectively present, it is clear that this number will increase as the number of values of k, for which M_k is substantial, increases; and that sinusoidal pre-modulation is therefore to be preferred to any other form.

It is still true that the ideal amount of pre-modulation is zero, as far as the original pole number is concerned. The improvement in m.m.f. waveform in the modulated connection which results from the use of pre-modulated windings is, however, so substantial as to offset the disadvantages of pre-modulation in the original connection. Accepting the necessity for pre-modulation, however, it is now clear that sinusoidal pre-modulation, which is actually desirable for the modulated connection, is the least undesirable form of pre-modulation in relation to the original unmodulated waveform. It only remains to determine a design technique for applying sinusoidal pre-modulation to any particular pole combination and slot number.

It should be noted that if a winding has $6m$ poles the individual phase windings cannot be expressed as

$$A \sin p\theta;\ A \sin p\left(\theta - \frac{2\pi}{3}\right);\ A \sin p\left(\theta - \frac{4\pi}{3}\right)$$

but only as

$$A \sin p\theta;\ A \sin\left(p\theta - \frac{2\pi}{3}\right);\ A \sin\left(p\theta - \frac{4\pi}{3}\right)$$

In the latter case it is impossible directly to apply pre-modulation, in the manner previously described, by three equally spaced pre-modulating waves. This is because three origins, spaced $2\pi/3$ geometrically, would all be in the same phase for a winding of $6m$ poles. Hence this pre-modulation technique is only applicable to pole numbers which are not multiples of 3, and the origins of the pre-modulation will only coincide with tho of the modulating envelopes if the modulated pole number al is not required to be a multiple of 3.

(4.4) Design Procedure for Sinusoidal Pre-Modulation

Consider one half-cycle of a sine wave, with origin at zer which is taken to correspond with a semi-perimeter of th machine, when the pole number is to be changed by 2. (Th extension to other differences is obvious.) Take the peak the wave to correspond with the 'centre-of-gravity' of the co groups of one-half of a phase-winding, and a quarter of a cyc of this wave, on each side of this peak, to represent one half-cyc of the modulating wave. The horizontal axis can convenient be scaled according to the slotting of the machine.

The e.m.f. per coil group can readily be determined, in term of V, the e.m.f. per coil, using the classical expressions for sprea factor. The e.m.f. per coil group is

$$V\frac{\sin \frac{n\theta}{2}}{\sin \frac{\theta}{2}}$$

where n is the number of coils per group, and θ is the electric angle, on the original main-pole scale, between successive coil The quantity $\sin (n\theta/2)/\sin (\theta/2)$ can conveniently be tabulated, f a series of values of n, for different values of θ; n will normal lie between 1 and 10, in most cases.

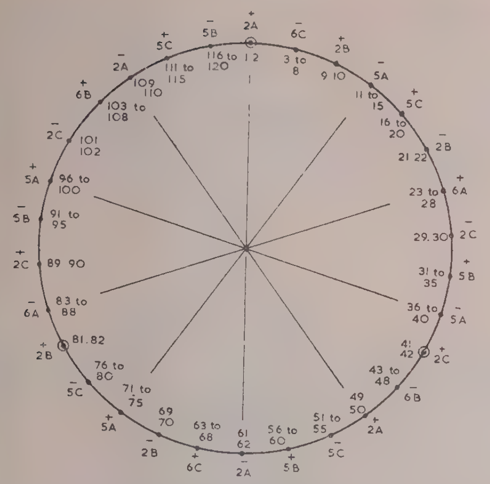
The e.m.f.s having been determined for each coil group in on half phase-winding, their values are set up, as ordinates, abscissae which correspond with the 'centre of gravity' of th respective coil groups. This 'centre of gravity' will lie at a sl if the number of coils in the group is odd, and intermediate between two slots if the number of coils in the group is even.

The heights of these e.m.f. ordinates are then compared wi the reference sine wave, the vertical scale of this wave bei adjusted to correspond with the e.m.f. of the centre coil grou If the ordinates follow this sine wave the distribution is ide In so far as they depart from it, the distribution is less than ide Several coil groupings can be quickly examined in this way, a it will usually be obvious which are the best one or two. Indee one can almost tell by eye which is best. It is believed that t coil-group sequence per phase-winding is much more obvious significant than that for the whole machine. Either sequen can, of course, be determined from the other.

The whole process can be well illustrated by reference design (*i*) of Table 2, giving a 3-phase 10-pole winding in 1 slots, grouped 2–5–6–5–2—2–5–6–5–2, according to the clo diagram of Fig. 9.

With respect to 10 poles one slot is 15°; and the e.m.f.s the coil groups of one phase-winding are thus $1{\cdot}98V$, 4·6 and $5{\cdot}42V$, etc., respectively. As will be seen from Fig. the centres of these coil groups for phase-winding A are, resp tively, between slot 1 and slot 2—called 'slot 1½'; at slot 1 and between slot 25 and slot 26. The latter point is the cen of the half-cycle of modulation, which extends over 30 s pitches on either side.

The nearness of the approach to sinusoidal coil-group d tribution is shown in Fig. 10. The ordinate $1{\cdot}98V$ at slot 1½ slightly above the ideal value of $1{\cdot}68V$; and similarly the ordin $4{\cdot}65V$ at slot 13 is a little greater than the ideal value, 4·30 Alternatively, one could say that the centre ordinate, $5{\cdot}42V$ slightly below its ideal relative value, and that the curve of co group distribution is thus a slightly flattened sine wave. If coil-group distribution 2–5–7–5–2 in 126 slots is used inste



ig. 9.—Quasi-fractional-slot winding in 120 slots, for pole-amplitude modulation: 10/8 poles.

Winding phase-sequence: ABC.
Phase-origin sequence: ACB.
⊙ Ideal phase origin.
10 poles: 120 slots: 4 slots per pole per phase.
Coil pitch 15-slots = 1·25 (full pitch) for 10 poles.
Coil-group sequence per phase: 2–5–6–5–2—2–5–6–5–2.
Overall coil-group sequence: 2–6–2–5–5, etc. (six times).
Modulation by reversal of half of each phase-winding.

t can be shown by similar analysis that the result is a slightly eaked sine wave of coil-group distribution.

Since slot numbers and the numbers of coils per coil group re necessarily integral it rarely happens that the ideal sinusoidal oil grouping is exactly attained, and the closeness of approxima-ion here reached is, in fact, very satisfactory. Good perfor-nance is given by many windings which depart far more widely rom the ideal than does this winding. Indeed, the m.m.f. nalysis given in Table 2 shows that this winding is one of the referred designs for 10/8 poles. Since, for this pole combina-ion, an exhaustive analysis of all possibilities has already been nade, the construction in Fig. 10 in this case merely confirms

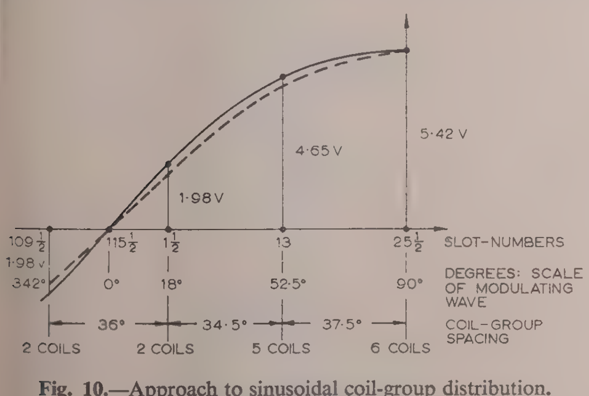


Fig. 10.—Approach to sinusoidal coil-group distribution.

10-pole winding in 120 slots.
30 slots = 90° on the modulating wave scale.
The wave $5\cdot42V\sin\theta$ is shown as a broken line.
$5\cdot42\sin 18° = 1\cdot68$. $5\cdot42\sin 52\cdot5° = 4\cdot30$.

the results of the general analysis. The general method of Fig. 10 is, however, now available to enable the best design for other pole combinations to be obtained by simple direct means.

In conclusion, it should be noted that 120 slots is the smallest number in which a standard single-speed integral-slot winding for either 10 poles or 8 poles can be wound, giving, respectively, 4 or 5 slots per pole per phase. The number 120 is, of course, three times the least common multiple (40) of 8 and 10. Two-speed double-winding motors for this pole combination have commonly been designed to have 120 slots. It would be possible, speaking arithmetically, to say that the 2-speed winding just discussed is an integral-slot winding for both pole numbers, though the authors believe that it would be a misuse of terms to do so. The new term suggested above—quasi-fractional-slot winding—is thought to be very much more appropriate.

(4.5) Simplified Approximate Procedure

The procedure explained in Section 4.4 can be radically simplified, in most cases, by the following considerations. Reference to Fig. 10 will show that the 'centres of gravity' of successive coil groups are almost equidistant, even in the winding there discussed, which has a very irregular coil-group distribution. Because of the interleaving of the phases this is always true; and, for all practical purposes, these 'centres of gravity' can be taken as equidistant. This avoids any necessity for drawing a clock diagram, as was done in Fig. 9.

Further, even in the larger coil groups, the vector sum of the e.m.f.s in one coil group is nearly equal to their arithmetic sum; and, as a first approximation, can be taken as equal. One then simply has to read off the values of the ordinates of a sine wave, representing the desired modulating wave, at intervals which correspond to the average coil-group spacing. The numbers of coils in the various groups have then to be taken as those integers which are most nearly proportional to the corresponding sine-wave ordinates.

For example, the average coil-group spacing in a 10-pole machine is 36°; and, by reference to Fig. 10, the corresponding ordinates for a half phase-winding are seen to be

$$\sin 18°;\ \sin 54°;\ \sin 90°;\ \sin 126°;\ \sin 162°$$

The coil grouping should therefore be proportional to

$$0\cdot309 : 0\cdot809 : 1\cdot000 : 0\cdot809 : 0\cdot309$$

or

$$1\cdot0 : 2\cdot61 : 3\cdot24 : 2\cdot61 : 1\cdot0$$

or, as nearly as small integers permit,

either $2 : 5 : 6 : 5 : 2$

or $2 : 5 : 7 : 5 : 2$

As Table 2 shows, these two are, in fact, the ideal coil groupings; and the result given in Section 4.4 has here been obtained with sufficient accuracy and much greater ease.

(4.6) The Theoretical Value of Layer Factor after Modulation

The winding factors for a number of particular examples of 10/8-pole windings, after modulation to 8 poles, are given in Table 2. Since the chording factor in every case is then virtually equal to unity, the layer factors are the same as the winding factors. (The layer factor of a fractional-slot winding has been defined in Reference 2 as the spread factor of a complete winding.)

There is a theoretical ideal value for the layer factor of a sinusoidally distributed winding after modulation, to which all these actual values are approximations. It can be deduced as follows.

The coil groups of a half phase winding after modulation are spread over 180°, but the distribution is not uniform. (The two halves of the winding are identical.) On the contrary, the conductor loading of the armature at the element δs in Fig. 11

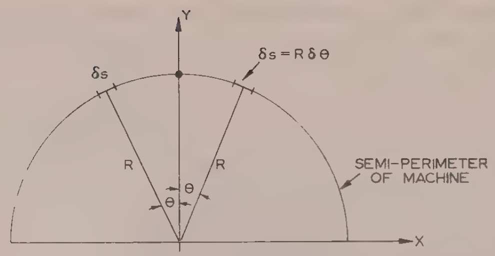


Fig. 11.—Ideal layer factor for sinusoidally distributed winding.

Conductor loading on Y axis: C_m.
Conductor loading at angle θ: $C_m \cos\theta$.

is ideally given by $C_m \cos\theta$, where C_m is the peak loading. Assume unit e.m.f. per conductor. Then the resultant e.m.f. of two symmetrically disposed elements, by vector addition, is $(2C_m \cos\theta\delta s)(\cos\theta)$. Now, $\delta s = R\delta\theta$; and the total e.m.f. for the half phase winding is therefore

$$2C_mR\int_0^{\pi/2}\cos^2\theta d\theta = \frac{\pi}{2}(C_mR)$$

But the total number of conductors, and the arithmetic sum of their e.m.f.s, is given by

$$2\int_0^{\pi/2}C_m\cos\theta\delta s = 2C_mR\int_0^{\pi/2}\cos\theta d\theta$$
$$= 2C_mR$$

The theoretical layer factor is therefore

$$\frac{\pi}{2}(C_mR)/2C_mR = \frac{\pi}{4} = 0\cdot785$$

This theoretical value (0·785) compares very closely with the actual values obtained by calculation for a 10/8-pole winding after modulation to 8 poles. A winding with excessive co centration of coils at the centre of each phase winding (such one grouped 2–5–8–5–2) has a slightly higher layer factor (0·81 than the theoretical value, whereas one which has too uniform distribution (2–5–6–5–2) has a slightly lower layer factor (0·75 For approximate computation, the value 0·785 may often assumed, and detailed calculation thus avoided. Any departu from sinusoidal distribution, though it may improve the lay factor, will result in a corresponding degree of deterioration the m.m.f. waveform.

(5) ACKNOWLEDGMENTS

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A GENERAL METHOD OF DIGITAL NETWORK ANALYSIS PARTICULARLY SUITABLE FOR USE WITH LOW-SPEED COMPUTERS

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SUMMARY

The concept of digital solutions of many of the network analysis problems associated with the design and operation of electrical power systems is now generally appreciated. There is, however, little indication of an increasing routine use of such methods in the United Kingdom, although many types of digital computer are now available. It is suggested that this is partly due to the fact that most of the existing programmes in this field employ techniques which have been prepared by skilled programmers for use with large high-speed computers, and digital analyses consequently tend to be restricted to the realm of special studies at computing centres.

The paper describes the network theory, programme details and application of a general method of digital network analysis particularly suitable for use with low-speed computers. The method has been proved using both high- and lower-speed computers and the results of sample load-flow and short-circuit studies are discussed.

LIST OF PRINCIPAL SYMBOLS

z = Impedance.
y = Admittance.
Y = Nodal admittance coefficient for load-flow studies.
Y' = Nodal admittance coefficient for short-circuit studies.
b = Shunt susceptance.
s = Sending-end busbar number.
r = Receiving-end busbar number.
V = Voltage of busbar above neutral, N.
d = Voltage difference of busbar relative to the reference busbar.
P, Q = Active and reactive powers (Q positive when lagging).
I = Injected nodal current.
i = Branch current for load flows.
i' = Branch current infeed to a faulted busbar.
E = Constant generator e.m.f.
t = In-phase transformer tap.
q = Quadrature transformer tap.
τ = Tolerance used as an iterative convergence criterion.
ϵ = Error quantity compared with τ.
(m) = Iterative state.

Subscripts.

0 = Reference quantities
g = Generation quantities.
l = Load quantities.
f = Fault quantities.
n = Equivalent shunt admittances.

(1) INTRODUCTION

It has often been demonstrated that new methods for the analysis of engineering problems rarely replace existing and well-tried methods in their entirety. The use of digital computers for power system analysis, which has hitherto been the field of specialized analogue computation,[1, 2] is unlikely to invalidate this principle, chiefly because numerical methods do not provide the physical correspondence of controls and parameters which is inherent in analogue representation. Whereas this facility is of prime importance to engineers concerned solely with system planning or with the training of operation personnel, it may be considered as secondary if the solution of network problems occurs as only part of the work of a group requiring other forms of computational assistance, for which a general-purpose digital computer is suitable.

The availability of small or medium-sized digital computers is such that there is no longer any need for digital network analysis to be confined to the large high-speed computers at computing centres where studies are inevitably carried out on a piecemeal basis. Provided that special consideration is given to the development of programmes suitable for smaller, slower computers, it is felt that they can be of considerable use in the solution of routine network problems and also as analytical tools in the investigation of conditions for which analogue representation may be inadequate, e.g. networks having low R/X ratios, low series impedances in conjunction with high shunt values, phase-shifting transformers, saliency, saturation, voltage regulator and governor effects. Many such studies are concerned only with local effects and the remainder of the system can be reduced to an equivalent for which a small or medium-sized computer would be adequate. Thus, scope exists for the employment of such computers even if access to a network analyser is normally available.

The most frequently occurring routine problems for network analysis are balanced load flow, transient stability and short-circuit investigations; unbalanced conditions comprise a separate and relatively minor group of studies. The method to be described is a general one which provides the basis for the treatment of the three former problems as a comprehensive digital investigation if required and which is inherently suitable for the inclusion of such special effects as listed above. The paper deals in detail only with the load-flow and short-circuit study programmes, but extensive desk-machine studies on small systems have been carried out to prove the method for all three cases.

(2) DESCRIPTION OF METHOD

(2.1) General

Early approaches to the digital solution of load flows[3, 4] employed mesh methods of analysis which were difficult to code in system terms. Succeeding investigations have generally established the superiority of the nodal iterative approach, with various means of solution of the linear equations involved according to the computer and the subroutines available, e.g. triangulation,[5] successive or simultaneous adjustments of the nodal voltages[6–8] and relaxation.[9] Short-circuit[10, 11] and transient-stability studies[12] have previously been considered as

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separate investigations using reduced networks in which physical correspondence of coefficients is destroyed.

The present paper advocates the use of an improved form of the Gauss–Seidel iterative process,[13] described in Section 10.3, for the solution of the nodal equations under all circuit conditions. The overall process is thus always iterative, and appropriate restraints can be continuously incorporated in the solution. By this means two primary requirements for routine programmes on smaller computers are fulfilled, namely: scaling for fixed-point computation is reduced to a minimum and programming made correspondingly easier; and network coefficients are unaltered. Thus physical correspondence is retained and data changes involve a minimum of disturbance and recomputation. This feature is further facilitated by the method adopted for the representation of tapped transformers during load flows.

The explicit use of Kron's connection matrix technique[14] for the formulation of the nodal equations and for the evaluation of the branch currents involves either the wasted storage associated with large matrices containing many zero elements or the extra time and programming effort of some interpretive scheme such as that described by Bennett.[3] Considerable simplicity and saving of time is achieved by supplying topographic information in the form of the sending-end and receiving-end numbers associated with a defined direction of flow for each branch. The formulae of Section 10.1 show that such information is sufficient for all purposes. This method of definition has also been used successfully for the formulation of other types of network equations, e.g. the differential equations for the determination of network transient recovery voltages.

Arithmetic in the programmes described is carried out in complex form using Cartesian co-ordinates with subroutines for conversion to polar form for output purposes. Such conversion and reconversion would be essential during the integration of the swing equations of a transient stability investigation.

(2.2) Load-Flow Studies

The equations of the nodal iterative method of analysis for network load flows are given in Section 10.1 and those for the representation of tapped transformers in Section 10.2. The method requires the definition of the basic load-flow problem in terms of:

(*a*) Nodal admittances, computed from branch impedance and topographic data.

(*b*) Loads and lumped susceptances at all busbars.

(*c*) Active and reactive power generation at all busbars except the reference busbar, B_0, which must take up the residue of generation and losses.

(*d* Voltage of the reference busbar to neutral, V_0.

(*e*) Transformer taps.

The network driving functions, i.e. the nodal currents $\boldsymbol{I}$, can be evaluated for an assumed or existing voltage distribution as shown and a better voltage distribution obtained by the solution of the equations $\boldsymbol{Yd} = \boldsymbol{I}$ for the difference voltages $\boldsymbol{d}$, there being n such equations for an $(n + 1)$ busbar system $B_0, B_1 \ldots B_n$. The process can be repeated, using the resulting voltage distribution to provide new starting values, until successive solutions agree to within a given tolerance, when the process is said to have converged.

The improved Gauss–Seidel method of solution of a set of linear equations with fixed driving functions can also be continued to a state of convergence. The load-flow programme, however, uses one application of this process to effect a better solution of the nodal equations at each step of the system iterations, the combined effect being an overall iterative process. Zero starting values are assumed for the difference voltages, i.e. all busbars are assumed to be at reference voltage, unless the results of a previous similar study are available, and the process is continued until a satisfactory convergence is obtained.

At each stage of the iterative process an error quantity, ϵ, is computed and compared with a specified tolerance, τ. Thus at the $(m + 1)$th stage,

$$\epsilon^{(m+1)} = [\epsilon_k^{(m+1)}]_{max} \quad k = 1, 2 \ldots n$$

where $$\epsilon_k^{(m+1)} = |d_k^{(m+1)} - d_k^{(m)}|$$

Convergence is complete when $\epsilon \leqslant \tau$, at which stage busbar voltages, branch currents, branch power flows and busbar power balances are evaluated and printed. The tolerance, τ, is an empirical criterion only and may vary with the system, the ultimate criterion being the permissible errors in the busbar power balances.

A complete system investigation usually requires the comparison of a variety of system operating conditions, each of which will be obtained as an optimum load balance, arrived at after a number of trial solutions with intervention by the system engineer between solutions to vary the generation or transformer taps, etc., in order to achieve the required state. It is uneconomic to specify fine tolerances for such intermediate studies and further time is saved in that the form of output developed permits ready interpretation of the load-flow results without transcription to a circuit diagram.

Conditions of convergence of the process and the use of acceleration techniques are discussed in Section 5.

(2.3) Short-Circuit Studies

The previous method of voltage definition, i.e. variable busbar difference voltages relative to a fixed reference busbar, can no longer be used unless the fault is invariably applied at busbar B_0, which is an unacceptable restriction involving redefinition of the system for each alternative fault location. This is avoided and the generality of the method preserved by replacing the difference voltages $\boldsymbol{d}$ by the busbar voltages to neutral, V_0, $V_1 \ldots V_n$, as the variables of the modified set of $n + 1$ nodal equations derived in Sections 10.1.2 and 10.2:

$$\boldsymbol{Y'V} = \boldsymbol{y_g E}$$

The nodal admittance coefficients, $\boldsymbol{Y'}$, correspond to a square matrix of order $n + 1$ which differs from the $\boldsymbol{Y}$ coefficients used for the load flow by: the addition of the elements of a row and column to allow for the reference busbar; the inclusion in the diagonal (self) admittances of lumped susceptances, generator admittances and loads, using for the latter a constant impedance representation which is automatically computed from the pre-fault load-flow conditions at each busbar; and the inclusion of the effects of transformer taps in appropriate self and mutual coefficients.

The new driving functions $\boldsymbol{y_g E}$ are constant, computed from the pre-fault generator terminal conditions and a list of appropriate generator impedances, the latter being sub-transient values for circuit-breaker ratings or transient values for protective gear applications and transient stability studies. This computation corresponds to the replacement of the specified constant power generations by equivalent voltages maintained behind appropriate generator impedances.

The modified nodal equations, which are now exact, i.e. they can be solved by a single application of any direct method, therefore represent the network in the pre-fault condition and their solution would give the voltages $\boldsymbol{V}$ already determined by the load flow. The solution of the same equations by means of

ie Gauss–Seidel process with one of the voltages, V_f, con-
nuously maintained at zero corresponds to the application of a
hort-circuit at busbar B_f. Zero starting values are assumed for
l voltages and the process is continued to convergence, generally
a wider tolerance than that accepted for the load flow. The
hort-circuit level and the infeeds to the faulted busbar are then
omputed and printed. The procedure may be repeated for any
umber of busbars to be faulted in turn, no reorganization of
ie coefficients being required.

Faults at locations other than busbars are allowed for by the
reation of fictitious nodes during the system definition, and
aults through impedance by modifications of the appropriate
elf-admittance coefficient with no applied fault restraint during
ie solution. The application of simultaneous faults is par-
cularly simple using the iterative method.

Short-circuit studies can be developed, alternatively, to follow
oad-flow studies with no intervention or as a separate programme
equiring the input of the converged difference voltages from the
oad flow. The choice depends on the type of computer and
he programme requirements, the former method being adopted
or the high-speed computer studies, where input and output
epresent an appreciable proportion of the study time, and the
atter method for the lower-speed computer to permit greater
lexibility.

(2.4) Transient Stability Studies

There are many arguments in favour of system reduction and the use of available integration subroutines for the digital solu-
ion of the swing equations, and this is perhaps the best approach for straightforward transient stability studies on large systems using high-speed computers. The use of low-speed computers for transient stability studies, however, is likely to be limited by the time factor to smaller equivalent systems, with perhaps the inclusion in detail of various secondary effects, in which circumstances it will be advantageous to retain the identity of busbars and branches. Programme development for this purpose might well be on the lines of the present network analyser procedure using a step-by-step integration technique with intermediate solutions of the network by the iterative process, incorporating restraints as required. The solutions at each step would provide good starting values for the next step and satisfactory accuracy could be obtained by variation of the integration time interval in conjunction with the tolerance of convergence.

It is felt that, with the advent of simulator techniques for the automatic solution of the transient stability problem,[15] routine studies of this type will continue to be carried out on network analysers thus equipped for some considerable time.

(3) SAMPLE SYSTEM STUDIES

(3.1) Description of Computer

The following information concerning the computer used for the studies is given to provide a basis of comparison for the solution times quoted.

The computer is a general-purpose machine using a one-address instruction system with facilities for address and instruction modification by means of eight quick-access 'B' stores. Main storage is by a magnetic drum whose capacity is 4096 words organized as 128 tracks of 32 words each. The word length is 32 bits, i.e. 10-decimal equivalent, with a double-length accumulator, and normal working is in fixed binary point with a maximum number magnitude less than one-half. If optimum programming is used, a period of 3·5 millisec is required for the operations of addition, subtraction and transfer, and 20 millisec for multiplication; otherwise, 10 millisec mean access time should be added to each instruction, this being the case for large general programmes of this type. Division is by subroutine and requires approximately 1 sec.

Input is by 5-hole paper tape read at 250 characters per second. Output is by standard equipment, either punched paper tape at 25 characters per second or directly from a teleprinter at 7 printed characters per second. All tapes utilize a preset parameter facility whereby temporarily faulty storage locations are omitted from the storage scheme.

(3.2) Problem Preparation

The preliminary preparation of system problems for digital analysis differs only in small details from that for a network analyser investigation. The procedure is:

(*a*) The system is defined by a one-line diagram as in Fig. 1, transmission lines being represented by their series impedances with susceptances lumped at busbars. The maximum size of

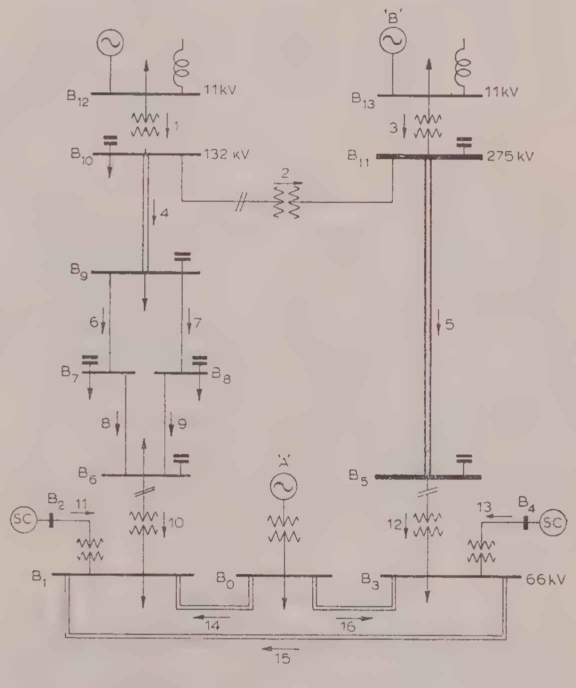

Fig. 1.—Definition of sample system.

network which can be solved using the present programme is 16 busbars, including B_0, and 32 branches of which 16 may be tapped transformers. There is no restriction on the number of branches in parallel between two busbars.

(*b*) A reference busbar, B_0, is chosen and the remaining busbars numbered sequentially to B_{13} for the 14-busbar system shown. The reference busbar must be able to provide the residue of system generation and losses. (Factors which affect the choice of B_0 are discussed in Section 5.)

(*c*) Branches are numbered consecutively, 1 to 16 as shown, without reference to the busbar numbers.

(*d*) Branch flow directions are allocated, thus specifying explicitly the *s* and *r* numbers associated with each branch. All flows are calculated with respect to the defined directions. For branches which are tapped transformers the flow directions must be such that the taps are represented at the sending end.

Table 1

System Data for Sample Study

Busbar data							Branch data					
Number	b	P_L	Q_L	P_g	Q_g	X''_d	Number	B_s	B_r	r	x	t
0	0	0·3925	0·2540	—	—	0·392	1	12	10	0·0356	0·3960	—
1	0	0·0925	0·0588	0	0	—	2	10	11	0·0412	0·6680	—
2	0	0	0	0	0·0825	2·096	3	13	11	0·0340	0·3484	—
3	0	0·2900	0·1850	0	0	—	4	10	9	0·1312	0·3860	—
4	0	0	0	0	0·1063	1·720	5	11	5	0·0456	0·3340	—
5	0·1602	0	0	0	0	—	6	9	7	0·3300	0·9688	—
6	0·0219	0·0255	0·0163	0	0	—	7	9	8	0·3500	1·0152	—
7	0·0121	0·0128	0·0080	0	0	—	8	7	6	0·3508	1·0400	—
8	0·0509	0·0458	0·0288	0	0	—	9	8	6	0·3020	0·9108	—
9	0·0236	0·0308	0·0190	0	0	—	10	6	1	0·0328	0·3108	—
10	0·0103	0·0253	0·0163	0	0	—	11	2	1	0·0228	0·5780	—
11	0·1602	0	0	0	0	—	12	5	3	0·0296	0·1912	−0·01
12	−0·0375	0·0150	0·0093	0·2250	0·0075	0·880	13	4	3	0·0112	0·1024	—
13	−0·1875	0·0300	0·0185	0·4303	0·1131	0·440	14	0	1	0·0460	0·2880	—
							15	3	1	0·0300	0·1876	—
							16	0	3	0·1376	0·8584	—

Reference voltage, $V_0 = 1{\cdot}0$ p.u.
Tolerance, τ, on $|d^{(m+1)} - d^{(m)}| = 0{\cdot}0002$ p.u.
All system quantities are in p.u. on a 400 MVA base.
X''_d = Source sub-transient reactance, resistance neglected.
$r + jx$ = Branch impedance.

(*e*) System data are expressed in p.u. values on nominal voltage bases and a convenient MVA base, chosen so that no series impedance exceeds 0·1 p.u. The maximum impedance which can be represented is 5 p.u. This allows for as wide a range of values as is normally encountered and ensures optimum scaling of the admittances formed within the computer. The data used for the studies on the sample system are shown in Table 1, where equivalent impedances are given for the number of circuits shown in parallel in Fig. 1.

(*f*) Data tapes are prepared on which system quantities are punched after being divided by a common factor of 10, this being the only further requirement for scaling purposes. Other data, such as the *s* and *r* numbers, are punched as integers. Separate tapes can be employed for each block of data, i.e. branch data, susceptances, loads, generation, taps.

(3.3) Sample Load Flow

The system of Fig. 1 has been used throughout the development of the present programmes since it contains all the typical system-elements, including shunt reactance, which is represented by negative susceptance. Further, the voltage distribution in this system is very sensitive to reactive power requirements, owing to the relatively large concentration of shunt susceptance associated with the 275 kV line, and thus unusually onerous conditions of convergence can be imposed during trial balances with various generation estimates.

The load flow programme is read in together with the prepared data tapes and a control tape containing the following information:

(*a*) The tolerance τ. (The result under discussion is an optimum load flow balance, and previous balances had shown that convergence to a tolerance of 0·0002 p.u. would give maximum busbar power-balance errors of the order of 0·5 MVAr which was considered acceptable for this study.)

(*b*) The interval between intermediate print-outs. Present practice is to print the existing solutions for the difference voltages, *d*, the error quantity, ϵ, and the busbar number corresponding to the largest error at intervals of, say, 10 iterations. It is thus possible, with little loss of time, to follow the convergence of the process; moreover, the printed values provide a useful point at which the computation may be stopped overnight

```
SE 20090                         LOAD FLOW                  400 MVA BASE

RUN 3.9                             NET               V         A
                              MW          MVAR       PU        DEG      BUS

 44
-.00090    +.00930            .38          .22      .9991       .53       1
+.04474    +.00780        -   .03     -    .00     1.0448       .43       2
+.00039    +.02246        -   .23          .54     1.0006      1.29       3
+.01118    +.02151        -   .01          .00     1.0114      1.22       4
+.01385    +.09366        -   .27          .04     1.0182      5.28       5

-.00705    +.02665        -   .36          .10      .9933      1.54       6
-.01175    +.08319            .05          .06      .9917      4.81       7
-.00752    +.05929            .05          .05      .9942      3.42       8
-.01686    +.14961        -   .11          .14      .9945      8.66       9
-.02225    +.22323        -   .15          .20     1.0029     12.86      10

-.00167    +.22399        -   .06          .20     1.0231     12.65      11
-.05575    +.30202        -   .02          .00      .9914     17.73      12
-.06932    +.35159        -   .02     -    .01      .9949     20.69      13


      SEND                      RECEIVE               I         A
   MW        MVAR            MW        MVAR          PU        DEG    BRANCH

  83.98    - 15.46         83.32    - 22.81        .215      28.16        1
   1.55    - 12.25          1.53    - 12.50        .031      95.66        2
 160.10    - 36.40        157.78    - 60.12        .413      33.50        3
  71.50    - 12.74         69.78    - 17.80        .181      22.95        4
 159.26    -  5.34        156.49    - 25.59        .389      14.57        5

  25.06    -  6.51         24.50    -  8.15        .065      23.20        6
  32.29    -  9.42         31.29    - 12.33        .085      24.91        7
  19.44    -  6.53         19.06    -  7.64        .052      23.38        8
  13.02    -  3.67         12.88    -  4.09        .034      19.17        9
  21.38    -  9.52         21.34    -  9.95        .059      25.52       10

-   .03      33.00       -   .08      31.56        .079    - 89.62       11
 156.22      40.88        154.40      29.09        .396    -  9.39       12
-   .01      42.52       -   .06      42.07        .105    - 88.80       13
- 12.40       3.24       - 12.42       3.12        .032    -165.37       14
  27.84    -  1.06         27.78    -  1.42        .070       3.47       15

- 10.23       1.46       - 10.27       1.23        .026    -171.88       16


  REF   GEN
  MW        MVAR

 134.37    102.69
```

Fig. 2.—Load-flow output.

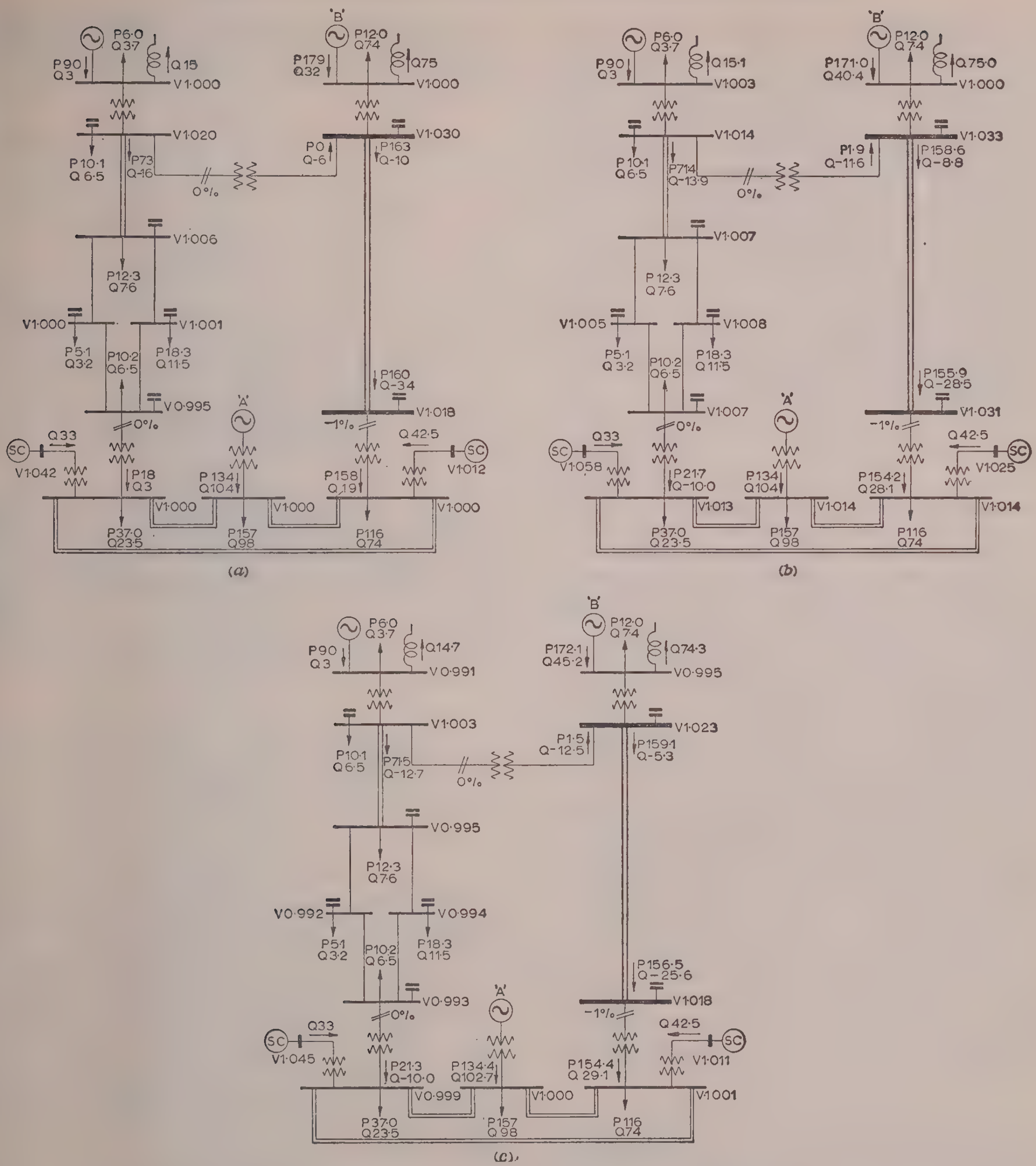

Fig. 3.—Comparison of network analyser and digital load-flow studies.

(*a*) Network analyser result.
(*b*) Digital result: reference 'B'.
(*c*) Digital result: reference 'A'.

or act as starting values in the event of a complete breakdown of the calculation for any reason.

(*c*) Acceleration factors. An empirical accelerating factor, A, can be applied as explained in Section 5. (In the case under discussion $A = 1\cdot 7$.)

(*d*) The maximum number of iterations to be performed before computation stops even if convergence is not complete. This may be necessary if, for instance, an unstable voltage condition has been posed in the data and the computation is completely unsupervised.

Fig. 2 is a reproduction of the final printed output for a load-flow study on the sample system using the given data. The number 44 immediately below the run reference number is the number of iterations required to converge from zero start-

ing values for the difference voltages to a tolerance of 0·0002 p.u., and the first two columns of the busbar data are the p.u. Cartesian co-ordinates of the converged difference voltages. Angles are given in degrees relative to the specified reference busbar voltage, and currents given for branches representing tapped transformers are the sending-end values. It should be noted that the generation at the reference busbar is that required to balance the net power flows, hence there is no busbar check available for B_0. The result of this study is also given in system diagram form as Fig. 3(*c*).

The overall study time for the results of Fig. 2 was approximately 2·3 hours divided as follows:

	min
Programme input	10
Data input and scaling	1
Coefficient formulation	2
44 iterations (2·5 min each)	110
4 intermediate prints (1 min each)	4
Computation of branch flows and balances	4
Final print	10
	141

Subsequent load-flow studies require no further programme input and take less time to converge, depending on the extent of the change to be made to the system. The time taken to obtain a new solution for a system condition which differs only in minor detail from an existing solution, however, remains the chief disadvantage of digital network analysis even though the iterative approach and the method of tap representation eliminate the need for further operations on the coefficient matrix.

Changes of system data, i.e. reference voltage, generation, loads and taps, are carried out by overwriting the appropriate previous data with the new values and proceeding with the first iteration. Sixteen further iterations were required for convergence from the result of Fig. 2 to a new balance corresponding to a tap change from −1 to −2% on branch 12, i.e. 55 min overall study time with a full print-out.

Network data changes, involving changes of branch impedance other than the removal of the only branch between two busbars, are carried out by the input of the new branch impedances and reformulation of the nodal coefficients. The effect on the last balance, i.e. with −2% tap on branch 12, of eliminating one of the two circuits comprising branch 4 required 24 iterations, i.e. 78 min with full print-out. The complete removal of branches can be treated alternatively by the renumbering of branches, which is facilitated by the initial allocation of high branch numbers to those to be removed or by overwriting the appropriate admittances with zeros. The latter method preserves the identity of the branch numbers and is to be preferred unless a large number of such changes are made simultaneously.

(3.4) Sample Short-Circuit Study

Short-circuit studies commence with the input of the programme tape followed by the same data tapes as were prepared for the corresponding pre-fault load flow together with generator impedance data, the list of busbars to be short-circuited and finally the converged difference voltages and the reference generation from the load-flow result.

Fig. 4 is a reproduction of the standard output for a short-circuit study based on the data of Table 1 and the load-flow result shown in Fig. 2, for faults applied at busbars B_{11} and B_5 in turn. The information printed for each fault case is:

(*a*) The number of iterations to convergence. (An accelerating factor of 1·4 was applied in each case and the tolerance used was 0·002 p.u., i.e. 10 times that for the load flow study.)

(*b*) The MVA short-circuit level.

(*c*) The p.u. fault-current contributions for all branches connected to the faulted busbar, including generator contributions which are zero in the cases shown. These currents are given as infeeds at the faulted end of each branch, their angles being relative to the previously specified V_0.

```
SE 20090              S/C   STUDY                        400 MVA BASE
RUN 3.9

                  BUS 11   1121 MVA                    17 ITERATIONS

            BRANCH          I PU        A DEG

               2            .584       - 69.23
               3           1.293       - 56.65
               5           1.008       - 87.60
            GEN.            .000           .00

                  BUS  5   1079 MVA                    14 ITERATIONS

            BRANCH          I PU        A DEG

               5           1.119       - 58.77
              12           1.648       - 85.10
            GEN.            .000           .00
```

Fig. 4.—Short-circuit study output.

Alternative output schemes may be considered necessary for particular protective gear applications, e.g. selected busbar voltages and currents in back-up zones could be printed following the input of a data tape to specify the requirements. For smaller systems, however, it is sufficient to select either a standard output scheme as described above or to output all busbar voltages and branch currents. No intermediate prints are considered necessary for short-circuit studies.

The time occupied by the short-circuit study was approximately 1·2 hours, divided as follows:

	min
Programme input	10
Data input and scaling	1
Formulation	3
31 iterations (1·9 min each)	59
2 prints (1·5 min each)	3
	76

(3.5) Study Times

It has been found that a very approximate comparison of the overall times for similar studies on networks of different sizes is given by the ratio of the number of busbars in each.

The times quoted above should be considered in relation to the details of the computer given in Section 3.1. The computation times for the same studies on a large high-speed computer using automatic programming techniques were reduced by factors of up to 50 : 1. After allowing for input and output, the overall reductions in study times are of the order of 15 : 1. The relative installed costs of the low- and high-speed computers and a large modern network analyser, respectively, are of the order of 1 : 7 : 5.

(4) COMPARISON OF NETWORK ANALYSER AND DIGITAL PROCEDURE

Since the need for and the scope of both methods of analysis have been previously stated, it is not intended to make comparisons between the merits of the methods but rather to compare the manner in which similar studies would be carried out using either a network analyser or a digital computer. For this purpose Table 2 shows the corresponding steps of both procedures.

The checks referred to in the digital studies procedure of Table 2 are carried out only for the first studies after programme

input, or if the programme has remained on the magnetic drum for some days since the last studies. The checks are made by means of test tapes containing data for a 4-busbar 4-branch

Table 2

COMPARISON OF NETWORK ANALYSER AND DIGITAL STUDY PROCEDURES

Network analyser	Digital computer
Load-flow study	
Reduce to equivalent circuit form and prepare p.u. data	As for analyser
Prepare plugging diagram and unit settings	Prepare coding diagram and data tapes
Plug-up and check	Programme input and check
Set units and loads	Data input, formulation of $\boldsymbol{Y}$
Trial balances by operator to system engineer's specification leading to optimum balance	Trial balances by computer to system engineer's specification leading to optimum balance
Record results	Final output and transcription
Short-circuit study	
Set generator impedances and reset outputs, record internal voltages	Programme input and check, data input, formulation of $\boldsymbol{Y}'$ computation of $y_g E$
Reduce analyser base voltage, apply fault and reset internal voltages	Solution of the nodal equations with the fault restraint applied
Record results	Output
Repeat previous 2 steps for another fault location	As for analyser

system. The results of either a load-flow or short-circuit study, as required, are subjected to a sum check after one iteration, the computer printing out the correctness or otherwise of both this test and the results of a further test of the unused storage locations up to the maximum capacity of a 16-busbar 32-branch system. The time taken is approximately 7 min for both the load-flow and short-circuit study programmes.

The chief difference between the use of the two methods for load-flow studies is due to the different manner of specifying the restraints, i.e. the balancing conditions. Whereas the greater accuracy of a digital solution is rightly considered to be of little importance in most studies, the fact that the digital method requires the reference busbar to supply all system losses means that, if all other inputs are equal, the generation at a reference busbar includes the net sum of the analyser errors and may often be quite different in the two solutions. The consequent voltage distribution in the digital method may thus vary appreciably in a sensitive system according to the choice of reference busbar, even though the remaining system data agree with those for a network analyser solution of the same problem. These remarks are illustrated in Fig. 3, which shows one analyser and two digital solutions of essentially the same system problem, which is that used for the sample studies.

Fig. 3(*a*) is the analyser solution achieved by modifying the generator reactive outputs to maintain certain selected voltage levels. (The results of this study provided the data shown in Table 1.) Fig. 3(*b*) shows the results of a digital study with station 'B' as reference ($V_0 = 1$) and other generation as for Fig. 3(*a*). The resulting generation at station 'B' is 171 MW, 40·4 MVAr compared with 179 MW, 32 MVAr in the analyser balance. The voltage level at the main load busbars in the area of station 'A' is of the order of 1·4% higher in the digital solution. An unsuccessful attempt was made to obtain a digital solution with station 'A' as reference ($V_0 = 1$) and other generation as for the analyser study, the system being critically unstable owing to a lack of reactive power, indicated by a steadily decreasing voltage level with increasing error, ϵ. Fig. 3(*c*) shows the results of one of several balances which could be obtained by increasing the reactive power generation at station 'B', where the input is 172·1 MW, 45·2 MVAr in this case. This study corresponds exactly to the sample data of Table 1 and the computer output of Fig. 2. The resulting voltage level at the main loads is a closer approximation to the analyser balance than that of Fig. 3(*b*).

These studies show the difficulties encountered in attempting to duplicate network-analyser balances, but it should be borne in mind that the chief aim of both types of analysis is to obtain comparative studies for different system conditions. The digital method appears highly suitable for such work, and the fact that its errors are not of a random nature but likely to be repeated in the same sense for successive studies may be considered to be an advantage in this respect. There is, however, undoubtedly a need to build up a background of experience in the use of the digital method with many types of system before general conclusions can be drawn.

(5) CONVERGENCE AND ACCELERATION

The criterion of convergence stated in Section 10.3 for the improved Gauss–Seidel process alone, i.e. as used for the solution of the short-circuit equations, appears to be satisfied by both sets of coefficients $\boldsymbol{Y}$ and $\boldsymbol{Y}'$ for normal networks. Athough the corresponding criterion for the overall iterative process used during load flows is difficult to formulate, it would again appear that convergence is assured for all normal system conditions. The definitions of 'normal' as used in these senses are: a normal network is one in which there is no tendency to resonance between inductive and capacitive elements to cause appreciable reduction of the nodal self-admittances; and by normal system conditions is meant the specification of sufficient reactive power generation to prevent the occurrence of voltage instability.

In connection with the first definition it may be noted that the use of the method to solve the exact equations for the distribution of the harmonic voltages in a network with large shunt capacitances for p.f. correction revealed an expected tendency to a divergent solution at frequencies such that some diagonal terms were appreciably reduced even though resonance did not occur.

An indication of the relative stability of the process is given by the curves of Fig. 5, which show the amounts of artificial acceleration that could be applied during the sample load-flow and short-circuit studies, respectively, in order to reduce the number of iterations required for convergence to a given tolerance. The curves show the effect of a factor A used to obtain an improved solution x by accelerating the differences between the results of an iteration $x^{(m+1)}$ and the starting values $x^{(m)}$. Thus $x = Ax^{(m+1)} - Ax^{(m)} + x^{(m)}$, where $x = d$ for load flows and $x = V$ for short-circuit studies. $A = 1$ corresponds to no acceleration, and the curves show reductions in the study time for values of A up to about 1·7 [Fig. 5(*a*)] and 1·4 [Fig. 5(*b*)]. Further increases cause uncertain behaviour of the errors, leading to slower convergence, and large values of A create artificial instability.

Experiments have been made with other methods of acceleration, e.g. variation of A during the process, different factors for real and imaginary parts, and delays before the application of the factors, but no significant improvement was obtained over the method described. If such iterative approaches are employed

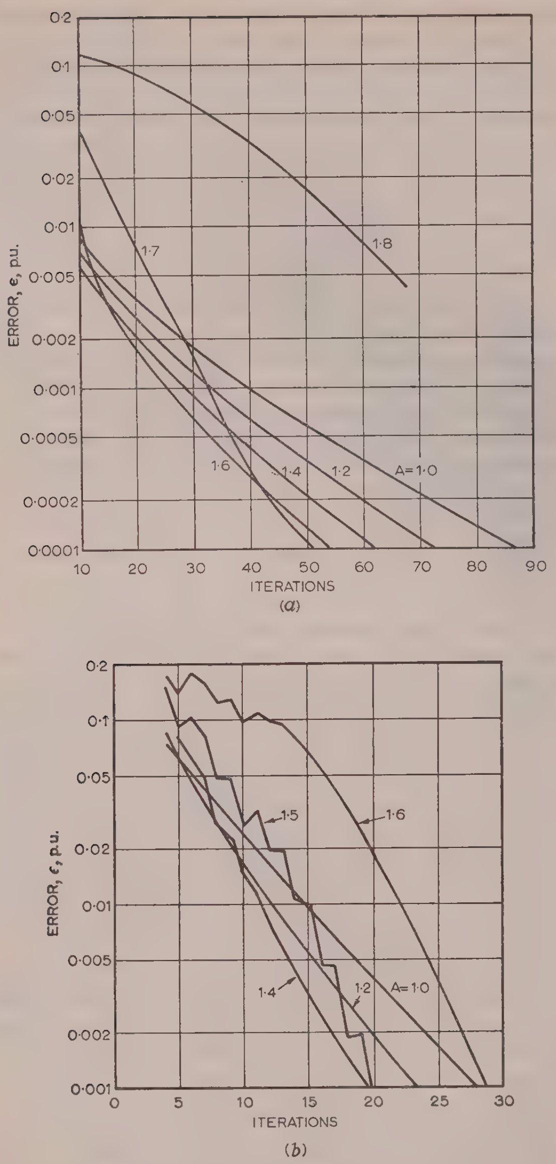

Fig. 5.—Variation of accelerating factor A.
(a) Load flow.
(b) Short-circuit.

for the solution of very large networks, further investigations concerning acceleration may be worth while, in particular the Aitken method,[7] although in this case care may be necessary to delay its application to avoid the oscillation of the variables which tends to occur during the early stages.

The rate of convergence for load flows is, to some extent, also affected by the choice of reference busbar. The fact that the latter either lags or leads most of the remaining busbars tends to create more uniform difference-voltage convergence patterns. It is suggested that, where possible, receiving-end busbars might be chosen as reference with standard accelerating factors of 1·6 for load flows and 1·4 for short-circuit studies. These recommendations may be subject to revision in the light of further experience.

The curves of Fig. 6 indicate the variation of the accuracy of the results with convergence to diminishing tolerances for the sample system. Tolerances of 0·0020 p.u. for intermediate and 0·0002 p.u. for final load-flow studies were considered acceptable, although the busbar balance errors in the latter case can be reduced by a factor of 2 if required by specifying a tolerance of 0·0001 p.u. at the expense of approximately 8 more iterations, which occupy 20 min. A tolerance of 0·0020 p.u. was considered acceptable for all short-circuit studies.

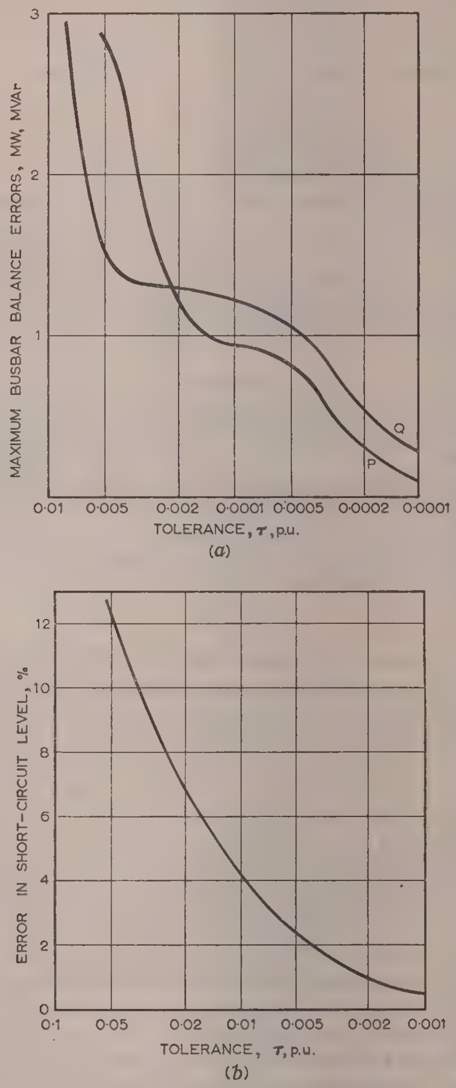

Fig. 6.—Convergence.
(a) Load flow.
(b) Short-circuit.

(6) SOME PROGRAMME DETAILS

Fig. 7 is a schematic of the load-flow programme which show the sequence of the operations previously described. Tw programming devices of interest are:

(a) To avoid repetition of the time-consuming division subroutin which is the final operation for the solution of each differenc voltage, at each step of the iterative process, the diagonal coefficien Y_{kk} are inverted outside the iterative loop to become Y_{kk}^{-1}.

(b) To avoid the use of a square-root subroutine, another relativel slow operation, the square of the convergence tolerance, τ^2, is forme for comparison with the maximum of the square of the error quant ties, ϵ^2, since the latter must be computed during the formation c the modulus of a complex quantity.

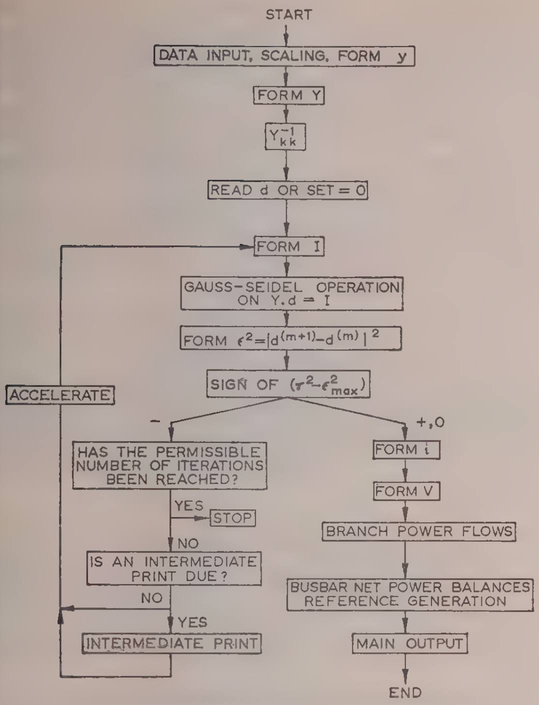

Fig. 7.—Schematic of load-flow programme.

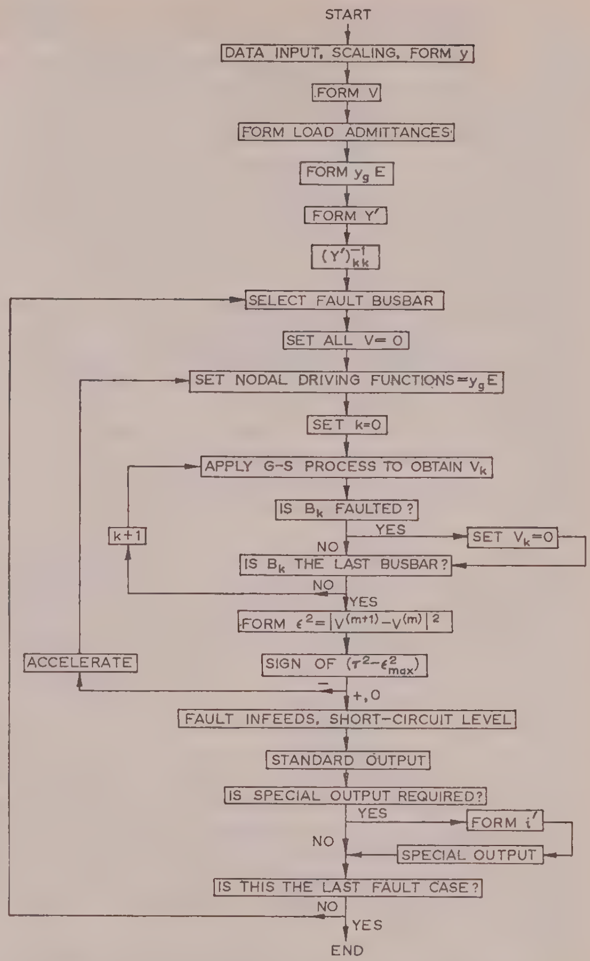

Fig. 8.—Schematic of short-circuit study programme.

Care was taken to provide storage space for the complete $(n+1)$th-order nodal coefficient matrix Y, even though the reference row and column are not needed for load flows. This is necessary as Y is formulated by scanning the s and r branch numbers where B_0 occurs. In fact, time would be wasted by the insertion of instructions to prevent the transfer of the admittance of branches connected to the reference busbar.

Fig. 8 is a schematic of the separate short-circuit study programme in which it can be seen that the modification of the nodal coefficients for shunt branches and tapped transformers to give Y' is carried out during their formulation. The diagonal coefficients Y'_{kk} are again inverted before the main solution, which differs from the load-flow process by the inclusion of the fault restraint.

Table 3

Computer Scaling of System Quantities

Quantity	Load-flow study		Short-circuit study
	Ratio of quantities computer/system	Permissible range of modulus of system quantity	Ratio of quantities computer/system
p.u.			
z	10^{-1}	0·1–5	10^{-1}
y	10^{-2}	0·2–10	10^{-2}
Y	10^{-2}	0–50	10^{-2}
Y^{-1}_{kk}	$2^{-10} \times 10^{-2}$	0·02–5	$2^{-10} \times 10^{-2}$
b	5×10^{-2}	0–10	10^{-2}
d	5×10^{-1}	0–1	10^{-1}
V	10^{-1}	0–5	10^{-1}
I	5×10^{-3}	0–200	10^{-3}
P, Q*	10^{-3}	0–500	2×10^{-4}
P, Q†	5×10^{-4}	0–1 000	—
i	5×10^{-3}	0–200	10^{-3}
t	10^{-1}	0–0·2	10^{-1}
θ (deg)	10^{-3}	0–180	10^{-3}

* Loads and generation.
† Branch flows and net balances.

Table 3 shows the fixed scaling arrangements adopted for the system quantities within the computer for both programmes. The per-unit values punched are all scaled down by a common factor of 10 for convenience, and the computer scaling is automatically applied to each block of data either as it is taken in or during its formation. Scaling is determined by the maximum values of the quantities to be represented by computer numbers of less than $\frac{1}{2}$, in conjunction with the requirement that a minimum number of scaling factors shall be associated with individual calculations during the programme. For example, the nodal equations for the load flow are (in p.u. form):

$$Yd = I$$

which can also be written

$$\frac{Y}{100}\frac{d}{2} = \frac{I}{200}$$

This equation in scaled computer quantities from Table 3 is thus

$$Y_c d_c = I_c$$

Some points of particular note in Table 3 will now be considered.

Whereas a scaling of $0 \cdot 1d$ per unit as used in the short-circuit programme is generally convenient, it is important to retain a maximum number of digits for the difference-voltage variables during load-flow studies to obtain accurate branch flows. Hence a scaling of $0 \cdot 5d$ per unit is used for the load flows with corresponding scaling for the other quantities associated directly with the iterative process. This scaling actually permits maximum values of ± 1 p.u. for both the real and imaginary parts of the difference voltages and is the optimum scaling to retain the maximum number of digits for d, while providing a sufficiently wide range of variation of busbar voltages and angles relative to reference for general studies.

A minimum value of $0 \cdot 1$ p.u. impedance is fixed by the choice of the MVA base, hence the maximum value of 5.

The admittance scaling permits a maximum nodal self-admittance coefficient of 50 p.u., which is the equivalent of five minimum impedance branches connected to the same node. Modification of the diagonal coefficients for shunt branches to obtain the Y coefficients may slightly affect these considerations, although load and generator admittances are generally lower than branch values and susceptance generally reduces the magnitude of the reactive part.

The unusual scaling factor associated with the inverse diagonal admittances, Y_{kk}^{-1}, is due to the fact that binary shift instructions are incorporated in their formation and also in the evaluation of d, and hence powers of 2 can be conveniently incorporated.

Two apparently different scalings are employed for power quantities during load flows, the factor of 5×10^{-3} for branch flows and power balances being the optimum arrangement to eliminate the use of separate factors in equations. The other factor of 10^{-4} is due to the fact that, after a net power input is evaluated for each busbar from the connected load and generation, the most frequent use of this quantity is as the numerator of an expression for the determination of the nodal currents during the iterative loop and the available division subroutine performs the operation $(x/2y)$. Thus a factor of 2 in the denominator makes the effective scaling the same as for the branch power quantities.

The permissible maximum values of parameters are fixed in some cases by the values of the parameters themselves, e.g. z, and in other cases by certain restrictions in the subroutines in which they are used, e.g. t, θ.

The s and r numbers associated with each branch are not scaled but are punched as integers for input and shifted by the programme into a digit position, where the result of arithmetic on them is immediately available as an address, e.g. for the purposes of eqns. (3).

The present programmes allow only for in-phase transformer taps, and thus the Y' matrix is symmetrical. (The Y matrix is inherently symmetrical owing to the iterative representation of both in-phase and quadrature taps.) Advantage is taken of the symmetry to reduce the storage required for the admittance coefficients which are stored as a triangular matrix, successive tracks corresponding to columns to facilitate the address system. This is preferred to the use of a serial storage system although the latter can be more economic. The overall storage utilized by the programmes to accommodate the largest systems, i.e. 16 busbars, 32 branches, is divided as follows:

	Tracks	
	Load flow	*Short-circuit*
Computer input and standard subroutines ..	23	23
Programme, including output subroutine ..	52	49
System data	12	12
Working space including nodal coefficients..	14	19
	101	103

(7) CONCLUSIONS

A method of digital network analysis has been presented whi should encourage the use of lower-speed digital computers f the routine network analysis of small- to medium-sized system Such problems occur on a day-to-day basis in the field of pow systems engineering, in circumstances where the speed of soluti may not be important if access is available to computation facilities requiring little supervision.

Ease of interpretation of intermediate and final study resul by means of planned output schemes such as those described considered to be of prime importance in the routine use of a method of digital network analysis.

Further developments in the application of digital compute in this field will undoubtedly be directed towards self-balanci programmes whereby large high-speed computers may succes fully compete with analogue methods for many classes problem. It is hoped that further experience in the use of t method described will demonstrate its inherent flexibility to such that it will prove suitable for use as a basis for su programmes.

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(10) APPENDICES

(10.1) Network Equations

The equations given in this Section refer to balanced networks with no mutual coupling between branches and with nominal fixed transformer ratios, the representation of tapped transformers being considered in Section 10.2.

(10.1.1) Load-Flow Studies.

A section of a typical system network with constant current sources is shown in Fig. 9(*a*). The nodal equations which represent this section are

$$\left.\begin{aligned}(y_b + y_c)d_j - y_b d_k &= I_j\\ -y_b d_j + (y_a + y_b)d_k &= I_k\end{aligned}\right\} \quad (1)$$

The general nodal equations for an $(n+1)$ busbar system ($\mathrm{B}_0, \mathrm{B}_1, \ldots \mathrm{B}_n$) are characterized by the matrix equation

$$\boldsymbol{Yd} = \boldsymbol{I} \quad (2)$$

$\boldsymbol{Y}$ is an *n*th-order symmetrical matrix whose elements can be formulated by clearing the appropriate storage locations and scanning the branch data to perform the operations

$$\left.\begin{aligned}Y_{rr} &= Y_{rr} + y_a\\ Y_{rs} &= Y_{rs} - y_a = Y_{sr}\\ Y_{ss} &= Y_{ss} + y_a\end{aligned}\right\} \quad (3)$$

for each branch in turn, where y_a is the branch admittance and s_a and r_a the particular sending-end and receiving-end busbar numbers.

The elements of the nodal current matrix $\boldsymbol{I}$ are related to the existing voltage distribution as shown in Fig. 9(*b*) by the equations

$$V_k = V_0 + d_k \quad (4)$$

$$\left.\begin{aligned}I_{gk} &= \left(\frac{P_{gk} + jQ_{gk}}{V_k}\right)^*\\ I_{lk} &= -\left(\frac{P_{lk} + jQ_{lk}}{V_k}\right)^*\\ I_{bk} &= -jV_k b_k\\ I_k &= I_{gk} + I_{lk} + I_{bk}\end{aligned}\right\} \quad (5)$$

Following the solution of eqn. (2) for the difference voltages $\boldsymbol{d}$, the branch currents and flows are

$$i_a = y_a(d_s - d_r) \quad (6)$$

$$\left.\begin{aligned}P_{sa} + jQ_{sa} &= V_s i_a^*\\ P_{ra} + jQ_{ra} &= V_r i_a^*\end{aligned}\right\} \quad (7)$$

The residual generation at the reference busbar is given by

$$\begin{aligned}(P_{g0} + Q_{g0}) &= (P_{l0} + Q_{l0}) - jV_0^2 b_0\\ &\quad + \Sigma(\text{branch power sent}) - \Sigma(\text{branch power received})\end{aligned} \quad (8)$$

The net busbar power balance is termed positive when the total power leaving the busbar exceeds the input, i.e. the net values correspond to the additional generation required at that point for balance:

$$\begin{aligned}(P_\kappa + jQ_k)_{net} &= \Sigma(\text{branch power sent}) - \Sigma(\text{branch power received})\\ &\quad + (P_{lk} + jQ_{lk}) - (P_{gk} + jQ_{gk}) - jV_k^2 b_k\end{aligned} \quad (9)$$

(10.1.2) Short-Circuit Studies.

Fig. 10(*a*) shows the network section of Fig. 9(*a*) with the constant-current sources replaced partly by constant-voltage sources, *E*, behind generator admittances, y_g, and partly by constant shunt impedances, y_n. The nodal equations representing this section are

$$\left.\begin{aligned}&(y_a + y_c + y_{g0} + y_{n0})V_0 && -y_c V_j && -y_a V_k = y_{g0}E_0\\ &-y_c V_0 && + (y_b + y_c + y_{gj} + y_{nj})V_j && -y_b V_k = y_{gj}E_j\\ &-y_a V_0 && -y_b V_j && + (y_a + y_b + y_{gk} + y_{nk})V_k = y_{gk}E_k\end{aligned}\right\} \quad (10)$$

The general equations for an $(n+1)$ busbar network using this method of representation are

$$\boldsymbol{Y'V} = \boldsymbol{y_g E} \quad (11)$$

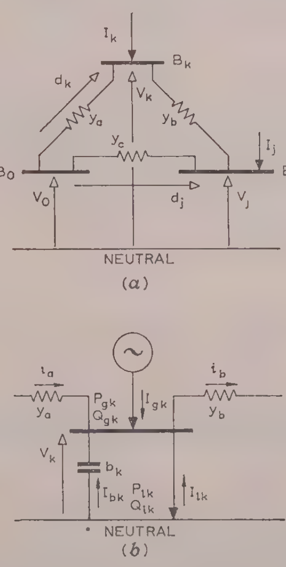

Fig. 9.—Network definition for current sources.

(*a*) Typical network section.
(*b*) Typical busbar B_k.

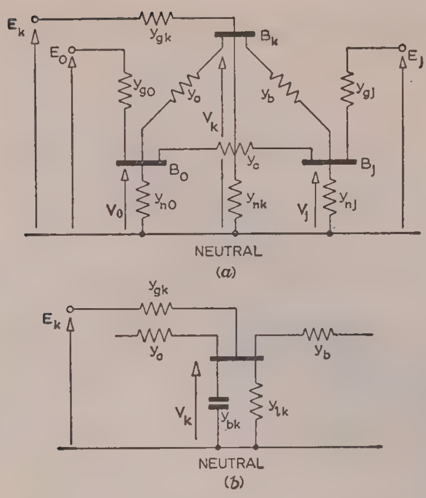

Fig. 10.—Network definition for voltage sources.

(a) Typical network section.
(b) Typical busbar B_k.

The elements of the matrix Y' are those of Y, after allowing for the reference busbar row and column, with the following modifications to the diagonal elements from Fig. 10(b):

$$Y'_{kk} = Y_{kk} + y_{gk} + y_{nk} \quad . \quad . \quad . \quad . \quad (12)$$

$$\left.\begin{aligned} y_{nk} &= y_{bk} + y_{lk} \\ y_{bk} &= jb_k \\ y_{jk} &= \frac{P_{lk} - jQ_{lk}}{V_k^2} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The driving voltages E are related to the pre-fault generated powers by

$$E_k = V_k + \frac{1}{y_{gk}}\left(\frac{P_{gk} + jQ_{gk}}{V_k}\right)^* \quad . \quad . \quad . \quad (14)$$

Following the solution of eqn. (11) for a fault at busbar B_f, i.e. with the restraint $V_f = 0$, the fault in-feeds are computed for each branch connected to B_f, irrespective of the previously defined directions of branch flow. For example, for a branch connected between busbars B_k and B_f,

$$i'_a = y_a V_k \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The generator in-feed, if any, is

$$i'_g = y_{gf} E_f \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The total fault current, i.e. the p.u. fault level, is

$$i_f = i'_g + \Sigma(\text{branch infeeds}) \quad . \quad . \quad . \quad (17)$$

(10.2) Representation of Tapped Transformers

(10.2.1) In-Phase Taps.

The analysis of multi-wound transformers can be facilitated by their reduction to equivalent star-type circuits and the creation of artificial nodes, the representation of any resulting negative impedances presenting no difficulties in a digital method. The general problem of the representation of transformers with in-phase taps can therefore be reduced to the case of a two-winding transformer with variable turns ratio. The accepted equivalent circuit for such a transformer, as shown i Fig. 11(a) consists of a pure admittance in series with an idea auto-transformer, the latter on the side of the tapped winding The tap value, t (per unit), may be positive or negative, but th

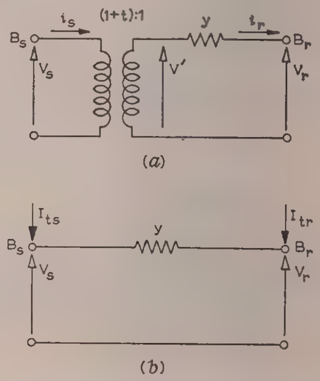

Fig. 11.—Representation of in-phase transformer taps.

(a) Equivalent circuit.
(b) Injected current equivalent.

standard convention adopted for digital analysis requires th busbar to which the tapped winding is connected to be th sending-end of the branch.

The transformer equations from Fig. 11(a) are

$$\left.\begin{aligned} V_s &= (1+t)V' \\ i_r &= (1+t)i_s \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (18$$

Hence

$$\left.\begin{aligned} i_r &= \frac{y}{1+t}V_s - yV_r \\ i_s &= \frac{y}{(1+t)^2}V_s - \frac{y}{1+t}V_r \end{aligned}\right\} \quad . \quad . \quad . \quad (1$$

The appropriate terms in a general set of nodal equations, f which the tapped transformer of Fig. 11(a) is a branch, are

$$\left.\begin{aligned} \ldots + yV_r - \frac{y}{1+t}V_s + \ldots &= I_r \\ \ldots \frac{-yV_r}{1+t} + \frac{y}{(1+t)^2}V_s + \ldots &= I_s \end{aligned}\right\} \quad . \quad : \quad (2$$

(10.2.1.1) *Load-Flow Studies.*

If there is no tap the terms in eqn. (2) corresponding to tho in eqn. (20) can be obtained either by putting $t = 0$ in eqn. (2 or from first principles,

$$\left.\begin{aligned} \ldots + yd_r - yd_s + \ldots &= I_r \\ \ldots - yd_r + yd_s + \ldots &= I_s \end{aligned}\right\} \quad . \quad . \quad . \quad (2$$

It is convenient for load-flow purposes to rewrite eqn. (20) that the nodal coefficients are the same as for eqn. (21). T consequent corrections are regarded as nodal current quantiti I_{tr} and I_{ts}, to be injected at B_r and B_s in addition to I_r and in order to simulate the effect of a tap in a branch which w previously a pure admittance. Thus the equivalent circuit Fig. 11(a) is replaced by that of Fig. 11(b), for which

$$\left.\begin{aligned} \ldots + yd_r - yd_s + \ldots &= I_r + I_{tr} \\ \ldots - yd_r + yd_s + \ldots &= I_s + I_{ts} \end{aligned}\right\} \quad . \quad . \quad (2$$

here
$$\left.\begin{aligned} I_{tr} &= -\frac{t}{1+t}yV_s \\ I_{ts} &= -\frac{t}{1+t}yV_r + \frac{2t+t^2}{(1+t)^2}yV_s \end{aligned}\right\} \quad . \quad . \quad (23)$$

These expressions are simplified by the definition of a para-eter $u(t)$ associated with the transformer:

$$u(t) = \frac{t}{1+t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

hich gives
$$\left.\begin{aligned} I_{tr} &= -uyV_s \\ I_{ts} &= -uyV_r + u(2-u)yV_s \end{aligned}\right\} \quad . \quad . \quad . \quad (25)$$

Thus the equivalent currents, I_{tr} and I_{ts}, can be determined om an existing approximation to the voltage distribution in the ame manner as the other components of I in eqn. (5).

Expressions for the winding currents of a tapped transformer, (t) and $i_s(t)$, have been derived from first principles in eqn. (19), ut it is again convenient for programme purposes to obtain xpressions for the corrections to be applied to $i(0)$, given by qn. (6), if there is no tap.

Thus
$$\left.\begin{aligned} i_r(t) &= i(0) - uyV_s \\ i_s(t) &= i(0) + uyV_r - u(2-u)yV_s) \end{aligned}\right\} \quad . \quad . \quad (26)$$

Both winding currents are necessary for the correct evaluation of the branch power flows using eqn. (7), but only i_s is printed as the branch current in the standard output scheme.

(10.2.1.2) *Short-Circuit Studies.*

The effects of taps in a separate short-circuit study programme can be allowed for directly in the formulation of Y' using eqn. (20). Alternatively, they can be expressed in terms of modifications to appropriate coefficients calculated according to eqn. (12):

$$\left.\begin{aligned} Y'_{rr}(t) &= Y'_{rr}(0) \\ Y'_{rs}(t) &= Y'_{rs}(0) + uy \qquad = Y'_{sr}(t) \\ Y'_{ss}(t) &= Y'_{ss}(0) - u(2-u)y \end{aligned}\right\} \quad . \quad . \quad (27)$$

The most convenient method of evaluating the fault in-feeds for a transformer branch $i'(t)$ is again to apply a correction to $i'(0)$, given by eqn. (15). Making either V_r or V_s zero in eqn. (19) and remembering that the in-feed is i_r in the former case and $-i_s$ in the latter, a common result is obtained:

$$i'(t) = (1-u)i'(0) \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

(10.2.2) **Quadrature Taps.**

An exact definition of a given quadrature tap is not generally available, so, for the purposes of this analysis, the effect of a positive quadrature tap of q per unit on the sending-end side of a phase-shifting transformer is defined as shown by the equivalent circuit and vector diagram of Fig. 12(a), in which an ideal phase-shifting unit is in series with a pure admittance. The use of this convention together with the requirement that vector powers on both sides of the ideal unit shall be equal [eqn. (29)] leads to the transformation equations (30):

$$V_s i_r^* = V' i_r^* \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

$$\left.\begin{aligned} V_s &= (1+jq)V' \\ i_r &= (1-jq)i_s \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Hence
$$\left.\begin{aligned} i_r &= \frac{y}{(1+jq)}V_s - yV_r \\ i_s &= \frac{y}{(1+q^2)}V_s - \frac{y}{(1-jq)}V_r \end{aligned}\right\} \quad . \quad . \quad . \quad (31)$$

The appropriate terms in a general set of nodal equations for which the phase-shifting transformer of Fig. 12(a) is a branch are thus

$$\left.\begin{aligned} \ldots + yV_r - \frac{y}{(1+jq)}V_s + \ldots &= I_r \\ \ldots - \frac{y}{(1-jq)}V_r + \frac{y}{(1+q^2)}V_s + \ldots &= I_s \end{aligned}\right\} \quad . \quad (32)$$

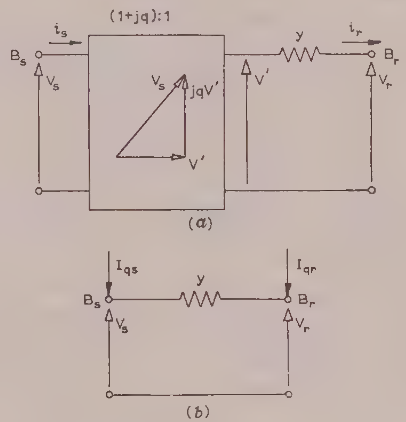

Fig. 12.—Representation of quadrature transformer taps.
(a) Equivalent circuit.
(b) Injected current equivalent.

The asymmetry of the nodal coefficients in eqns. (32), which is due to the non-reciprocal transformations (30), affects the logic of both standard programmes and also the storage requirements of the short-circuit study programme but not the generality of the iterative approach.

(10.2.2.1) *Load-Flow Studies.*

Fig. 12(b) shows an equivalent circuit in which the effect of the quadrature tap is replaced by the injection of nodal currents, I_{qr} and I_{qs}, into a network whose nodal coefficients are independent of tapped transformers. Expressions for these currents, derived in the same way as for the in-phase tap case, are

$$\left.\begin{aligned} I_{qr} &= -\frac{jq}{(1+jq)}yV_s \\ I_{qs} &= \frac{jq}{(1-jq)}yV_r + \frac{q^2}{(1+q^2)}yV_s \end{aligned}\right\} \quad . \quad . \quad (33)$$

These expressions are simplified by the definition of a parameter $p(q)$ associated with a phase-shifting transformer.

Thus
$$p(q) = \frac{jq}{(1+jq)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

$$\left.\begin{aligned} I_{qr} &= -pyV_s \\ I_{qs} &= -p^*yV_r + p^*pyV_s \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (35)$$

The corrections to be applied to branch flows in this case are

$$\left.\begin{aligned} i_r(q) &= i(0) - pyV_s \\ i_s(q) &= i(0) + p^*yV_r - p^*pyV_s \end{aligned}\right\} \quad . \quad . \quad . \quad (36)$$

(10.2.2.2) *Short-Circuit Studies.*

The effects of quadrature taps on the coefficients Y' are given directly by eqns. (33), or in terms of corrections as

$$\left.\begin{aligned} Y'_{rr}(q) &= Y'_{rr}(0) \\ Y'_{rs}(q) &= Y'_{rs}(0) + py \\ Y'_{sr}(q) &= Y'_{sr}(0) + p^*y \\ Y'_{ss}(q) &= Y'_{ss}(0) - p^*py \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (37)$$

The corrections to the fault in-feeds in this case depend on the defined direction of flow as

$$\left.\begin{aligned} i'_r(q) &= (1 - p)i'(0) \\ i'_s(q) &= (1 - p^*)i'(0) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (38)$$

(10.3) Improved Gauss–Seidel Method for the Solution of Linear Equations

The feasibility of relaxation-type methods for the solution of linear equations depends on the presence of strong coefficients to which the behaviour of certain residuals can be related. In order that such methods can be conveniently programmed for use as general processes on an automatic digital computer these coefficients should occur in predictable locations, preferably as diagonal elements. The direct Gauss–Seidel iterative method[13] uses the diagonal terms alone to obtain improved solutions from driving functions which are modified by the off-diagonal terms in each equation.

In the improved Gauss–Seidel method a forward substitution process is used for the solution of a set of coefficients with a lower triangular coefficient matrix and driving functions modified by the existing approximations to the remaining upper off-diagonal terms in each equation. Thus, given the equations

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\ \vdots \qquad\qquad & \quad \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n &= b_n \end{aligned}\right\} \quad . \quad . \quad (39)$$

$x^{(m+1)}$ is obtained as a better solution than $x^{(m)}$ by solving

$$\left.\begin{aligned} a_{11}x_1^{(m+1)} \qquad\qquad\qquad\qquad\qquad &= b'_1 \\ a_{21}x_1^{(m+1)} + a_{22}x_2^{(m+1)} \qquad\qquad\quad &= b'_2 \\ \vdots \qquad\qquad \vdots \qquad\qquad\qquad & \quad \vdots \\ a_{n1}x_1^{(m+1)} + a_{n2}x_2^{(m+1)} + \ldots + a_{nn}x_n^{(m+1)} &= b'_n \end{aligned}\right\} \quad (4$$

where

$$\left.\begin{aligned} b'_1 &= b_1 - [a_{12}x_2^{(m)} + \ldots + a_{1n}x_n^{(m)}] \\ b'_2 &= b_2 - [a_{23}x_3^{(m)} \ldots + a_{2n}x_n^{(m)}] \\ &\vdots \\ b'_n &= b_n - [0] \end{aligned}\right\} \quad . \quad . \quad (4$$

Convergence, i.e. agreement of successive solutions of x within a specified tolerance, is assured if $\lambda \leqslant 1$, where λ is ai root of the equation

$$\begin{vmatrix} a_{11}\lambda & a_{12} & \ldots & a_{1n} \\ a_{21}\lambda & a_{22}\lambda & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1}\lambda & a_{n2}\lambda & \ldots & a_{nn}\lambda \end{vmatrix} = 0 \quad . \quad . \quad . \quad (4$$

This criterion is more easily fulfilled and the rate of co vergence generally faster than the corresponding features of t direct Gauss–Seidel process, because less information is neglect in the evaluation of each improved variable. Although forwa substitution takes longer than the simple multiplication in t direct process, no additional scaling difficulties are introduced.

[The discussion on the above paper will be found on page 398.]

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DIGITAL COMPUTERS IN POWER SYSTEM ANALYSIS

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SUMMARY

Programmes have been developed for carrying out power-system load studies and transient stability studies on an automatic electronic digital computer. The methods employed are similar to those used at present on network analysers, but improvements have been made where possible. In the method of solving load studies, loads and generation are represented by equivalent admittances to neutral, and nodal voltage equations are used. A control programme which automatically adjusts the reactive-power generation and transformer taps to keep the voltages within satisfactory limits has been developed.

A programme to carry out transient stability calculations for a multi-machine system, using the standard step-by-step method, has been written, together with a programme to calculate the power-equation coefficients needed to compute the power transfer at the end of each interval. Significant generator characteristics like saliency, flux decrement, voltage-regulator action, damping and governor action, which are normally neglected in network-analyser studies, have been included in the programme.

It is shown that the use of computers is a highly efficient and economic method for the solution of power-system problems.

LIST OF SYMBOLS

Y = Admittance.
$S = P + jQ$ (to represent load or generation).
P = Active power.
Q = Reactive power. (Magnetizing reactive power and active power flowing in the same direction are assumed to have different signs for all mathematical work inside the computer, but the opposite sign convention is used for the input and output of data to conform to the normal practice of power-system engineers.)
V = Voltage.
Z = Impedance.
I = Current.
A, B, C, D = Control parameters.
θ = Rotor angle.
X'_d = Direct-axis transient reactance.
X_q = Quadrature-axis synchronous reactance.
X_d = Direct-axis synchronous reactance.
V_q = Voltage behind quadrature-axis synchronous reactance.
V_i = Voltage corresponding to field currents.
V' = Voltage behind transient reactance.
V'_q = Quadrature-axis component of V'.
V_x = Driving voltage.
Z_x = Mesh impedances.
I_x = Mesh currents.

Any symbol in square brackets represents a matrix of that quantity, e.g. [Z] represents the impedance matrix of the system.

(1) INTRODUCTION

The paper presents some methods of using an electronic digital computer for the solution of problems in power-system analysis of the types which are at present investigated by network analysers. Particular attention has been paid to the manner in which the established methods can be improved when a computer is used. An important feature of an automatic computer is its ability to make simple, logical decisions and thus work through a maze of alternative paths in a programme. Thus, besides the improvement in the accuracy of representation of the system, the logical facilities afforded by a computer can do much of the work which is normally performed by the engineer operating the analyser, provided, of course, that it is given the necessary rules.

Programmes have been written for power-system load studies and transient stability studies. It has been found that these can be carried out with as much efficiency and economy on a digital computer as on a network analyser, but further improvements are possible in methods of formulating the problems and controlling the solution. The programmes have been developed on the Pegasus I Computer (with a 7168-word store).[1] This is a medium-size computer, comparable in cost with a network analyser, but it can also be used for many other classes of work.

Digital computers have now been in use for over ten years in power-system analysis. The earliest work was carried out by Jennings and Quinan,[2] using ordinary business machines for solving the power-flow problems. Later Dunstan,[3,5] Henderson,[6] Bennett,[4,7] Ward and Hale,[8] Brown and Tinney,[9] Glimn and Stagg[10] and others have presented methods of solving power-system load studies. Johnson and Ward[11] also used the computer for stability studies. They all laid great stress on methods of solution for a given set of conditions, rather than on the use of the computer to calculate an optimum condition. It is felt that digital computers will have a great capacity for automatic synthesis and optimization as well as analysis. As larger computers become available it is to be expected that more general methods will be developed for such problems as power-system design.

(2) PROGRAMMES FOR POWER-SYSTEM ANALYSIS

The paper describes three programmes which have been developed for problems of power-system analysis:

(a) Power-system load studies.
(b) Power-system transient stability studies—Part 1. Calculation of power equation coefficients.
(c) Power-system transient stability studies—Part 2. Rotor-angle calculation.

All these programmes have been tested on a variety of problems and an account of the experience gained is given in later Sections.

The load-study programme is used to calculate the busbar voltages and transmission-circuit loadings resulting from a specified loading condition for a system. If these voltages are not satisfactory, the reactive-power generation at appropriate busbars or transformer tap settings are changed to bring the voltages within acceptable limits. The programme is so arranged that further studies on the same system for different loading conditions or changes to the network can be carried out automatically by the computer.

The transient stability programme is used to compute, by the usual step-by-step method, the swing curves of the generators

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following a disturbance in the system. This programme is divided into two parts; in Part 1, the power-equation coefficients and other necessary data for the machines in the system for each of the different switching conditions are calculated. These coefficients are required to calculate the power output of each machine from the rotor angles after every time interval. Part 2 consists of the rotor-angle calculation by the step-by-step method. As in the load-studies programme, stress has been laid on fully automatic operation of the computer so that a series of studies can be carried out without any manual intervention. An elementary form of control programme has been developed which can be used to determine either the critical switching time for the system or its stability for a given sequence of fault and switching operations.

Significant generator characteristics which are normally neglected in network-analyser studies have been included in the programme. Simple representation of saliency, flux decrement, voltage-regulator action, damping and governor action can all be taken into account.

(3) REPRESENTATION OF PLANT

(3.1) Generators and Loads

One of the basic problems in the digital-computer solution is the best representation of the loads and generation in the system. For the purpose of the load study, the load at a busbar can be regarded as a sink consuming certain real and reactive power, and, similarly, a generator may be regarded as a source. Loads may thus be represented by a current flowing out of the busbar to the neutral of the system or as an admittance connected between the busbar and the neutral of the system. The r.m.s. current I or the admittance Y depends on the voltage V at the busbar and is given by the following equations:

$$I = S/V^* \qquad (1)$$

$$Y = S/|V|^2 \qquad (2)$$

where V^* is the complex conjugate of V, and S is the complex apparent power.

On a network analyser a generator is usually represented as a voltage source behind an equivalent reactance. It can easily be shown that, so far as the rest of the system is concerned, this can be replaced by a current source across the same impedance, and this is more common when a digital computer is used. However, for load studies it is sufficient to represent a generator either by an equivalent current fed into the network or by an equivalent admittance. (This normally consists of negative resistance and reactance since S in the above equations is negative.)

Admittances computed from eqn. (2) may be called 'equivalent load and generator admittances'.

(3.2) Transformer

There is normally no difficulty in representing transformers with nominal ratios. As on a network analyser, they can be represented by an equivalent impedance. Special representation is required when the effect of tap settings has to be considered. A transformer with an off-nominal ratio can be represented by an ideal auto-transformer and an equivalent impedance arrangement as shown in Fig. 1(a). The transformer is connected between busbars A and B, and Z is the equivalent impedance. The off-nominal ratio is represented by n in per-unit values. Thus a tapping of +5% corresponds to $n = 1{\cdot}05$, and a tapping of −5% to $n = 0{\cdot}95$. Equivalent currents can be used to represent the effect of the tapping on the voltages and currents in the rest of the system, as shown in Fig. 1(b). Appendix 10.1 shows how these are calculated.

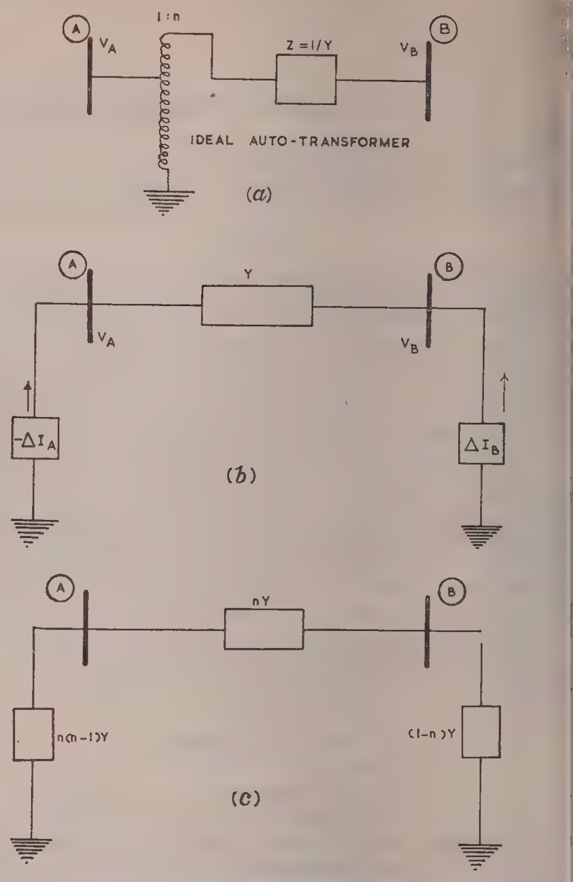

Fig. 1.—Transformer representation.
$\Delta I_A = (n-1)Y[(n+1)V_A - V_B]$
$\Delta I_B = (n-1)YV_A$

Another method of representation is by equivalent admittances as shown in Fig. 1(c), and is similar to that suggested by Ward and Hale.[8] This is used for the power-equation coefficients programme as the tap setting is fixed.

(4) NETWORK EQUATIONS

(4.1) General Theory

When a network analyser is used for the solution of a power-system problem, a network model is set up for the purpose. For solution by a digital computer it is necessary to represent the system analytically by formulating equations defining the problem and using appropriate methods to solve them.

Equations defining the voltage/current relationship in a network, whether active or passive, can be written down easily to satisfy Kirchhoff's laws. There are two main types of equations, namely mesh current and nodal voltage. The mesh current equations can generally be represented by the matrix equation

$$[V_x] = [Z_x][I_x] \qquad (3)$$

Similarly, nodal voltage equations can be represented by the matrix equation

$$[I] = [Y][V] \qquad (4)$$

In eqn. (4) all driving voltages are replaced by the equivalent

current sources. More detail about methods of formulating and solving the equations can be obtained from Reference 12.

Nodal-voltage equations have four distinct advantages over mesh current equations:

(i) The number of equations, particularly in a large power system, is always less.
(ii) Parallel branches can be treated separately without adding to the number of equations.
(iii) No difficulty is encountered in the formation of the admittance matrix, if cross-over branches are present.
(iv) Off-nominal turns ratios in transformers are more easily represented.

Moreover, the solution gives the required voltages directly. Thus it has been found advisable to use nodal voltage equations. A particular feature of the present load-study programme is that equivalent admittances are used to represent loads and generation. This is discussed in the following Section.

(4.2) Equivalent Admittance Method

An approximate set of voltages is assumed at all the busbars in the system except one, called the 'floating busbar', where the voltage is specified and kept constant. The load and generation at all the other busbars, with an assumed starting value for the reactive loading of generators, are specified.

A direct solution cannot be obtained, because, for accurate representation of loads and generation, the voltages at all the busbars should be known exactly, and thus an iterative method of solution becomes necessary. Load and generator admittances can be calculated using the assumed busbar voltages. If the floating busbar is numbered zero and other busbars and junctions are numbered consecutively from 1 to n, nodal-voltage equations of the following type, taking the neutral as the reference busbar, can be written down for every busbar in the system:

$$\left.\begin{aligned} I_0 &= Y_{00}V_0 + Y_{01}V_1 + \ldots + Y_{0m}V_m + \ldots + Y_{0n}V_n \\ I_1 &= Y_{10}V_0 + Y_{11}V_1 + \ldots + Y_{1m}V_m + \ldots + Y_{1n}V_n \\ &\cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot \\ &\cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot \\ I_m &= Y_{m0}V_0 + Y_{m1}V_1 + \ldots + Y_{mm}V_m + \ldots + Y_{mn}V_n \\ &\cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot \\ I_n &= Y_{n0}V_0 + Y_{n1}V_1 + \ldots + Y_{nm}V_m + \ldots + Y_{nn}V_n \end{aligned}\right\} \quad (5)$$

It is desired to solve these equations to obtain the voltages $V_1, V_2, \ldots V_n$ at the n busbars. As there are $n + 1$ equations, one of them is redundant and the equation at the floating busbar must be neglected since I_0 is not known. The terms $I_1, I_2, \ldots I_n$ are all zero because there is no active source connected to any junction, except the floating busbar, and as the voltage V_0 at the floating busbar is known, it follows that the first column on the right-hand side of eqn. (5) consists of known quantities which have the dimensions of current. These may be transferred to the left-hand side, giving a total current source which may be defined as follows:

$$I'_m = I_m - Y_{m0}V_0 \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where m takes all values from 1 to n.

Eqn. (5) can then be rearranged by introducing these new quantities as follows:

$$I'_m = \sum_{p=1}^{n} Y_{mp}V_p \quad . \quad . \quad . \quad . \quad . \quad (7)$$

which can be represented by the matrix equation

$$[I'] = [Y][V] \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

This equation can be solved to give a first approximation to the busbar voltages, which can be used to calculate the appropriate load and generator admittances. Further iterations can be carried out until the change in busbar voltages in two successive cycles is less than an agreed limiting value. The set of voltages so obtained can then be assumed to be the solution to satisfy the specified loading conditions of the network. This would require a fresh inversion of the system matrix for each iteration. A better method, which is normally employed in this programme, is to modify the equivalent current matrix of eqn. (6) instead of the system admittance matrix, in the following manner: At any busbar, suppose the desired load is S; then assuming an initial voltage V_A, the admittance Y_A is, from eqn. (2),

$$Y_A = S/|V_A|^2$$

If the voltage at the same busbar in a later cycle is found to be V_B, the admittance represents a load of

$$S_B = Y_A|V_B|^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Thus the admittance does not represent the load accurately, but if additional generation equal to the difference in load is introduced, the net load at the busbar will be correct. The error ΔS is given by

$$\Delta S = S - S_B \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Hence an equivalent current source ΔI can be introduced between the busbar and the neutral to represent ΔS, and correct the load. Its value is

$$\Delta I = \Delta S/V_B^* \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

This can be added to the equivalent current source at the busbar as defined by eqn. (6). These corrected equivalent currents can then be used to obtain a new set of voltages and the process repeated as before.

The programme for load studies has been developed using this method. A flow diagram of the programme and methods of use are described in the next Section. With a 7168-word store and no magnetic tape, the programme can handle systems with 33 busbars and 120 lines.

(5) LOAD-STUDIES PROGRAMME

The load-studies programme is divided into four parts:

(a) Input programme.
(b) Initial solution.
(c) Control and adjustments.
(d) Output programme.

A flow diagram of the programme is shown in Fig. 2. Step 1 corresponds to the input programme, while steps 2–9 correspond to the calculation of the initial solution. The same programme is repeated for later iterations. Steps 10–13 comprise the control programme, and step 15 indicates the output programme. The diagnostic programme is called in if a solution is not obtained. This programme prints information which is useful to the system engineer in finding out why the programme failed to function normally.

(5.1) Input Programme

The aim in writing the input programme has been to keep data preparation as simple and straightforward as possible. The data should be accepted in a form convenient to system engineers, preferably one in which it is generally available. The organization of the input data has been planned to meet these objectives as closely as possible.

As a number of studies are normally required on the same system, the data have been divided into two groups—system data and problem data. System data comprise all the permanent parameters of the system. Problem data comprise those particular to each study.

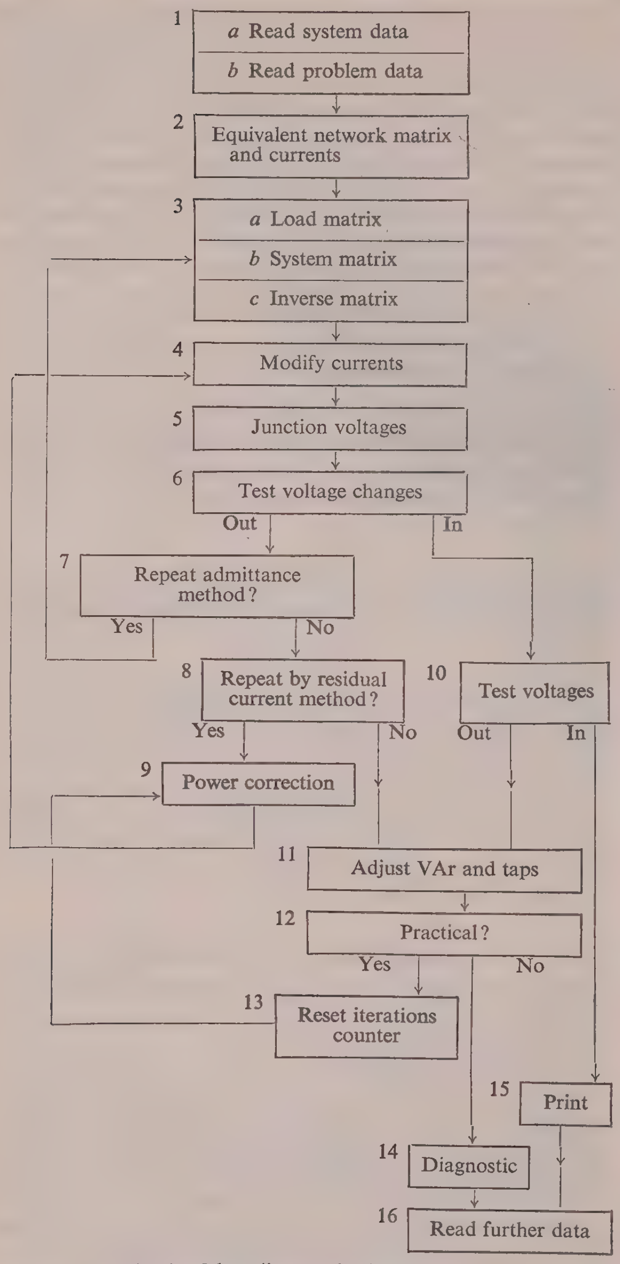

Fig. 2.—Flow diagram for load studies.

Before commencing a series of studies, it is necessary to give a reference number to each line, cable, series reactor, transformer and generator. The system data comprise the following:

(i) Resistance, reactance, shunt susceptance and the maximum continuous thermal rating of each series element in the network.
(ii) Equivalent impedance of each transformer with tappings.
(iii) Upper and lower limits on the settings, and the tapping step for each transformer.
(iv) Limits on real and reactive power output of each generator.

A system-data tape is prepared for a particular system giving the data for each element preceded by its serial number. A special input programme has been written to read this and store the data appropriately.

A fresh problem-data tape has to be prepared for each study or series of studies to be carried out. For each problem the junctions in the system have to be numbered. The junction numbers are allocated to suit the convenience of the programme for each series of studies; they do not necessarily correspond to the busbar numbers. The floating busbar is always numbered zero, and all the other junctions are then numbered consecutively. The following tables of data are then prepared and punched on a tape:

(*a*) Connection table.
(*b*) Table of load and generation.
(*c*) Table of initial transformer tap settings.
(*d*) Table of initial busbar voltages.
(*e*) Table of control parameters.
(*f*) Programme parameters.

The connection table defines the configuration of the network. It consists of the line numbers, each followed by the junction numbers connected by that line, and also the generator numbers, each followed by the junction number to which it is connected. The table of load and generation consists of the junction numbers followed by the load or generation in megawatts and reactive megavolt-amperes. Generated power is assumed to be positive. Although for mathematical calculation inside the computer magnetizing reactive and active power flowing in same direction have different signs, for the purposes of input and output they are given the same sign. This is to facilitate data preparation by power-system engineers. The table of control parameters is needed for the control and adjustment of generator reactive power and transformer tappings. It is discussed in greater detail in Section 5.3. The programme parameters consist of the total number of junctions in the system, the highest transformer number and, if any line outage is to be considered, the number of that line. The system-data tape together with the problem-data tape contain all the data necessary to define the problem completely. For simplicity of preparation and tape punching, almost all numbers are in an unscaled form. All the tables and sets of data are independent of each other and may be put in any order on the tape. If some data are not specified the computer assumes an appropriate starting value, e.g. $1 + j0$, for all unspecified junction voltages. Once the data tapes have been fed in, only short problem-data tapes specifying the changes in data from the previous study are required for later studies. Thus the computer can automatically carry out a series of studies from a single tape.

(5.2) Initial Solution

The first operation to be carried out, after the data have been read in, is to compute the equivalent network admittance matrix and the equivalent currents as defined in eqn. (6). The computations needed for this purpose are shown in Fig. 3. A π-representation of transmission lines is employed, and the connection table is used to compute the equivalent lumped shunt susceptances at the junctions. In a network with no mutual admittance between any of the branches, the connection table is also used to form the network admittance matrix. The mutual inductance between a pair of lines can be represented by a Y-network, which introduces an extra node. The lumped susceptances are added to the corresponding diagonal terms of this matrix, which can then be reduced by eliminating the first row. The first column, negative, provides the equivalent current sources, and the remainder is the equivalent network matrix. The load and generator admittances are then computed using the table of initial voltages. To obtain the load matrix, which is diagonal, the sum of load and generation at each busbar is used. The load matrix so formed is added to the network matrix to form the system matrix of step 3(*b*) in Fig. 2. This corresponds to the matrix $[Y]$ in eqn. (8). The system matrix is

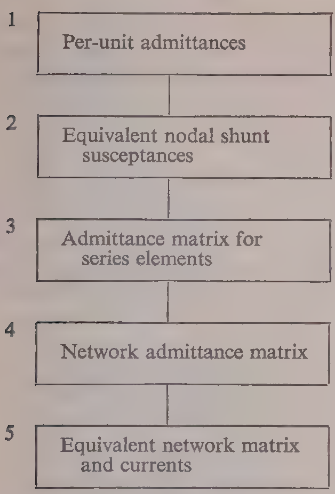

Fig. 3.—Calculation of the equivalent matrix.

inverted and the inverse matrix $[Z]$ multiplied by $[I']$ to give the first approximation to the voltages as shown in eqn. (12):

$$[V] = [Z][I'] \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

As indicated in Section 4.2 further iterations can be carried out by using this set of voltages to compute the load matrix again. Since this involves a matrix inversion every cycle, it is a time-consuming process and is used only occasionally. Normally the loads are corrected by the residual-current method by computing the residual currents according to eqn. (11) and adding them to $[I']$. This is shown in step 9 of the flow diagram. Step 4 in the flow diagram includes the modification of $[I']$ to represent transformer tap settings. The equivalent currents as shown in Fig. 1(*b*) are added vectorially to the corresponding terms of $[I']$. If tappings were represented by equivalent admittances, as in Fig. 1(*c*), a matrix inversion would be necessary whenever tap settings were changed. Iterations are carried out until the voltages converge to a solution or a predetermined number of cycles is exceeded. The control programme is then entered.

(5.3) Control and Adjustment

To enable the programme to be flexible, a count is kept on the number of iterations by each method. It is thus easy to carry out iterations by the equivalent admittance method, if desired, by setting the limiting number of cycles for this process (step 7 in Fig. 2) to be higher than unity, which is the usual condition. Similarly, the limiting number of cycles by the residual-current method (step 8) is also variable.

A solution by the residual-current method is usually obtained within ten cycles, but it is possible that, if the system is badly conditioned or if the initial reactive-power generation specified is not appropriate, convergence may not be obtained. If an acceptable solution (step 6) is not obtained within the specified limit of cycles, usually 10 (step 8), the control programme is entered.

A detailed flow diagram of the control programme is shown in Fig. 4, and comprises steps 10, 11, 12 and 13 of Fig. 2. This is discussed briefly below. For each junction in the system an upper and lower limit on voltage and a list of four, or less, control parameters *A*, *B*, *C* and *D* are specified. These could be either junction numbers at which it is possible to vary reactive power generation or a transformer at which a tap setting may

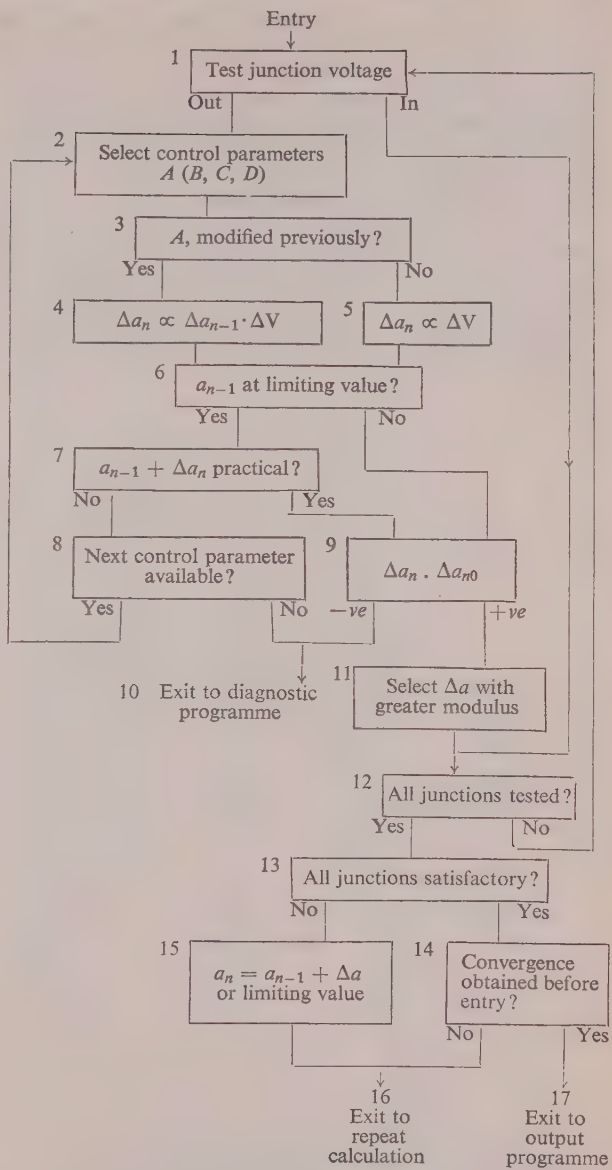

Fig. 4.—Flow diagram of the control programme.

a_{n-1} = Value at *A* at entry to control programme.
a_n = Value at *A* at exit from control programme.
Δa_n = Calculated change in the value at *A*.
Δa_{n-1} = Change made at *A* at the previous entry.
ΔV = Desired change of voltage at busbar under test.
Δa_{n0} = Calculated change in value at *A* for busbars tested previously in the same cycle.

be changed. The voltages at the busbars are tested, and if all the voltages are satisfactory and convergence had been obtained before entry, the output programme (step 15) is entered.

If the voltage at any busbar, say junction 2, is outside the limits, the first of the control parameters is examined. An appropriate change, proportional to the change in voltage needed at junction 2, is then made at *A* to try to bring the voltage within limits. If changes had been made previously at *A*, these are taken into account. If it is found that the parameters at *A* are at a limiting value and no further change is possible, the same process is carried out using *B*, *C* and *D* in turn.

It is possible that *A* may be the control parameter for more

than one busbar in the system. The change made is then determined by the conditions at the busbar whose voltage needs maximum adjustment. However, if it is found that, for two busbars, the changes needed at A are in different directions, diagnostic results (step 14) are printed, since contradictory changes indicate an error in specifying the control parameter. When all the busbars have been tested, iterations are restarted at step 9 to try to converge to a new solution.

Experience with the programme showed that it was advisable to enter the control programme every iteration thereby checking the voltage levels and making appropriate adjustments to the system conditions. It was found that this accelerated convergence and avoided complete failure in some problems for which the conditions were critical and the initial assumptions were not sufficiently close.

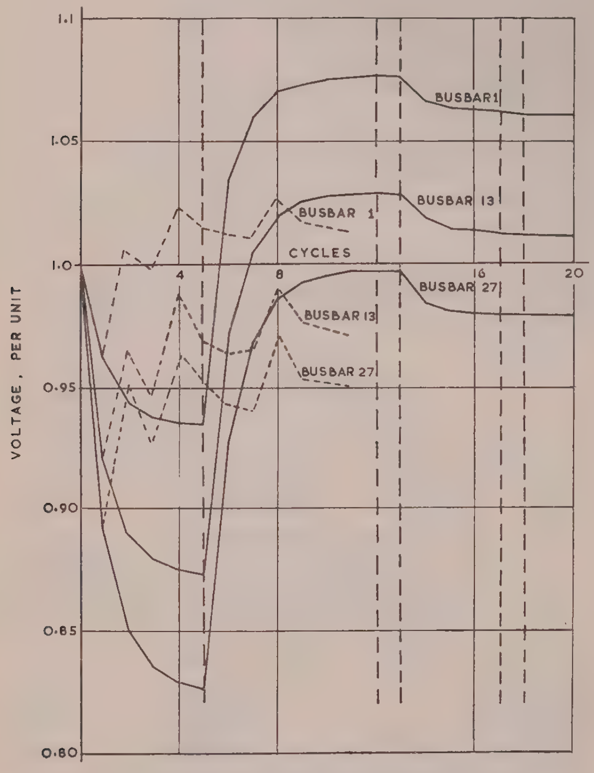

Fig. 5.—Convergence of voltage.

The vertical broken lines indicate the instants when the control programme was called in to modify parameters.

Fig. 5 shows the manner in which the voltage at three busbars in a 28-busbar system converged. The continuous lines show the results obtained when the control programme was entered only when convergence had been obtained. A solution to satisfy the initially specified loading and generation was obtained in five cycles. As the voltages were low, the control programme was used to make appropriate correction, and the final solution was obtained after 20 cycles. The broken lines show the results when the control programme was entered every cycle. This led to convergence within 10 cycles and gave a better solution.

This programme provides great flexibility in the solution of problems. It is possible to control busbar voltages to within fine limits. No attempt has been made so far to accelerate convergence of the problem, and this does not appear to be necessary. The control programme will vary the reactive-power generation if the initial loading was such as to cause instability. If a solution is not obtained the results are usually printed at the last stage reached so that changes may be made in the input data which may help convergence. It should be noted that each cycle is a solution to the system, although not the desired one, and results may be printed after any cycle to indicate the behaviour of the system.

The above control programme attempts to adjust the reactive-power generation and transformer tappings to obtain voltages within satisfactory limits. However, in systems with very long transmission lines, the reactive-power flow may be critical and it is possible that a solution to satisfy the initially specified reactive-power generation may not be obtained. It may then be found more advisable to try to get a solution with specified voltages at the generator busbars. A version of the programme has been written in which residual currents (calculated as shown in Appendix 10.2) are injected every iteration to correct the voltage magnitude and the power at the generator busbars and the real and reactive power at the load busbars. The reactive-power generation is obtained from the solution and may be examined to see whether it is within acceptable limits. Changes may be made in the specified voltages or other parameters, if necessary.

It has been found from experience on some problems that the choice of the floating busbar may affect convergence. The computer could change the floating busbar if convergence is not obtained, but this would require additional space in the computer store.

To make the most economic use of the computer, it is desirable that a series of studies should be carried out successively. It is easy to consider changes in loading conditions and others of similar type by modifying the generation and load data and carrying out further iterations by the residual-current method, without altering the system matrix.

(5.4) Output

Provision has been made in the programme for optional printing of the voltages obtained after any cycle by depressing one of the hand switches on the computer. This would indicate the progress of the problem but increase the computing time. To keep track of the changes in the control parameters, these are indicated whenever changed.

When a satisfactory solution is obtained, the output programme is called in to print relevant results. The line flow is computed at both ends of each line and printed, and attention is drawn to any overloaded lines. This is followed by the results at the junctions, where the junction number, the voltage (in percentage), the angle (in degrees) and the net load or generation (in megawatts and reactive megavolt-amperes) are printed. The total net load, losses and net generation are also printed out. The final transformer tap settings are also printed.

It is expected that this will provide all the information that is required, but any other information can be printed or deleted if so desired. It is possible to print the load flow on to a system diagram, but this would need more output time.

(5.5) Example

The data and results obtained from a study on a 28-busbar system are now given. A line diagram of the system is shown in Fig. 6. Table 1 gives the line parameters and a connection table for the system. Table 2 gives the initial loads and generations, and a summary of the results obtained. The first solution was obtained after one cycle of the equivalent-admittance method and three cycles of the residual-current method. At this stage the voltages were found to be too low, the reactive-

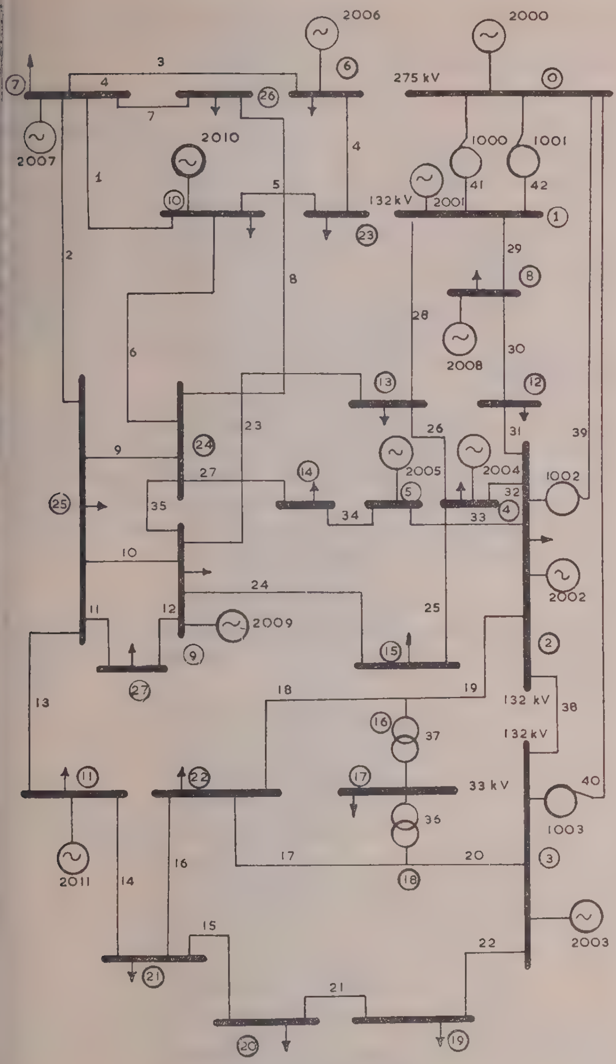

Fig. 6.—System diagram.

Junction numbers are encircled, thus (5)

Lines are numbered 1 to 42, transormers (with tappings) 1 000 to 1 003, and generators 2 000 to 2 011.

For convenience in specifying control parameters, for the computer study, transformers 1 000 and 1 001 (lines 41 and 42) were lumped together and replaced by an equivalent transformer 1 000 (line 41).

power generation was altered and also tappings were reset. The final solution was obtained after six more cycles.

The total time used by the computer for this study was about 14½ min, which consisted roughly of input, computation and output as follows:

Input of programme tape	2 min
Input of data tapes	1 min 20 sec
Computation	9 min
Output	2 min 10 sec
Total	14 min 30 sec

The matrix inversion in the first cycle took about 4½ min and each iteration approximately 15 sec.

Table 1

LINE DATA

Line No.	Resistance	Reactance	Shunt susceptance	Connection table from	to
	per unit	per unit	per unit		
1	0·0343	0·095	0·0206	10	7
2	0·01035	0·0285	0·0250	25	7
3	0·039	0·104	0·0235	7	6
4	0·0142	0·0397	0·0086	23	6
5	0·0265	0·0725	0·0160	23	10
6	0·0034	0·0096	0·0021	24	10
7	0·0252	0·0693	0·0205	26	7
8	0·0159	0·0437	0·0095	26	24
9	0·014	0·038	0·0082	25	24
10	0·014	0·038	0·0082	25	9
11	0·0097	0·0266	0·0058	27	25
12	0·0165	0·0452	0·0097	27	9
13	0·023	0·0633	0·0137	25	11
14	0·02	0·0547	0·012	21	11
15	0·0059	0·0161	0·0035	21	20
16	0·00245	0·0067	0·006	22	21
17	0·0088	0·0241	0·0053	22	18
18	0·0088	0·0241	0·0053	22	16
19	0·017	0·0362	0·0102	16	2
20	0·017	0·0362	0·0102	18	3
21	0·0142	0·0395	0·0085	20	19
22	0·0131	0·0362	0·008	19	3
23	0·0082	0·0226	0·0049	13	9
24	0·0053	0·0145	0·0032	15	9
25	0·0171	0·0473	0·0102	15	4
26	0·0122	0·0226	0·0074	13	4
27	0·0138	0·038	0·0083	24	14
28	0·00765	0·045	0·0396	13	1
29	0·0184	0·0506	0·0112	8	1
30	0·0226	0·0621	0·0136	12	8
31	0·0087	0·024	0·0052	12	2
32	0·0104	0·0285	0·0064	4	2
33	0·0104	0·0285	0·0064	5	2
34	0·0011	0·003	0·0006	14	5
35	0	0·2	0	24	9
36	0	0·643	0	18	17
37	0	0·643	0	17	16
38	0	0·2	0	3	2
39	0	0·125	0	2	0
40	0	0·125	0	3	0
41	0	0·0625	0	1	0

Base MVA = 100.

(6) TRANSIENT STABILITY STUDIES

(6.1) Power-Equation Coefficients

The power output of the mth machine, P_m, in a system with n machines, is, as shown in Appendix 10.3, given by the following equation:

$$P_m = \sum_{p=1}^{n} B_{mp} \sin(\theta_{mp} - \alpha_{mp}) \quad . \quad . \quad . \quad (13)$$

B_{mp} and α_{mp} are constants depending on the voltage behind the transient reactance of the generator and the network parameters, and are the power-equation coefficients to be calculated.

The results of the load study are used as the initial conditions for this calculation. The loads are represented by constant equivalent admittances. Since the number of equations corresponds to the number of generators it is convenient to reduce the network matrix so that the equations related to the inactive busbars are eliminated. This corresponds to a partial inversion process and reduces the number of terms in eqn. (13).

A flow diagram of this programme is shown in Fig. 7. The computation of the coefficients consists in the formation of the admittance matrix of the network, its reduction to active busbars and finally inversion of the reduced matrix. As nodal voltage

Table 2

TABLE OF LOADS, GENERATION AND VOLTAGES

Junction No.	Power	Reactive power Specified	Reactive power Computed	Voltage
	MW	MVAr	MVAr	%
Floating generation at fixed voltage				
0	178·6	—	16·9	96·0
Generator busbars				
1	197	66	66·1	104·9
2	383	297	298·1	105·3
3	146	50	50·0	105·2
4	220	110	111·5	102·3
5	32	19	20·0	103·4
6	296	170	171·7	105·8
7	167	90	96·7	101·9
8	36	21	21·7	104·1
9	100	59	60·0	99·4
10	190	92	100·0	100·4
11	283	168	170·1	101·2
Loads				
2	164	127		105·3
4	199	98		102·3
6	209	99		105·8
7	93	50		101·9
8	65	38		104·1
9	162	86		99·4
10	316	140		100·4
11	294	156		101·2
12	29	14		104·6
13	166	90		100·5
14	35	17		103·1
15	66	36		99·4
17	36	12		98·4
19	33	13		103·1
20	36	15		101·9
21	29	13		101·8
22	72	28		102·0
23	28	13		103·3
25	59	30		99·9
26	51	28		99·9
27	66	36		98·7

Transformer tap settings = − 9% on the h.t. side, i.e. 0·91 per unit.

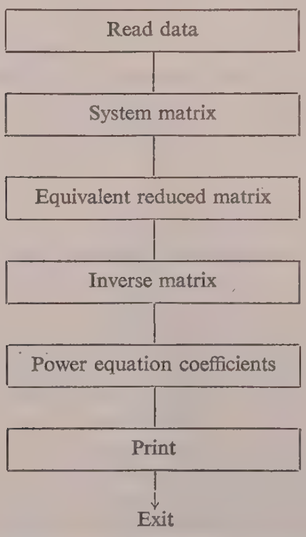

Fig. 7.—Flow diagram for the programme to calculate power-equation coefficients.

equations are used, the voltage behind the transient reactanc is replaced by an equivalent current source across the sam reactance. The constant B_{mp} is given by

$$B_{mp} = |Z_{mp}||I_m||I_p| \quad . \quad . \quad . \quad . \quad (14$$

where Z_{mp} is the appropriate element of the inverse matrix an I_m, I_p are equivalent currents, while α_{mp} is the negative of th complement of the angle of Z_{mp}.

These have to be calculated for the various network condition like prefault, faulted and fault cleared, etc. The data inpu programme is very similar to that for load studies. The systen data and problem data are fed in for the first condition. It i only required to change the relevant data for the other sets o the same system. Different types of faults are represented b connecting the fault point to the neutral through an equivalen fault impedance.

Unfortunately, owing to the restricted storage space, it is no possible to store the coefficients, and they are thus printed ou after each set is calculated. They could also be stored o magnetic tape. This output tape is suitable for direct input t the computer as data for the rotor-angle calculation. The tim taken to calculate and print out one set of coefficients for 33-busbar system with 14 machines was 7 min.

(6.2) Rotor-Angle Calculation

The flow diagram of this programme is shown in Fig. 8. It i self-explanatory and the method of calculation is well know

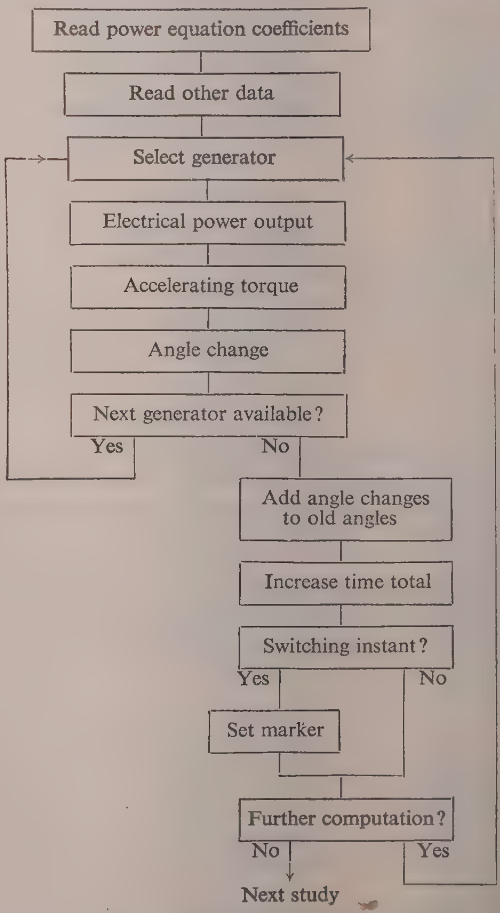

Fig. 8.—Flow diagram for the rotor-angle calculations.

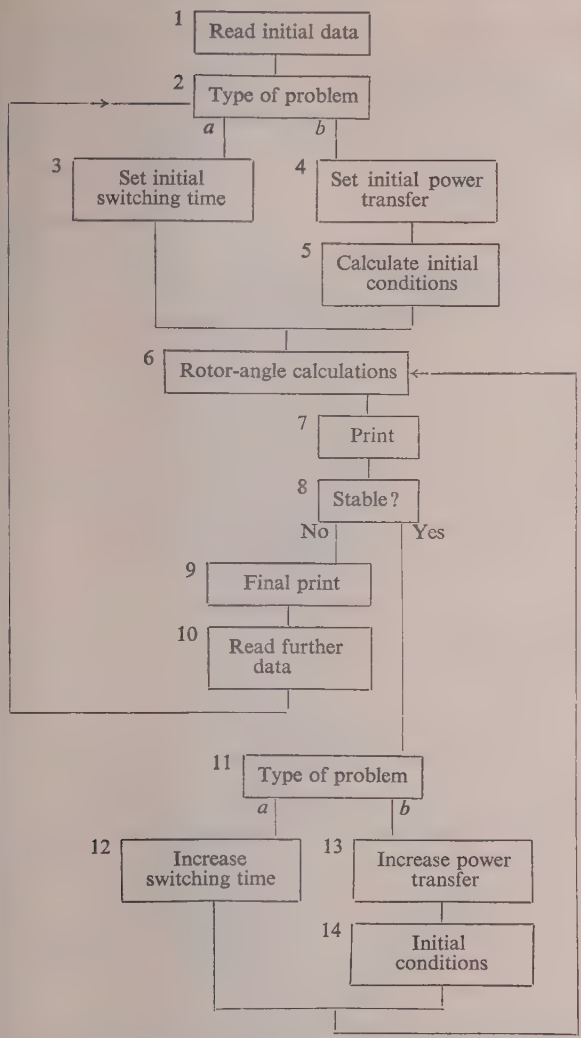

Fig. 9.—Flow diagram for a simple control programme for transient stability studies.

Type of problem
a. To calculate critical switching time.
b. To calculate maximum power transfer.

The power output of each generator is computed after each interval using the coefficients, and it is found that the step-by-step calculation gives reasonably accurate results, since much shorter intervals than those used for analyser studies can be employed.

The control of this problem is much simpler than that of load studies. The flow diagram of a simple control programme is shown in Fig. 9, and the programme can be used to determine the critical switching time or simply to test the stability of the system for a particular fault and switching sequence. To test whether the system is stable, it is convenient to consider the rotor angle of each machine relative to a chosen reference machine in the system. Then the system may usually be assumed to be stable if $\Delta\theta$, the relative angular velocity of each machine, changes sign. The calculations are carried out until stability of the system is established or specified limits of angle, velocity or time are exceeded, when the system is assumed to be unstable. To find the critical switching time, an initial value is specified (this may be zero) which is increased in steps until the system becomes unstable. The penultimate value may be assumed to be the critical switching time.

Another aspect of system analysis is the investigation of methods of improving stability. Variation of parameters such as machine reactances, inertia constants, voltage-regulator or governor characteristics, etc., can be considered. In order to carry this out automatically on the computer the parameters will have to be specified and an order of priority in which they are to be varied indicated. It should be possible for the programme to be extended to consider all these and then to decide on the most efficient and economic system if the necessary cost data are available. The computer reads more data after each study, and changes in parameters may be made without difficulty.

The normal output is to print either the absolute or the relative rotor angle at specified intervals, and these have to be plotted to indicate stability. As printing is a slow process in the computer, attempts have been made to reduce the output. Before considering this any further, it may be pointed out that another way of plotting the curves is to plot the values of $\Delta\theta$ against θ for every machine. These are called the 'acceleration loci' of the machines. The two types of curves for a single machine connected to an infinite busbar are shown in Figs. 10

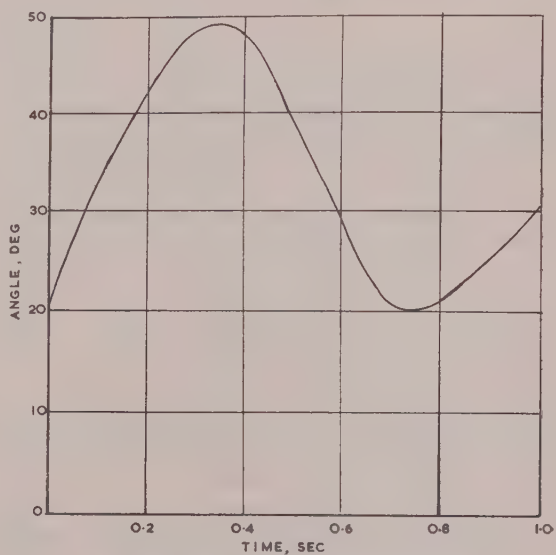

Fig. 10.—Swing curve for a single machine.

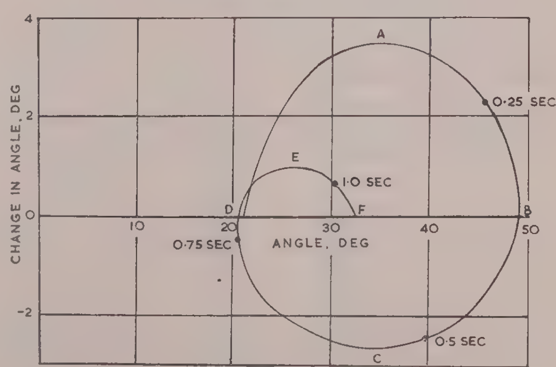

Fig. 11.—Acceleration locus for a single machine.

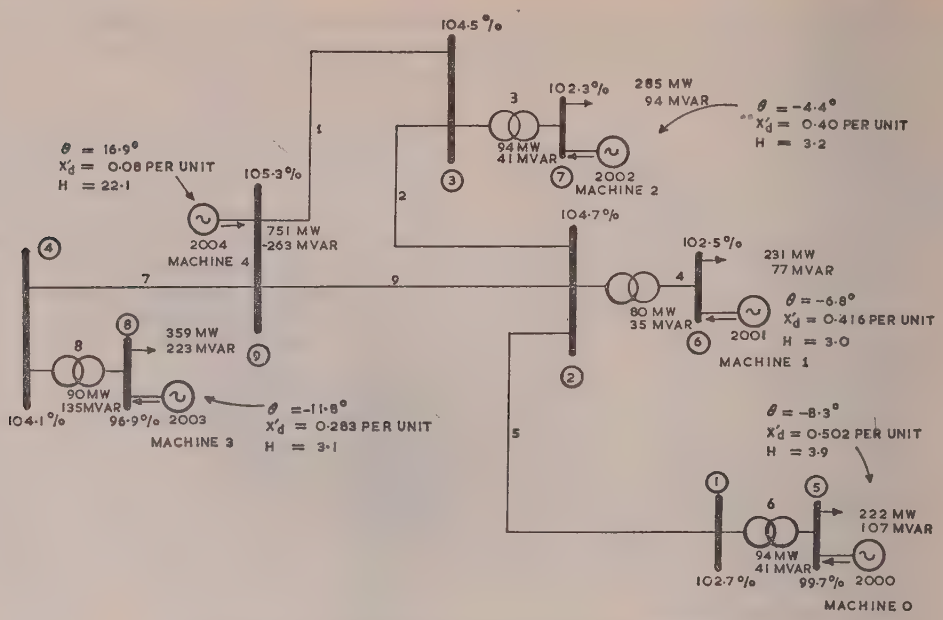

Fig. 12.—Five-machine stability study system diagram.

and 11. It can be seen from Fig. 11 that the shape of the curve may be approximately determined if the points A, B, C, D, E, F, etc., are known. These are the points at which $\Delta^2\theta$ or $\Delta\theta$ changes sign. If this method is preferred, the computer can print the time, θ and $\Delta\theta$ every time $\Delta^2\theta$ or $\Delta\theta$ changes sign. Again, it is more convenient to print the relative angles and their rates of change with respect to the reference machine. The computer can be used to print the swing curve directly but not to print acceleration loci. However, this could be done on a digital plotter. These forms of output may be used if it is desired to study in detail the behaviour of the machines, otherwise the computer need only print whether the system is stable or not for each particular condition.

The programme can handle up to 24 machines, and the time taken for each study depends on the number of machines and the output required. It takes about 17 sec to calculate θ and $\Delta\theta$ per interval for a 14-machine study, and about 15 sec to print them.

(6.3) Example

Table 3 and Fig. 12 give the system data and initial conditions for a simple 5-machine problem. A stability study was carried out for a 3-phase fault on line 9 (near busbar 9) followed b simultaneous openings of the circuit-breakers at both ends c the line after 0·125 sec. The results relative to machine 4 ar plotted as acceleration loci in Fig. 13.

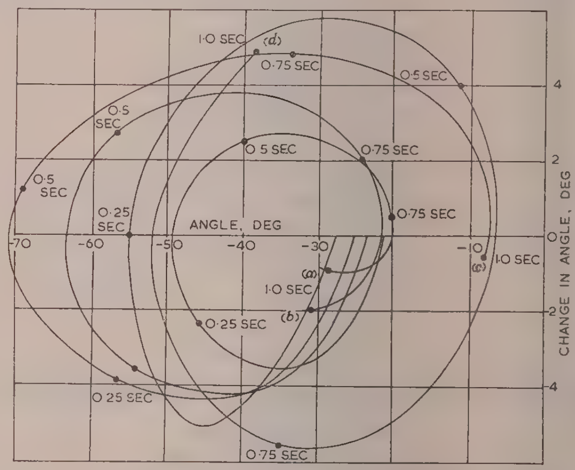

Fig. 13.—Five-machine stability study. Acceleration loci relative to machine 4.

(a) Machine 2.
(b) Machine 1.
(c) Machine 0.
(d) Machine 3.

Table 3

SYSTEM DATA FOR 5-MACHINE STABILITY STUDY

Line number	Resistance	Reactance	Shunt susceptance
	per unit	per unit	per unit
1	0·018	0·159	0·478
2	0·01	0·085	0·256
3	0·003	0·067	0
4	0·004	0·084	0
5	0·014	0·119	0·36
6	0·005	0·112	0
7	0·015	0·132	0·592
8	0·003	0·052	0
9	0·022	0·198	0·596

Base MVA, 200.

(6.4) Representation of the Generator

Provision has been made in the programme for the inclusion of significant generator characteristics which are normall neglected on network analysers. In order to get the schem into operation, these improvements have been introduced in th simplest form. They can be replaced without difficulty by mor accurate methods as and when the need arises and the necessar data are available.

Sub-routines have been included which provide for saliency, flux decrement, positive-sequence damping, voltage-regulator action and governor action. The representation of flux decrement and damping is mainly based on the methods suggested by Crary.[13] A brief review of these is given below. For simplicity they are discussed in terms of a single machine connected to an infinite busbar.

(6.4.1) **Governor Action.**

A simple linear representation with a dead band as shown in Fig. 14 may be used. It is assumed that the governor does not

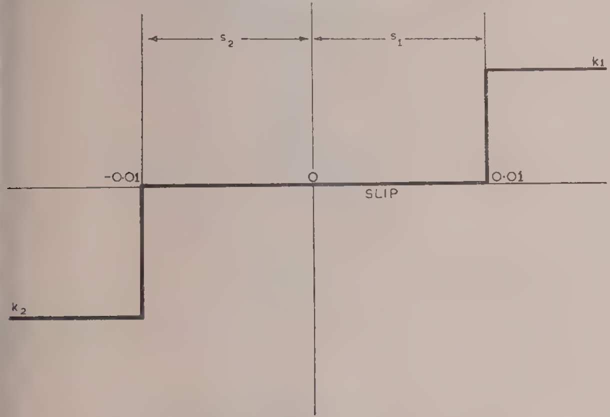

Fig. 14.—Representation of governor action.

act for slips between s_1 and s_2. At other times the mechanical power input at the end of any time interval Δt is given by the equation

$$P_n = P_{n-1} + k\Delta t \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where P_n = Power at the end of the interval.
P_{n-1} = Power at the start of the interval.
$k = k_1$ or k_2 depending on the slip.

This representation assumes a constant rate of opening or closing and neglects the effect of speed of the machine on the rate of governor response. A limiting value of the mechanical power input beyond which it cannot be increased is also specified.

(6.4.2) **Saliency, Flux Decrement and Voltage Regulator.**

Fig. 15 shows the vector diagram of a synchronous machine. This diagram and the notation are well known. The following

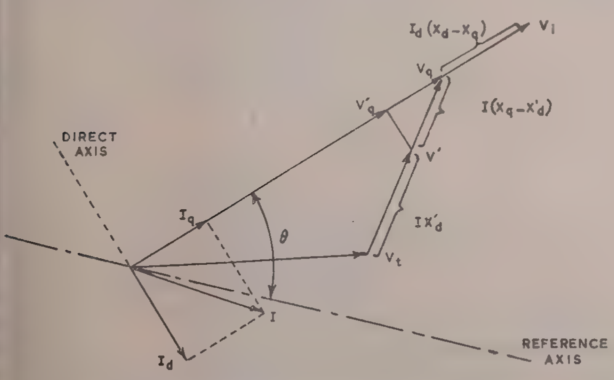

Fig. 15.—Vector diagram of a synchronous machine.

equations can be written for a salient-pole machine connected to an infinite busbar with voltage V, and if θ is the rotor angle with respect to the infinite busbar

$$V_q' = \{[1 + (X_d' - X_q)\sin\beta_{11}]/Z_{11}\}V_q - [(X_d' - X_q)V\sin(\theta + \beta_{12})]/Z_{12} \quad . \quad (16)$$

$$V_i = \frac{X_d - X_d'}{X_q - X_d'}V_q - \frac{X_d - X_q}{X_q - X_d'}V_q' \quad . \quad . \quad . \quad (17)$$

From eqn. (16)

$$V_q = \frac{V_q' - [(X_q - X_d')V\sin(\theta + \beta_{12})]/Z_{12}}{1 - [(X_q - X_d')\sin\beta_{11}]/Z_{11}} \quad . \quad (18)$$

In the above equations, Z_{11}, Z_{12}, etc., are the transfer impedances of the circuit including the value of X_q for the machines and β_{11}, β_{12}, etc., are the respective angles of these impedances.

The value of V_q before the fault can be obtained from the results of the load study. Eqn. (16) can then be solved to give the initial value of V_q'.

The equation for the electrical torque, T_e, of the machine is as follows:

$$T_e = (V_q^2/Z_{11})\sin\alpha_{11} + (V_qV/Z_{12})\sin(\theta - \alpha_{12}) \quad . \quad (19)$$

where α_{11} and α_{12} are the complements of the angles β_{11} and β_{12}, respectively. Substituting Z_{11}, Z_{12}, α_{11} and α_{12} corresponding to the network with fault, V_q and T_e can be computed using eqns. (18) and (19), respectively. The usual step-by-step method can then be followed to compute θ at the end of the interval. If the field flux linkage is assumed to remain constant, V_q can be recalculated using the same value of V_q', and the process is continued as before.

The above representation thus allows for the saliency of the machine. To represent saturation, voltage-regulator action or flux decrement, a variation in V_q' is calculated as follows:

$$\Delta V_q' = [(V_{ex} - V_i)/\tau_{d0}]\Delta t \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where V_{ex} = Per-unit exciter voltage.
τ_{d0} = Direct-axis open-circuit field time-constant, sec.

V_i can be calculated from eqn. (17). Saturation is represented by calculating the voltage behind the Potier reactance and hence appropriately correcting V_i from the open-circuit saturation characteristic of the generator. For simplicity, the voltage regulator is assumed to have an exponential response with a dead band, from which the variation in V_{ex} with time may be calculated. If it is to be ignored, V_{ex} may be assumed constant and equal to V_i before the fault. Knowing V_{ex} and V_i, the change in field flux linkage can easily be determined and $\Delta V_q'$ added algebraically to V_q' to give the value for the next interval. For greater accuracy, the values of V_{ex} and V_i at the middle of the time interval should be used in eqn. (20). They can be obtained by extrapolation.

In this way a simple representation of saliency, flux decrement, saturation and voltage-regulator action can be obtained.

(6.4.3) **Damping.**

The damping torque is proportional to the slip and can be defined by the equation

$$T_{damp} = T_d s \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

where T_d is the damping constant and s is the slip. The damping constant can further be shown to be

$$T_d = a\sin^2\theta + b\cos^2\theta \quad . \quad . \quad . \quad . \quad . \quad (22)$$

where a and b are constants depending on machine parameters. Usually, it is not necessary to consider the variation of T_d with the rotor angle, and a constant value may be assumed. The slip can be expressed in terms of $\Delta\theta$ and the damping torque calculated for each interval.

(6.5) Stability Studies with Improvement

Results obtained from studies carried out on a single machine connected to an infinite busbar and on a 7-machine system are given below to demonstrate the effect of these improvements.

(6.5.1) Single Machine System.

The line diagram for the system is shown in Fig. 16. Table 4 gives the data for the problem. Three-phase faults at the machine terminals were considered. The fault was removed and the circuit-breakers were reclosed after a specified time. Two studies were carried out, one with full-load power transfer

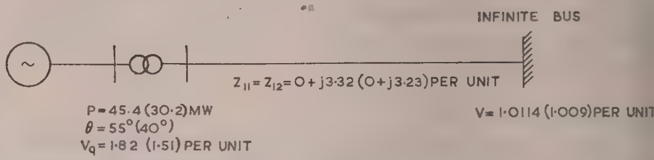

Fig. 16.—Line diagram of system for single-machine stability study.

Base apparent power = 100 MVA.
Unbracketed values refer to study 1.
Bracketed values refer to study 2.

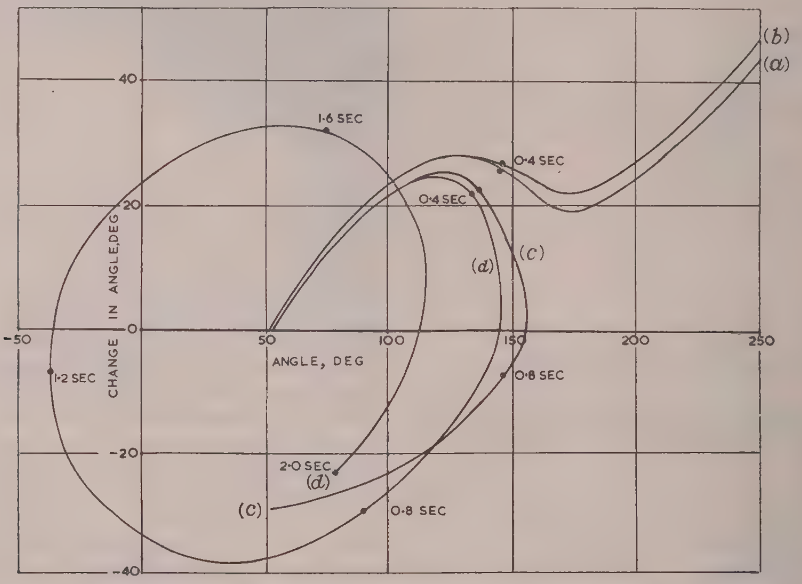

Fig. 17.—Single-machine transient stability study 1.

(a) No improvement.
(b) With flux decrement.
(c) With governor action, damping and flux decrement.
(d) With governor action and damping.

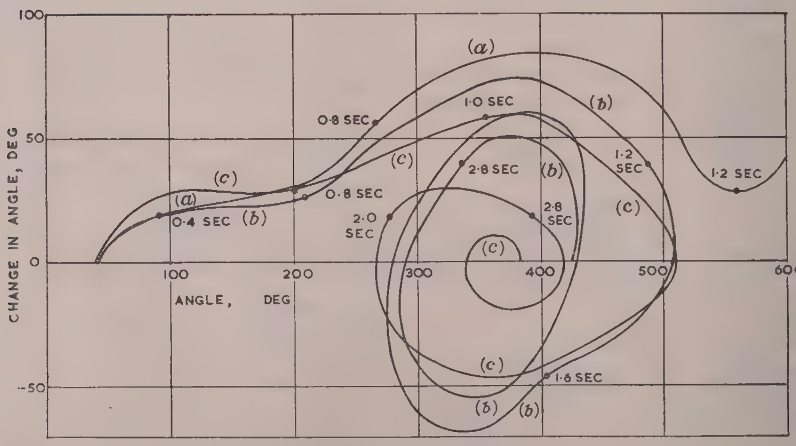

Fig. 18.—Single-machine transient stability study 2.

(a) Slower governor action.
(b) Faster governor action.
(c) Test results.

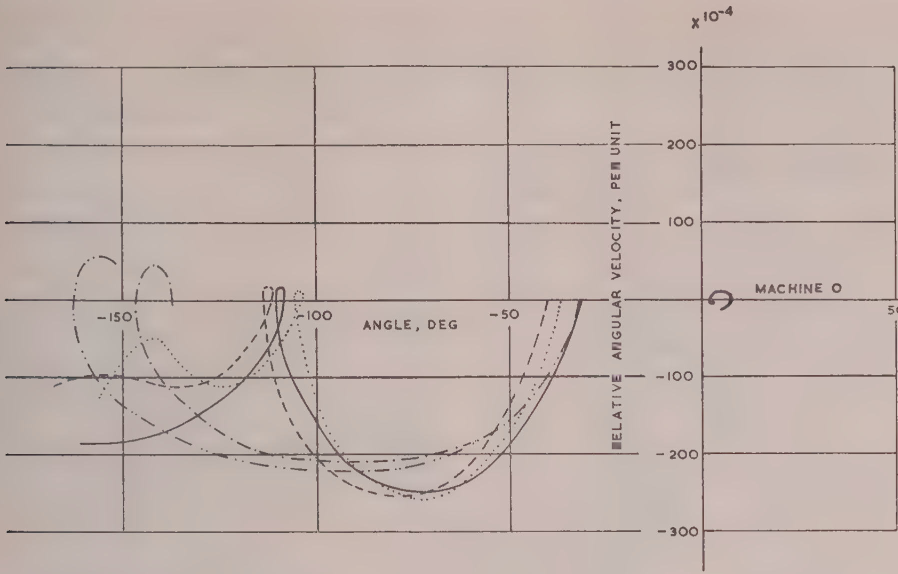

Fig. 19.—Seven-machine transient stability study (no improvements). Acceleration loci relative to machine 6.

——— Machine 1.
– – – – Machine 2.
· · · · · · Machine 3.
—·—·— Machine 4.
—··—·· Machine 5.

Table 4

DATA FOR SINGLE-MACHINE TRANSIENT STABILITY STUDY

Generator rating	56·3 MVA, 45 MW, 0·8 power factor
Synchronous reactance X_d	164% on 56·3 MVA
Transient reactance X_d'	17·4% on 56·3 MVA
Inertia constant H	5·85 kWs/kVA
Open-circuit time-constant τ_{d0}	9·9 sec
Governor characteristics	
Dead band s_1	0·001
s_2	−0·001
Constant k_1	0·2
k_2	−0·2
Damping constant	7·5

and the other at reduced load, each with a different switching time. In these studies saturation and voltage-regulator action were neglected, but the parameters for the other improvements were approximately correct.

Fig. 17 shows the acceleration loci for a clearing time of 0·32 sec for initial full-load power transfer. Fig. 18 shows the acceleration loci for the reduced load condition with switching at 0·68 sec. With the initial representation of governor, damping and flux decrement, the system is unstable, but if the rate of governor action is doubled, the machine slips a pole pair and is then pulled in. Actual results obtained on the field test from which data for these calculations were derived are given in curve (*c*).

(6.5.2) Seven-Machine System.

Results obtained from studies on a 7-machine 132 kV system with a total loading of 100 MW following a double line-to-earth fault are indicated in Figs. 19 and 20. One end of the faulty line was cleared after 0·12 sec and the other after another 0·06 sec. Fig. 19 indicates instability when the machine is represented by constant voltage behind the transient reactance and governor action is neglected. Fig. 20 indicates what happens when the improved method of representation including voltage regulator and governor action is used. The changes are mainly due to the governor action, and the curves indicate the danger of the present high-speed governors acting in anti-phase to the requirements of the system and causing instability of the system after a disturbance.

(7) CONCLUSIONS

The main objective in undertaking the work which led to the development of the programmes described in the paper was to examine the suitability of existing digital computers for carrying out power-system studies of the types normally performed on network analysers. The authors believe that the work which has been done demonstrates that, once suitable programmes have been prepared, digital computers are more economical and efficient, and that various improvements can be made in the methods used. Similar conclusions have been reached from the programmes which have been developed in the United States.

Special emphasis has been laid on the development of the logical control of the programmes so as to make the operation of the digital computer as automatic as possible and reduce the number of decisions required from the engineer. The load-study control programme has shown that programmes can be developed which can adjust conditions to make the system operate satisfactorily. Only simple methods have been used at present, owing to the restrictions imposed by storage capacity and speed.

Although these programmes are being used successfully for the solution of the problems for which they are written, the authors do not regard them as more than prototypes. Many future improvements are possible in the methods of representation, analysis and control, and these can be introduced to meet the particular requirements of those who are concerned with the solution of these problems in practice. With larger computers, a more general mathematical formulation of the whole power-system problem to cover both steady-state and transient conditions simultaneously will lead to a more powerful general method which can be linked with economic loading and minimization of losses.

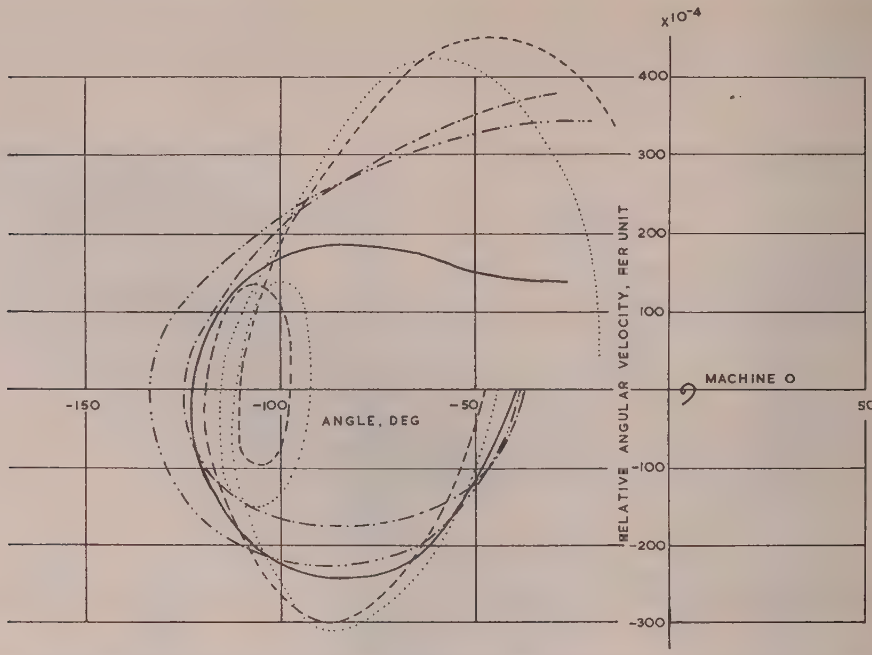

Fig. 20.—Seven-machine transient stability study (with improved generator representation). Acceleration loci relative to machine 6.

——— Machine 1.
------- Machine 2.
········ Machine 3.
—·—·— Machine 4.
—··—·· Machine 5.

(8) ACKNOWLEDGMENTS

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(10) APPENDICES

(10.1) Representation of Transformers

Special representation is required when the effect of transformer tappings has to be considered. If tappings are not considered and V_A and V_B are the voltages at the two busbars A and B [see Fig. 1(*a*) and (*b*)] the current I flowing in the transformer is given by the following equation:

$$I = (V_A - V_B)Y \quad . \quad . \quad . \quad . \quad . \quad (23)$$

where $$Y = 1/Z$$

but with the tappings set at off-nominal ratio n, the actual current flowing through the transformer into busbar B is

$$I_B = (nV_A - V_B)Y \quad . \quad . \quad . \quad . \quad . \quad (24)$$

The current flowing out at A is

$$I_A = n(nV_A - V_B)Y \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Hence, to represent the effect of the tappings on the currents and voltages in the rest of the system, additional currents $-\Delta I_A$

and $+\Delta I_B$ must be injected at A and B, so that the net currents flowing out at A and in at B are I_A and I_B, respectively. Thus

$$\Delta I_A = I_A - I \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

$$= n(nV_A - V_B)Y - (V_A - V_B)Y$$

$$= (n-1)Y[(n+1)V_A - V_B] \quad . \quad . \quad (27)$$

and

$$\Delta I_B = I_B - I \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

$$= (nV_A - V_B)Y - (V_A - V_B)Y$$

$$= (n-1)YV_A \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

Thus the additional currents introduced at A and B, together with the equivalent admittance included in the system admittance matrix, can be used to represent transformers with off-nominal ratios for the load studies.

These additional currents are calculated every cycle and added to the residual currents at the respective busbars. Although ΔI_A and ΔI_B may not be correct for the first cycle, the final voltages and currents will be correct to within the required accuracy as an iterative process is used for the solution.

(10.2) Calculation of Residual Currents

The load study programme calculates the first approximation to the voltage by computing and inverting the system admittance matrix. A current is injected to correct for the real and reactive power loading at every load busbar. The following scheme is used at generator busbars to try to keep the voltage at a fixed value and a predetermined power generation.

Suppose the voltage at a busbar after the nth cycle is V_n, and that a current I_{n-1} was injected in the previous cycle, the power, P_y, represented by the admittance Y, as calculated by eqn. (2), is

$$P_y = \mathscr{R}(Y|V_n|^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

The current I_{n-1} contributes

$$P_i = \mathscr{R}(I_{n-1}V_n^*) \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

The total power P_n, represented by the admittance and the current I_{n-1}, is

$$P_n = P_y + P_i = \mathscr{R}(Y|V_n|^2 + I_{n-1}V_n^*) \quad . \quad . \quad (32)$$

Therefore ΔP (the correction to power at the busbar) is

$$\Delta P = P - P_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

where P = desired power at the busbar.

Therefore a current ΔI should be further injected at the busbar to provide this additional power ΔP according to the following equations:

$$\Delta P = \mathscr{R}(\Delta I . V_n^*) \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

or

$$\Delta P = \Delta I_p V_p + \Delta I_q V_q \quad . \quad . \quad . \quad . \quad (35)$$

where

$$\Delta I_p + j\Delta I_q = \Delta I \quad . \quad . \quad . \quad . \quad . \quad (36)$$

$$V_p + jV_q = V_n \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

For any junction m, we have, from eqn. (12):

$$V_m = Z_{m1}I_1' + \ldots + Z_{mm}I_m' + \ldots + Z_{mn}I_n' \quad . \quad (38)$$

where Z_{m1} to Z_{mn} constitute the mth row of inverse of $[Y]$ [of eqn. (8)].

Ignoring the effect of changes at other busbars, a change in voltage ΔV will be produced by this change in current ΔI such that

$$\Delta V = \Delta V_p + j\Delta V_q = \Delta I Z_{mm}$$

Therefore $\quad \Delta V_p + j\Delta V_q = (\Delta I_p + j\Delta I_q)(R + jX) \quad . \quad . \quad (39)$

where $\quad Z_{mm} = R + jX$

Therefore

$$\left.\begin{aligned} \Delta V_p &= \Delta I_p R - \Delta I_q X \\ \Delta V_q &= \Delta I_q R + \Delta I_p X \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (40)$$

As this change in voltage is to be such that the final voltage has a magnitude $|V|$:

$$|V_n + \Delta V| = |V|$$

or

$$|V_p + jV_q + \Delta V_p + j\Delta V_q| = |V| \quad . \quad . \quad (41)$$

or $\quad V_p^2 + \Delta V_p^2 + 2V_p\Delta V_p + V_q^2 + 2V_q\Delta V_q + \Delta V_q^2 = |V|^2$

Neglecting ΔV_p^2 and ΔV_q^2 and noting that

$$V_p^2 + V_q^2 = |V_n|^2$$

$$2V_p\Delta V_p + 2V_q\Delta V_q = |V|^2 - |V_n|^2 = \delta \quad . \quad (42)$$

substituting for ΔV_p and ΔV_q from eqns. (40)

$$2V_p(\Delta I_p R - \Delta I_q X) + 2V_q(\Delta I_p X + \Delta I_q R) = \delta \quad . \quad (43)$$

From eqn. (35)

$$\Delta I_p = \frac{\Delta P - \Delta I_q V_q}{V_p} \quad . \quad . \quad . \quad . \quad . \quad (44)$$

substituting for ΔI_p in eqn. (43), and solving for ΔI_q

$$\Delta I_q = \frac{2\Delta P(V_p R + V_q X) - \delta V_p}{2|V_n|^2 . X} \quad . \quad . \quad . \quad (45)$$

ΔI_p can then be calculated using eqn. (44). ΔI should then be added to I_{n-1} to give I_n. It should be noted that the value of I' used in eqn. (38) is the sum of I_n and the equivalent current source computed before the matrix inversion, using eqn. (6).

(10.3) Derivation of Power-Equation Coefficients

A method based on nodal voltage equations to calculate the power-equation coefficients has been developed so that the sub-routines written for the load-studies programme could be used. A voltage behind an impedance can be replaced by an equivalent current source across the same impedance, as shown in Fig. 21.

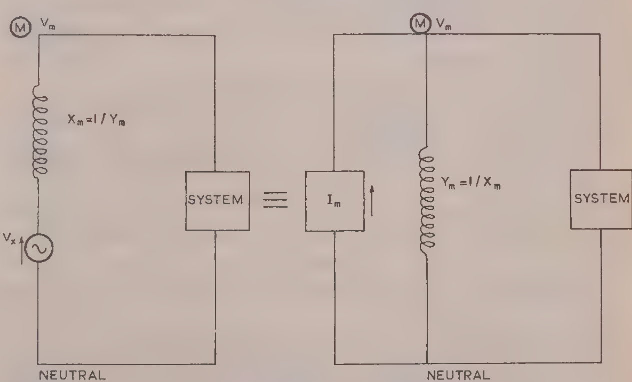

Fig. 21.—Computer representation by equivalent current source.
$I_m = V_x Y_m$.

The power output, P, of the machine, if it is represented by the voltage V_x behind the reactance X_m, is given by the following equation:

$$P = \mathscr{R}[V_x^*(V_x - V_m)Y_m] \quad . \quad . \quad . \quad (46)$$

where $Y_m = 1/jX_m$

$$\text{Thus } P = \mathscr{R}(|V_x|^2 Y_m - V_x^* V_m Y_m) \quad . \quad . \quad (47)$$

Now $\mathscr{R}(|V_x|^2 . Y_m)$ is zero, as the equivalent impedance of the machine is assumed to be a pure reactance. In any case the resistance is negligible. Hence eqn. (47) may be rewritten as

$$P = \mathscr{R}(-V_x^* V_m . Y_m) \quad . \quad . \quad . \quad . \quad (48)$$

Again, as Y_m is a pure susceptance,

$$Y_m^* = -Y_m$$

and hence

$$P = \mathscr{R}(V_x^* Y_m^* V_m) \quad . \quad . \quad . \quad . \quad . \quad (49)$$

$$= \mathscr{R}(I_m^* V_m) \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

as $I_m = V_x Y_m$.

Substituting for V_m in eqn. (50),

$$P = \mathscr{R}[I_m^* . (Z_{m1}I_1 + \ldots + Z_{mm}I_m + \ldots + Z_{mn}I_n)] \quad . \quad (51)$$

Now if $Z_{mp} = |Z_{mp}| \angle \beta_{mp}$ and $I_p = |I_p| \angle \delta p$, eqn. (51) can be reduced to

$$P = \sum_{p=1}^{n} |Z_{mp} I_m I_p| \sin(\delta_{mp} - \alpha_{mp}) \quad . \quad . \quad . \quad (52)$$

where $\alpha_{mp} = \beta_{mp} - 90°$, $\delta_{mp} = \delta_m - \delta_p$.

Eqn. (52) gives the power output of any one machine connected to the system. Similar equations are available for all the other machines. It must be noted that the rotor angle θ may be directly used instead of δ in eqn. (52) as $\delta = \theta - 90°$.

(10.4) List of Main Sub-Routines

All the sub-routines operate on complex numbers unless otherwise mentioned.

(10.4.1) General Sub-Routines.

All the general sub-routines perform general mathematical functions and can be used for other programmes. The explanations, however, are given in terms of the programmes developed.

N 1: *Computation of admittance matrix.*—Uses a connection table to form the admittance matrix of a network and store it as a lower triangular matrix. Scalar numbers only.

N 2: *Load and generator admittances.*—Given tables of net load and generation and the voltages at the junctions, computes equivalent admittances.

N 3: *Matrix inversion part* 1.—Matrix reduction.

N 4: *Matrix inversion part* 2.—Back substitution. Used to invert a symmetrical matrix.

N 5: *Matrix multiplication by vector.*—Multiplies a rectangular matrix by a vector.

N 6: *Network reduction.*—Reduces the admittance matrix of a system with n busbars to give the admittance matrix of an equivalent system with m busbars, eliminating the first $(n - m)$ busbars.

(10.4.2) Load-Flow Sub-Routines.

The load-flow sub-routines, although general in nature, were written particularly for the load-study programme and may not find wide application to other problems.

N 20: *Formation of equivalent network matrix.*—Reduces the admittance matrix formed by N 1, by eliminating the first row and first column to give the equivalent network matrix.

N 21: *Residual currents.*—Computes the residual currents at the junctions to correct the load or generation.

N 22: *Transformer tap correction.*—Computes the equivalent currents to represent the effect of transformers with off-nominal ratios.

N 23: *Line outage.*—Computes the equivalent currents to represent the effect of a line outage.

N 24: *Power flow in lines.*—Computes the power flow in lines given the line parameters, a connection table and a set of voltages at the junctions.

N 25: *Power at junctions.*—Computes the net load or generation at the junctions.

N 26: *Load study control.*—Used to vary the reactive-power generation at busbars and transformer tap settings to bring the voltages within limits.

(10.4.3) Stability Sub-Routines.

The stability sub-routines are used in the programmes for transient stability calculations.

N 30: *Electrical power output.*—Calculates the electrical power output of a generator using eqn. (13).

N 31: *Generator representation*

(*a*) *Damping approximation.*—Calculates the damping constant.

(*b*) *Flux decrement and saliency.*—Used to consider the effect of flux decrement and saliency.

(*c*) *Voltage regulator.*—Used to consider the effect of voltage regulator.

(*d*) *Governor action.*—Calculates the mechanical power.

N 32: *Rotor angle calculation.*—Calculates the rotor angle of each machine.

(10.4.4) Input and Output Sub-Routines.

Sub-routines N 40–N 43 are used for load studies, while N 44 is used for the transient stability programme.

N 40: *System data input.*—Reads and stores system data.

N 41: *Problem data input.*—Reads and stores problem data.

N 42: *Load studies output.*—Prints the line flows, results at the junctions, numbers of overloaded lines, transformer tap settings, and total net load losses and generation.

N 43: *Print voltages.*—Prints voltages at the end of every cycle.

N 44: *Print rotor angle.*—Prints the time, rotor angle, and change in rotor angle after every interval.

DISCUSSION ON THE ABOVE TWO PAPERS BEFORE THE SUPPLY SECTION, 18TH JANUARY, 1961

Mr. J. G. Miles: The papers deal with techniques for using digital computers to carry out some processes of power-system analysis which in the past have been done almost wholly on network analysers—namely load flow, fault and transient stability studies.

It is only four years since the publication of the first Institution paper on this subject, but in this period digital computation for such problems has become respectable, even if not yet a fully accepted alternative to network-analyser calculations. Progress in computation is fast, and since the present papers unavoidably describe work which was done some years ago there need be no surprise if the discussion includes mention of subsequent developments and improvements. However, this work has undoubtedly prepared solid foundations for future progress.

Since both papers approach the same end by different means it will be of interest to observe where they correspond, and more particularly where they differ. Both describe programmes written for computers which, by present-day standards, are small, and in both cases this factor has to some extent affected the form of the programme. In particular, it led Mr. John to adopt the

Gauss–Seidel iterative process as the basis of his load-flow programme, as opposed to the matrix inversion scheme used by Dr. Gupta and Prof. Humphrey Davies. Mr. John's paper is therefore naturally concerned to some extent with procedures for artificially accelerating the convergence of this process, whereas the other authors have given more attention to automatic control of various types.

Mr. John has hinted at the various factors which cause difficulty in obtaining digital solutions. We have now gained considerable experience in the use of programmes of this type, and these factors are clearly recognizable. Not surprisingly, they can also cause difficulty in obtaining analyser balances. The first is any network arrangement which gives rise to low values of self-admittance at one or more nodes. This arises particularly on systems with large capacitance compensation, either series or shunt, and also on systems having very unequal line impedances. Under such conditions Gauss–Seidel programmes may fail to converge, and the matrix inversion process may suffer from accuracy troubles.

The second factor is equally important. It is any system operating condition which approaches the region of voltage instability, whether this arises by design or incorrect specification of the reactive generation. Such a condition produces weak or negative convergence of the iterative process for constant power, whichever method is used for the solution of the network equations.

Operating experience with both programmes shows that many systems in fact exhibit one or other of these factors. This leads one to consider the features which a fully general digital load-flow programme should possess:

(*a*) It must be able to deal adequately with small nodal self-admittances and with operating conditions near the voltage stability limits. This may require the use of a solution process different from either of those discussed in the papers.

(*b*) It must deal rapidly with the introduction of small changes to an established balance, e.g. changes in generating specifications, transformer taps, or line outages. Provision should be made for some degree of self-balancing by the automatic insertion of the first two of these. This is essential to avoid difficulties arising from incorrect specification of reactive generation.

(*c*) If these programmes are to be used on large computers, and used efficiently, all possible variations in system arrangement should be programmed in advance and the computer allowed to select the optimum according to some given criterion. We cannot any longer work on a piecemeal basis, as in present network-analyser procedure.

I would be very interested to have the authors' opinions on these suggestions, particularly since fast and reliable digital computation would be an essential feature of any scheme for automatic system control.

Mr. D. G. Hawkins: The C.E.G.B. have had digital computers installed for over $2\frac{1}{2}$ years, including an IBM 709 computer for the last six months. We have found that, when an engineer approaches a complex transmission problem, he asks, 'Can we make a model to represent it?' He then thinks of possibly a d.c. representation and, following that, an a.c. analyser. I would make a plea that the first thing we should do in digital work is to look at it in isolation from the network analyser. Too often we keep comparing the methods of using the network analyser with those of the digital computer, and also trying to operate the latter in a similar way to the former.

The first step in using a digital computer is to express the problem in the simplest possible terms. Usually we find that this is not done for one reason or another, mainly lack of time. The initial time taken would be easily saved by one's being able to use less skilled staff to carry out the subsequent work using digital-computing methods.

Engineers tend to want their results immediately, but in most cases they will have to plan the work and possibly wait until the next day for the final results of network studies if the data have to be amended, particularly when using large and costly computers, and here I am thinking of the next generation of digital computers. We have had to face transmission design problems for a very long time, and I am sure that a delay of a few more hours would not make very much difference to their solution.

Approximately 300 hours of transmission work are carried out in a year using a Deuce computer, and we are beginning to carry out transmission design work on the IBM 709 computer. We have found that it takes about 4 min on the IBM 709 computer to solve an a.c. load flow of 50 busbars and 50 lines, and this includes a detailed print-out of the results.

Mr. K. C. Parton: I believe that the general lack of the routine use of digital computers for power-system calculations in the United Kingdom is due not so much to the lack of suitable programmes or computers, but mainly to the fact that up to now most organizations have been quite satisfied with the performance and results obtained from ordinary network analysers. Mr. John's suggestions that only digital computers can tackle exacting networks needing low R/X ratios, phase-shifting transformers, etc., are in any case not correct if one has a large network analyser of the Blackburn type.

However, a case is now becoming clear for carrying out certain types of power-system calculations on the digital computer, and thus the papers have great value.

My own experience has been with a large Blackburn analyser, on which we have for interest repeated some studies using the programme prepared by Dr. Gupta and Prof. Humphrey Davies. In using this programme, however, we have found that convergence has often been difficult, and the choice of the floating busbar has been important. More investigation appears necessary on these points.

There are several essential features that would appear to be necessary in computer programmes before they are likely to be used for all routine work. These are as follows:

(*a*) Continual access by the system study engineer to the computer is vital. It is unlikely that any major network studies can be carried out unless the system designer can be personally present to keep modifying the programme input throughout the study.

(*b*) Careful attention must be given to the organization of the programme so that an operator can easily scan given results and send in modified input instructions whenever he wishes. This aspect does not as yet appear to have received much attention, but is now becoming practical with computers capable of automatic 'time-sharing'.

The idea of a fully comprehensive programme that will always correctly solve a network automatically, even if it is an ill-conditioned network, does not seem to me to be either practical or advisable.

Dr. P. D. Aylett: We must regard these as pioneering papers. The art is still in a very rapid state of development, and thus there are two ways of looking at the application of digital computers to power-network problems. First we can consider just how we stand at this moment in time, and secondly we can consider how things are likely to develop over the next few years. We are, in fact, in the midst of a revolution in methods of engineering computation and analysis, brought about by the development of high-speed automatic digital computers.

We see from Mr. John's paper that he had great difficulties

and has done a very good job with what is a very slow machine indeed. A machine now being constructed will, for example, execute an instruction 10 000 times faster than the machine with which he had to work. This immense development must have a great influence on the way in which engineering problems are approached. The use of computers has already transformed design methods in many industries, and this is beginning to happen in the electricity supply industry.

The papers show an interesting contrast in the two methods used for carrying out load-flow studies. We have been studying the relative merits of the two types of programme on a Pegasus computer. Where the problem is storage limited, the use of the purely iterative Gauss–Seidel method described by Mr. John is advantageous. It is possible to solve a 130-busbar problem on Pegasus computer using the Gauss–Seidel method compared with the 35 busbars possible for the matrix-inversion method. On the other hand, it is likely that convergence to a satisfactory solution will be more rapid with the matrix-inversion method. We have used the matrix-inversion programme extensively, and although we have had difficulties with convergence this has generally been with systems operating close to the stability limit. In these cases the selection of the slack busbar is important, and in order to give ourselves freedom in this respect, we are developing a method of distributing system active and reactive losses over a number of points.

What is now wanted is to develop power-network optimization criteria. We require a network which transmits and distributes electricity with the best security level and minimum losses, at the lowest possible cost. These factors could be included in computer programmes in the future.

Monsieur C. Marique (*Belgium*): I would like to make some remarks on what we have done in Belgium on the load-flow programmes. I will not go into mathematical explanations since we use the Gauss–Seidel iteration process, but I will deal with the four main kinds of control which we have in the programme to obtain a good solution. First, the network is divided into zones, a zone being a set of busbars linked together by lines and linked with other zones exclusively by transformers. Thus, in a zone, we have no transformer. In each zone there is one busbar at which the voltage must be kept within given limits to control the general level of voltages of that zone. Periodically, if the voltage of that particular busbar in each zone is not within the limits, the taps of the surrounding transformers are automatically changed.

Secondly, the programme controls the flow of reactive power through the transformers between two zones. It can happen that a final solution is obtained with reactive power circulating between two or more transformers, and, of course, that solution is not acceptable. Thus, for all transformers linking two zones, the programme can correct the reactive power flowing through the transformers by altering the tappings. Thus we can arrange to have at will, the same power factor, or different ones, for all the transformers.

The third kind of control maintains the voltage on any busbar at a particular value. This is done by correcting the reactive power of a nearby generator, or by tappings on a transformer. For each of the busbars, the corrections can be made at up to three points (generators and transformers) in a given order of preference.

The last control concerns the slack busbar which takes up the losses. Regardless of the losses, its output (or input) can be kept within given limits by modifying, during the calculation, the output (active, reactive or both) of other chosen generators. In that manner the slack busbar can be chosen in the best way for the convergence of the problem.

Finally, I would like to mention the representation of transformers. We use an equivalent circuit, with a series impedance for the load losses and a shunt impedance for the no-load losses. In most transformers the impedance changes appreciably with the tapping. The programme takes this into account, provided that the equivalent series impedance can be represented by a simple equation (of second degree at the most) as a function of the variable ratio.

Mr. D. G. Taylor: An interesting feature of the two papers is that two well-known but quite different methods have been used for the load-flow problem. It has been suggested that the so-called Gauss–Seidel method might be suitable for small computers, and that, looking ahead, the matrix method might be more suitable for the large machines of the future. I find this a little surprising. The storage requirements and the time per iteration both increase with the number of busbars for the Gauss–Seidel method (assuming a fairly constant number of lines per busbar) and with the square of the number of busbars for the matrix method. The time for the matrix inversion, which is a necessary part of the matrix method, varies approximately with the cube of the number of busbars. Our experience suggests that the Gauss–Seidel method is faster for systems with more than 20 or 30 busbars.

In our Gauss–Seidel programme we have found it expedient to store the admittance coefficients in packed form, each coefficient having an associated address. This reduces both the storage required and the time per iteration.

Fig. 5 of Mr. John's paper shows the effect of accelerating factors on convergence. The problem of predetermining the optimum accelerating factor has received much attention during the past few years in the context of the solution of partial differential equations by relaxation methods. I am interested to learn of experiments using different factors for real and imaginary parts.

In Section 6 of the paper by Dr. Gupta and Prof. Humphrey Davies a description is given of a method for deriving the B and α coefficients required for transient stability studies. It is the matrix-inversion operation (Fig. 7) which I find surprising, since it is usual to derive the coefficients directly from the reduced admittance matrix.

In transient-stability studies we have found it worth while to use a more sophisticated method for forward integration than the conventional step-by-step method used on network analysers, particularly when dealing with induction motors when we would otherwise have had to use a very small step. In Section 6.4.2 there is an analysis of a synchronous machine connected to an infinite busbar. It is not clear how this analysis was extended for the 7-machine problem described later. Was an iterative procedure used?

Mr. T. W. Berrie: Much of the subjects of 'conversion' and 'voltage stability' can be summed up in 'network conditioning'. Existing network analyser studies produce a well-conditioned network. One presents to existing digital programmes a well-conditioned network and the results are good.

For a small system, network-analyser simulators are probably the best device to use for transient-stability studies. For more than about six busbars one should use a digital solution for studies involving the usual simplifying assumptions. It is very dangerous to add parameters piecemeal to transient-stability studies. Experience shows that the turbo-alternator unit must be represented completely or the results are misleading. It will be many years before the digital computer takes over from the true analogue computer for complex transient-stability studies.

I was surprised to read that the installed cost of a modern network analyser approaches that of a high-speed computer, since the former is probably the cheapest of all analytical equipment.

For the future we must tap the resources of the system design engineer and also take advantage of the logical nature of the computer.

Finally, since over 30% of our overall study time is at present taken up with the processing of power-system data, the programmes of the future will make the computer process its own data, extracting it automatically from the main data store.

Mr. E. Marchand: A digital computer is basically a mathematical tool, and any use made of it for network analysis will usually be incidental to its other functions in any organization. This is another way of saying that if one has a computer one might as well use it for network analysis. I am firmly convinced that organizations such as the Electricity Boards would find it more suitable to have separate analogue equipment for each type of problem.

These opinions are largely the result of investigating systems in which only the lines and loads are fixed and where the output of generators can be specified only within wide limits. This means, in effect, that on a network analyser all generator busbars can be assumed to be floating without appreciably adding to the work. In the example given in the paper by Mr. John, the outputs P_g and Q_g of two generators are fixed, leaving the third generator at the floating busbar to supply some power for the loads and all the network losses both real and reactive. No mention is made in the paper of a method of specifying the maximum reactive output of all generators, or possibly re-allocating the losses.

My criticism of the test problem is that, although it emphasizes minor errors in network analysers, it is not a typical problem since most networks have better symmetry. However, when the approximate nature of the starting information is taken into account, these small errors fade into insignificance.

For comparison, the following times refer to a similar study on a 50 c/s conjugate impedance analyser:

Setting up and connecting: 40 min.
Balancing: 15 min.
Single-end reading: 10 min.
Total: 65 min.

The low-speed computer would take 141 min. For subsequent studies the respective times would be 15 and 100 min. These times can be achieved on a machine costing about one-third of the price of the low-speed computer.

Mr. D. A. Newey: It is relevant to consider the application of the computer to other routine aspects of power-system analysis, apart from those dealt with in the papers. The following examples show how a Mercury computer is being used in conjunction with a Blackburn analyser to make the most economical use of both.

With both digital and analogue computations it is necessary to refer the system to a base value. A digital-computer programme has been written to deal with this routine calculation.

The initial input data to the computer consists of line, load and generator details. In the permanent data the impedances of several types of cables and lines are fed in. It is therefore only necessary to refer to the type of line and its length for the programme to calculate its impedances and susceptance to the base. The line susceptances are combined at the nodes to give a π representation. The load admittances are calculated given any combination of data, i.e. active, reactive or apparent power, power factor, etc. Generators are dealt with similarly, the transient details being evaluated for stability studies. Fig. A shows the teleprinter print-out of the final system data. The advantages of this method are as follows:

(*a*) All the constants can be stored easily for use on the analyser of the digital computer.

(*b*) Irrespective of the person carrying out the study the data are in a standard form, which makes back-checking easier.

(*c*) Time is saved since several man-days are usually spent in preparing data. This study data took 3 min computer time.

Another tedious task is obtaining transient-stability swing curves by a step-by-step method. The present papers deal with

```
BASE   MVA = 200
BASE   KV  = 132          11
BASE   Z   = 8.71         0.605


LINE       P.U.  IMPEDANCE

1 - 2      0.0171 J 0.2802 T
2 - 3      0.0332 J 0.1439
2 - 4      0.0332 J 0.1439
4 - 5      0.0972 J 1.0650 T

BUS        LOAD            SUS.          GEN.
              (P.U.  ADMITTANCE)

 1    0.000 -0.000     J0.000     -0.140 0.105
                                         0.000
 1                                -0.140 0.105
                                         0.000
 3    0.024 -0.009     J0.000
 4    0.088 -0.048     J0.000
 5    0.050 -0.015     J0.000

DT = 0.025
F  = 50

BUS    P.U. K           P.U. XD

 1     11.48        0.000  1.057
 1     11.48        0.000  1.057
 7      2.43        0.007  0.530 T
```

Fig. A.—Teleprinter data print-out.

a complete programme for this, but at present we are finding a combination of analyser and digital methods to be economical when, as is usual, stability studies follow power-flow studies. The initial data are obtained from these power flows on the analyser, and the impedance matrices for various circuit conditions are measured directly from the analyser.

The Blackburn analyser with digital metering is ideally suited to these measurements, a 20 × 20 matrix being obtained in under three hours. The final total computer time for a 23-machine problem was 13 min.

The organization of these programmes is being carefully watched in order that they can be married with programmes mentioned in the papers to give a complete digital-computer programme for carrying out power-system analysis whenever this becomes economical.

Mr. M. D. Bakes: Before contemplating the extension of the present work to more comprehensive programmes, it is important to consider carefully whether to employ system-reduction techniques or to retain the basic network equations, and also whether to employ the Gauss–Seidel iterative scheme or to perform an explicit inversion.

I have been concerned with the preparation of programmes for use in conjunction with a Blackburn network analyser, and owing to the use of certain matrix manipulations the explicit inversion of the basic network matrix appears to have a number of important advantages.

For example, in the paper by Dr. Gupta and Prof. Humphrey Davies, it appears that, in the calculation of power-equation coefficients, the process of system reduction followed by matrix inversion is repeated for each fault condition. This can be avoided if we omit the system reduction and invert the full system matrix for the pre-fault condition.

If we call the pre-fault admittance matrix Y and the modified fault condition matrix Y', we can write Y' as

$$Y' = Y + uv^T,$$

where u and v are single-column matrices used to define the system modification, and v^T is the transpose of v. We can then find $(Y')^{-1}$ from

$$(Y + uv^T)^{-1} = Y^{-1} - (Y^{-1}u) \quad (I + v^TY^{-1}u)^{-1} \quad (v^TY^{-1})$$

As an example, suppose we have a single short-circuit on node j.

Let x_j be the jth column of Y^{-1}, y_j^T be the jth row of Y^{-1}, and y_j^{-1} be the jth diagonal element of Y^{-1}. Then it can be shown that

$$(Y')^{-1} = Y^{-1} - \frac{x_jy_j^T}{y_j^{-1}}$$

Since we already know Y^{-1}, the only work involved in finding Y'^{-1} is the matrix multiplication of a column matrix by a row matrix followed by the subtraction of this matrix product from Y^{-1}. Any other type of fault can be dealt with using similar procedures, i.e. involving only column-by-row matrix multiplication followed by matrix addition.

The advantage of this scheme is that the computer only has to invert the network matrix once, and all subsequent fault conditions can easily be derived from this inverse. This means that the computer time is reduced by a factor of the order of N, where N is the number of fault conditions applied, and this results in significant reductions on large networks where several fault conditions are to be studied.

Mr. A. Brameller: Operating experience with the load-flow programme described by Mr. John has shown that, whilst the method gives satisfactory results for many systems, in some cases slow or no convergence is obtained. We have therefore attempted to develop a more general method of solution.

The first aim was to reduce the difficulties caused by systems having weak diagonal terms in the admittance matrix. The Gauss–Seidel process in the original programme was replaced by direct matrix inversion for both the load-flow and short-circuit studies. The computer storage required for the direct inversion programme is the same as that for the Gauss-Seidel process. Difficulties were still encountered with the load-flow programme for systems operating close to the voltage stability limit, and attention was therefore given to improving the method and carrying out iterative calculations maintaining constant voltage and power at specified busbars.

In the original programme by Mr. John the basic single iterative calculation consists of the following:

(*a*) Calculations of a set of nodal injected currents for specified active and reactive powers and a particular voltage distribution.

(*b*) Calculation of a new set of voltages using the current obtained in (*a*).

Modification to this procedure consisted in evaluating the voltage and the associated nodal current simultaneously. This method gives the fastest convergence yet obtained, and it has the added advantage that it enables generator outputs to be maintained as constant active and reactive power or constant voltage and active power without additional computation time.

Table A illustrates the relative times of computation for the various methods on four different networks. Systems A and B operate under normal conditions, system B exhibits weak diagonal terms in the admittance matrix, system C operates at conditions approaching voltage instability, and system D combines the bad features of systems B and C.

Table A

DIGITAL LOAD FLOW SOLUTIONS BY DIFFERENT METHODS

Network characteristics	Number of iterations		
	Method (*a*)	Method (*b*)	Method (*c*)
A. Normal	70	17	11
B. Weak diagonals	900	6	4
C. Voltage sensitive	Divergent	700	15
D. Weak diagonals and voltage sensitive	Divergent	Divergent	150

In each case the number of iterations required to obtain convergence to the same accuracy with the various methods is shown. Method (*a*) is the Gauss–Seidel process described by Mr. John. Method (*b*) has the direct inversion in place of the Gauss–Seidel process. Method (*c*) uses direct matrix inversion and solves for voltage and current simultaneously. In each case the time required per iteration is more or less the same except for method (*a*), which is a little longer.

Dr. J. R. Mortlock: During the discussion various claims have been made with regard to load studies for systems involving varying numbers of nodes. With a Pegasus computer, which costs about £50 000, installed, with 28 or 30 nodes the time taken for a solution is 900 sec. Going up the scale, a Mercury computer (installed cost of about £150 000) will deal with 60 nodes and take 1 000 sec. This is about the same order of time, but twice as many nodes are involved. With an IBM computer, with a six-figure capital cost, 50 nodes can be dealt with in 60 sec. Various figures, squares and cubes have been given for the time taken as a function of the number of nodes, and so for 60 nodes the time could be 100 sec. Therefore, to double the number of nodes dealt with in the same time a computer would cost three times as much. If we retain the same number of nodes and reduce the time to one-tenth, the cost of the computer could be increased by a factor of ten. In the future we want to control a system by a computer. Let us take a 50-node system and assume, quite simply, that each node has only one interconnection, and that, with sources plus loads, there are 100 circuits in all. We want to be sure that, if we lose any one of these circuits, the operator knows about it, and is warned that if he has an additional outage the system will be in trouble. How long would it take for him to know? In our assumed 50-node system, which is fairly small, there are 100 circuits. We can assumed that it will take 100 min to scan the system on the IBM, for 100 studies. Thus, with a computer having a six-figure capital cost, the operator would know after about two hours whether the system was risky or not. Obviously we have a long way to go.

Advanced system analysis was largely a closed book in this country until about 12 years ago. Our approach on the digital side may be conditioned quite materially by the fact that all our background is on a network-analyser basis. It may be better to start again and approach the subject from a purely mathematical angle, to see if we can reduce materially the time taken to carry out a study.

Mr. K. J. Eales: I do not claim to be an expert on this subject, but this does not mean that I am unable to make good use of a digital computer for power-system studies, since I recently proved to my own satisfaction that this device is a powerful tool

in the hands of an engineer who has had little or no experience in power-system studies. With this method it is the computer which does the actual detailed analysis of the system and not the engineer.

In Section 7 of the paper by Dr. Gupta and Prof. Humphrey Davies it is stated that with larger computers a more general mathematical formulation of the whole power-system problem to cover both steady-state and transient conditions simultaneously will lead to a more powerful general method. This statement implies that they have already covered the steady-state condition and hence, presumably, the steady-state stability. However, B.S. 2658 states that the test for steady-state stability is the behaviour of the system in the presence of small changes. As it is well known that the load-flow study does not take into consideration any such changes, I would like to ask the authors whether a programme could be devised to cater for 'steady-state stability' and, if so, would they be able to obtain both the 'inherent stability limit' and the 'constant flux stability limit' as defined by the same British Standard?

Mr. H. I. J. Goldberg: With digital solutions, the actual time taken and the cost of doing the study both depend on the computer used and the coding or facilities of the programme. For a Gauss-Seidel programme on Pegasus I we have had, for a 7-busbar system, a time per iteration of 3–4 sec. This is the same iteration time as that for the programme in the paper by Dr. Gupta and Prof. Humphrey Davies with the same system, while with the inverse matrix method additional time is required for inverting the matrix.

For Sirius, which is a smaller and cheaper computer, the time per iteration was 12 sec. Speed was sacrificed so that one could study a larger system on the computer. The size of the system which one can study depends not only on the computer, but on how the programme is coded. With the programme of Dr. Gupta and Prof. Humphrey Davies, on a Pegasus I, which has an 8 000-word drum, and a re-entry facility, one can deal with 33 busbars. On Sirius, which has a 4 000-word store, with a Gauss–Seidel programme and no re-entry facility, one can deal with up to 130 busbars.

One of the advantages of the Gauss–Seidel system is the facility of linear acceleration factors, which can speed up the programme time by reducing the number of iterations. My experience with the small 7-busbar system was that, with no acceleration factor, it took 28 iterations to get a solution, but with the optimum acceleration factor, it took only 11 iterations. The acceleration factor varies with the system, and ultimately the computer should be able to compute automatically the optimum acceleration factor, thereby producing the quickest solution first time.

THE AUTHORS' REPLIES TO THE ABOVE DISCUSSION

Mr. M. N. John (*in reply*): It would appear to be generally accepted that the eventual use of digital computers for routine network analysis is now regarded as fact. The actual date of this happening is chiefly a question of economics, i.e. the present generation of network analysers will, with some exceptions, be replaced by digital computers or by the facility of hiring computer time. In the latter case, the possibility of data transmission links in conjunction with large time-shared computers operating at relatively low per-unit costs will completely alter the future perspective.

Much of the discussion has been concerned with the important question of the best means of using digital methods and how this affects the programmes to be developed. Mr. Miles feels that there is a case for a fully general load-flow programme with all possible variations programmed in advance on the grounds of efficiency, whereas Mr. Parton sees the need for continual access by the system engineer to be vital. My own feeling is that these two points of view are irreconcilable, and that a case clearly exists for two types of programme:

(*a*) The most numerically efficient programme requiring least external interference for use on well-conditioned systems, or for parameter surveys including the economic effects mentioned by Dr. Aylett. Such a programme would also fulfil the requirements for automatic system control by a digital computer.

(*b*) A programme which will perform in such a way and provide such information that the user, who can interrupt it if required, will obtain a maximum amount of information for the investigation of new or difficult systems.

A comparison between my paper and that of Dr. Gupta and Prof. Humphrey Davies provides an example of two different approaches to these diverse aims. Here, again, the advent of time-shared computers in which interruption is not an undue economic liability will modify future developments.

I differ with the opinion that digital methods should always be looked at in isolation from network-analyser methods, especially in the case of the second type of programme described above. By definition, the analyser is an analogy of the physical system, and this so-called 'network analyser approach' is often no more than the user is required to think in system terms, which is surely a process to be encouraged.

Mr. Parton correctly states that the Blackburn analyser has certain advantages in some of the types of study listed in the paper as difficult on the a.c. analyser. This does not, however, affect the case for the ease with which even a low-speed digital computer can carry out these and other studies involving subsidiary hand calculations.

In reply to Mr. Marchand, first, there is no difficulty in an iterative programme in allowing the generator outputs to vary within prescribed limits or to incorporate any similar restraints. Secondly, it is stated in the paper not that the sample problem is typical but that it contains many typical system elements, and further that it provides pessimistic conditions for the convergence studies. It is only in the latter connection due to voltage sensitivity that the differences between the analyser and digital solutions are emphasized.

The network-analyser cost in the comparison queried by Mr. Berrie was based on that of a comprehensive modern analyser of some 20 generator units fitted with simulator facilities for transient stability studies.

In reply to Mr. Taylor on the use of different accelerating factors for the real and imaginary parts of the difference voltages, these experiments were not carried far. Since overall convergence was dictated by the more slowly convergent real parts, it was found that one could not exceed more than a critical value of the factor for these, and that different factors applied to the imaginary parts produced little overall reduction in the time to converge. The position might be altered for different systems.

I am grateful to Mr. Brameller for the information on improvements to the Gauss–Seidel programme. In particular, the simultaneous evaluation of voltage and associated nodal current, as well as reducing the number of iterations and permitting the incorporation of prescribed generating schedules, would seem to assure a convergent solution except in cases of true system-voltage instability. This is a most significant effect.

Dr. P. P. Gupta and **Prof. M. W. Humphrey Davies** (*in reply*): It is very gratifying that the old controversy between network analysers and digital computers has not been brought up and there is wide agreement about the usefulness and advantages of computers. Although Mr. Marchand still feels that analogue devices are better, by the time a simulator was installed for every job, the cost would be prohibitive.

Messrs. Berrie, Parton and Newey recommend the combined use of analysers and computers. In organizations where large network analysers are already installed, it is natural that the analyser will be used until such time as easy access is available to digital computers with programmes that represent an advance on past analyser techniques. Dr. Mortlock gives some interesting figures to illustrate the cost of using these methods. A sufficiently large computer can carry out all the work that an analyser can do and have much time available for other classes of work. If proper use is made of the computer we are convinced that it would be uneconomic for any organization to install an analyser.

Mr. Berrie suggests that it is dangerous to add parameters piecemeal to transient studies and recommends the use of analogue computers. While analogue devices may still be useful for the design of the control system of individual units, we believe that, for the study of the transient behaviour of systems containing many units, more accurate methods can be developed for the digital computer. The present programmes are only a step towards more complex representation of the machine, but they may be used to investigate the effects of individual parameters, and this can be very useful in system analysis and design.

We agree that all data processing should be carried out automatically. In this respect we are grateful to Mr. Newey, who indicates some of his procedures and their advantages.

We are in full agreement with Mr. Miles about the aims of a digital-computer load-study programme. Our main objective even at this stage has been to develop a suitable control programme which can satisfy all his criteria. The latest control scheme in which initially specified values may be varied automatically helps the convergence even of ill-conditioned networks. We are grateful to Mr. Brameller for his comparative study of various methods, which shows the rapid convergence obtained by inverting the admittance matrix.

We must disagree with Mr. Parton about his view of the needs of a load study on a computer. It will generally be uneconomic to try to treat the computer like an analyser and expect to have facilities for continuous access and manual intervention. The logical facilities of modern computers enable programmes to be developed which can make most of the engineering decisions. It is relatively easy to repeat a particular study if additional data are subsequently required.

Convergence of the iterative process is an essential requisite, and ways to achieve this should be an important feature of the control programme. The choice of floating busbar at present does play rather an important part in this respect, but the computer may be made to select another junction automatically without any difficulty. We are sorry that Mr. Parton did not get satisfactory convergence on all his studies. Most of the studies to which he refers gave solutions at the first attempt using the old control programme.

Another question which has been attracting attention is the choice of the methods. We feel that for larger and faster computers the matrix inversion method is more suitable. It has been shown that it leads to convergence much earlier than the Gauss–Seidel method. The matrix inversion has to be carried out once only. Methods are now available to alter the inverse matrix to represent changes in network conditions, thereby facilitating the study of different system conditions. We are grateful to Mr. Bakes for indicating one such method and hope to incorporate it in our programmes at a later stage.

One of the important features of the matrix method is that the number of iterations needed for convergence is independent of the size of the system. Thus although, as Mr. Taylor points out, the time for matrix inversion will be much longer, the total time for a series of studies will be of the same order if not less for larger systems on larger computers.

Mr. Taylor also questions the derivation of the B and α coefficients required for transient-stability studies. The matrix inversion is necessary for the method adopted and enables the rotor angles to be used directly in the calculation without involving any transformation of co-ordinates which may make the actual swing-curve calculation slower. We feel that this should be kept as fast as possible, even if the preliminary calculation takes slightly longer.

We have found the step-by-step method fairly satisfactory and accurate. More sophisticated methods of forward integration can, of course, be used but they take up more time and space on the computer. The detailed analysis of the synchronous machine can easily be extended to a multi-machine system by solving simultaneous equations relating V_q' and V_q for every interval. An iterative procedure is then unnecessary.

Our statement that a general mathematical method is possible reflects our hopes for the future and only implies that then it will be possible to have one programme for all the system performance calculations. It does not mean that we have already a programme for stead-state stability. However, as Mr. Eales points out, the test of steady-state stability is the behaviour of the system with small changes. The present transient-stability programme may be used with slight modifications to study this. We also expect the answer to Mr. Eales's last question to be 'yes'.

In conclusion we would emphasize the importance of the control programme, which should tend to make the studies automatic and enable systems of all types to be solved without any manual intervention at all.

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Oct. 1961

CALCUTTA BRANCH: CHAIRMAN'S ADDRESS

By M. DATTA, M.Sc.Tech., Ph.D., Member.

'EXPLOITATION OF WATER POWER IN WEST BENGAL'

(ABSTRACT *of Address delivered* 14*th December*, 1960.)

Although West Bengal claims to be the pioneer of hydro-electric generation in India, having started operation as early as 1897, the overall development of water power in this truncated State is not spectacular. Of the total power of 610 MW developed hitherto from various sources, only 6·7 MW account for hydro-electric power.

This is due to the comparatively high cost of developing the water sites, and the great distances of the sources of water power and the proximity of the coal-fields to the load centres. Until the enactment of the Electricity (Supply) Act, 1948, and the formation of an autonomous Electricity Board, no attempt had been made to develop the water-power resources on anything but a negligible scale. The two sources of power are now considered as complementary rather than competitive, and ways and means are being devised to harness cheaper water resources.

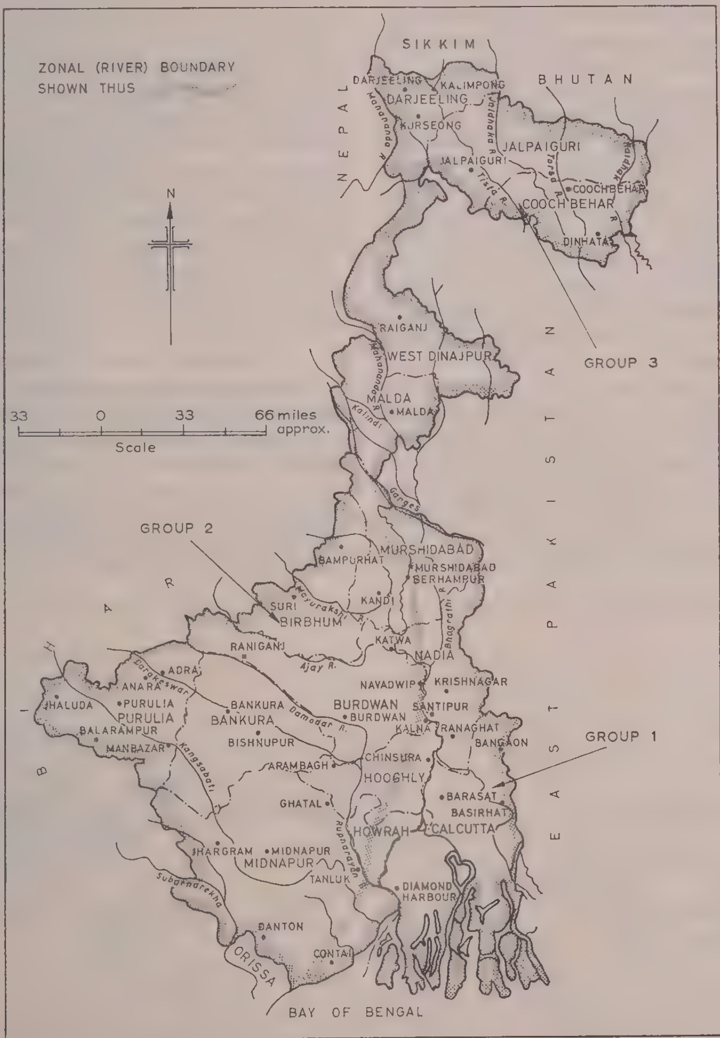

Fig. 1.—The rivers of West Bengal.

The Rivers of West Bengal

For a proper appreciation of the value of the rivers in West Bengal (Fig. 1) for hydro-electric generation, it is convenient to classify them into the three following groups.

Dr. Datta is with the West Bengal State Electricity Board, Calcutta.

Group 1.—In this group may be classed the rivers of the Gangetic plains, which have their sources in the distant mountains and flow for hundreds of miles before entering the plains of Bengal, maintaining a more or less perennial flow and navigable all the year round. The Ganges is the largest, draining an average annual rainfall of 42 in over a catchment area of 397 500 sq. miles and with a recorded discharge of about 1 000 000 cusec. But the Gangetic plains being absolutely flat, the rivers in this region cannot be utilized for hydro-electric generation.

In the lower deltaic region, however, there are a number of estuaries where the possibilities of obtaining power from the tides may be explored. But, even in this regard, a distinguished engineer has said, 'Power by rise and fall of tide will always be alluring but disappointing'. Only four modern proposals to use the tides are known, but none of them has made much headway, and consequently tidal power is not being looked upon as a source of energy at present, at least in this country.

Group 2.—This group consists of rivers such as the Subarnarekha and Cossye in Midnapore district; the Silai and Dwarkeswar in Bankura district; and the Damodar, Rupnarayan, Ajoy, Mayurakshi, Dwarka, Pagla, Brahmani, etc., in Birbhum and Murshidabad districts. They run more or less from west to east, each independent of the other, and have their sources in the Chotonagpur and Santhal Parganas hills. With the advent of the monsoon, lasting from June to September, the rivers bring in enormous volumes of water, at times causing destructive floods. They, however, dwindle to a mere trickle, sometimes even during the rainy season, and during the dry season there is practically no flow.

The Damodar is typical of this type of river, its flow varying from ½ cusec (March) in the dry season to 60 000 cusec or more in the monsoons, which frequently cause devastating floods resulting in great loss of life and property, one of the worst being that of 1943. The Ajoy, which lies further north, becomes completely dry in May but discharges over 12 000 cusec in August.

It is, however, to be noted that the floods do not always synchronize with the times when irrigation demand arises, especially after the middle of September, when the rainfall is hardly sufficient to meet the requirements of the paddy crop. Even in normal years, artificial irrigation is thus a necessity in these districts to ensure a normal harvest. It is therefore apparent that, to meet even the irrigational needs of the area, storage is a necessity. But, owing to the flatness of the country, it is difficult to obtain suitable sites for storage dams within the boundaries of the State. Good sites are, however, selected in the upper valleys of the rivers lying within the hilly regions of Chotonagpur and Santhal Parganas, where, for storage of flood waters from the rainfall of the catchment area, large reservoirs and dams have been built in connection with the Damodar and the Mayurakshi Projects.

Group 3.—The rivers in this group, rising from the Himalayan region, bordering the north, for the most part, with Sikkim and Bhutan, flow into the districts of Jalpaiguri and Darjeeling. The Tista and all others which flow east of it, namely the Great Rangit, Torsa, Jaldhaka, Raidak and Gangadhar, belong in this group. Here the rainfall is heavy and the thickly wooded mountains retain much of the monsoon waters, so that considerable discharge can be obtained in the dry season also. These rivers have great potential for hydro-electric generation. Some of the larger ones, which are snow fed, give good discharge during the summer, the flow being least during December and January. It is therefore possible to utilize this group for power generation without having to construct dams to form large reservoirs but based on minimum flow only.

Damodar Valley Corporation Project (Group 2)

The rainfall in the catchment area of the Damodar Valley feeds the Damodar, the Barakar and their tributaries, Konar and Bokaro. To harness these sources for flood control, eight storage dams were proposed. It was also realized that the control of the rivers would not only prevent the recurrence of devastating floods but would also provide for regular irrigation, navigation and production of hydro-electric power.

The master plan prepared for the utilization of the water

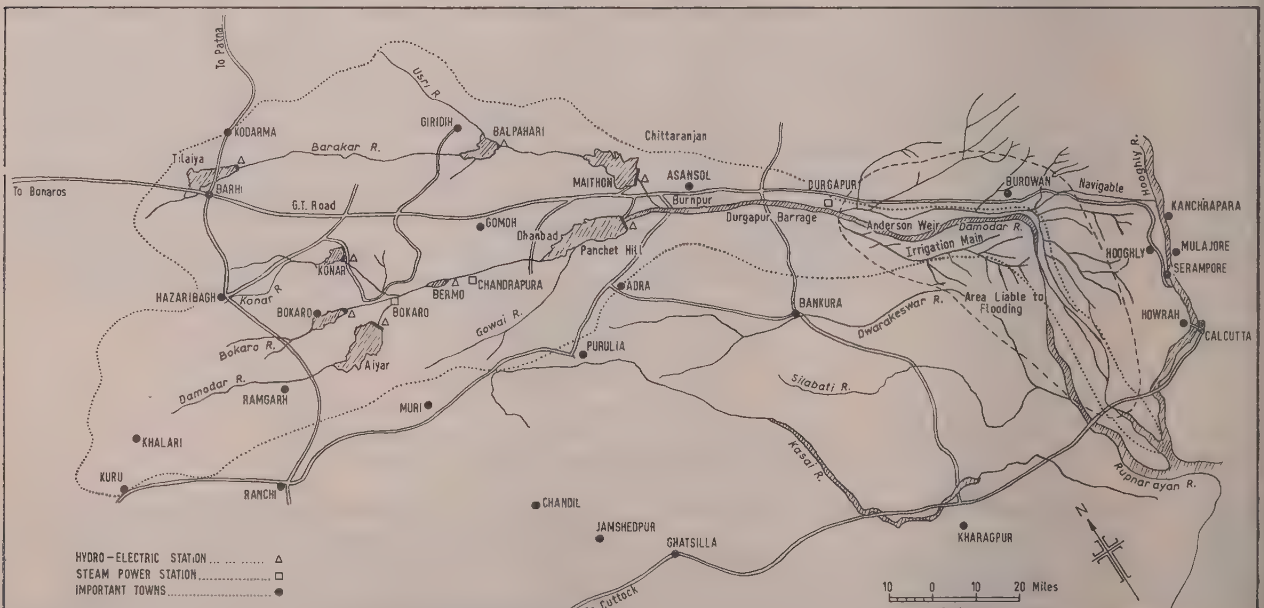

Fig. 2.—Damodar Valley Corporation Project.

resources of the Damodar River (Fig. 2) recommended installation of the following hydro-electric stations:

	MW
Damodar River	
Aiyar	45
Bermo	30
Panchet Hill	40
Barakar River	
Tilaiya	4
Balpahari	20
Maithon	60
Other Tributaries of the Damodar	
Konar	40
Bokaro	3

Of the above, the Tilaiya, Maithon and Panchet Hill power stations have already been completed.

The River Damodar and its tributaries being flashy in nature and the dry-period discharge very low, the combined firm capacity of the above hydro-electric stations is much less than the total installed capacity. A thermal power station at Bokaro with an initial installed capacity of 150MW was therefore designed and put into operation in 1953.

The present demand for power is, however, far in excess of that available. The generating capacity and transmission facilities of the D.V.C. are therefore being augmented. The following thermal power projects are now well advanced, the first two being already under trial:

Two 75 MW units at Durgapur.
One 75 MW unit as an extension to the Bokaro thermal station.
Two 125 MW units at Chandrapura.

In addition, there were proposals for the construction of a hydro-electric station at Konar dam with one 20MW reversible pump turbine unit, and the addition of one 40MW unit at Panchet Hill, where the necessary provision for this addition exists.

The D.V.C. power system has now become predominantly thermal, and thus it is advantageous to develop the hydro-electric stations for operation at very low annual load factors, thus making a large peak contribution. The incremental cost resulting from reduced load factor, i.e. increased amount of installed plant, larger conduit and transmission, is less than the capital cost of increasing the installed capacity of a thermal station. Therefore, in this predominantly thermal system, it is preferable to develop water power at the lowest practical load factor at which the system can absorb energy. A further advantage due to increased installed capacity is that spillage has been considerably reduced at times of high run-off by operation of the plant for long periods, thereby economizing in the use of fuel in thermal stations.

Jaldhaka Project (Group 3)

Jaldhaka River, which has its origin in Sikkim from snow-bound altitudes of the order of 15 000 ft, makes its course via Bhutan to Bindu, where it is met by two major tributaries, Nichu and Bindu Khola. Thereafter it constitutes the 16-mile boundary between Bhutan and Bengal. It then enters the plains of North Bengal, where meets a number of tributaries, and joins the Brahmaputra in East Pakistan.

The reach of the river between Bindu and Naksal has a fairly good gradient, 1 in 40 on the average, and a total drop of about 800 ft. It is proposed to harness the river in steps in the existing river bed. The first step will utilize a head of 550 ft between Bindu and Biru confluence with Jaldhaka, and the second stage, another drop of about 230 ft to generate 54 MW at 50% load factor (both stages combined and including seasonal generating capacity). In the third stage it is proposed to utilize another drop of 1 300 ft on the Nichu River, a tributary of the Jaldhaka forming the northern boundary of West Bengal with Bhutan. The power available at 50% load factor from this stage would be 23 MW. Fig. 3 exhibits this stagewise development.

Hydrology and General Aspects of the Project.—The river, up to its confluence with Bindu Khola, drains a total area of 172 sq. miles, of which about 150 sq. miles falls within the territory of Bhutan, and agreement has been reached with that country for utilization of its water resources.

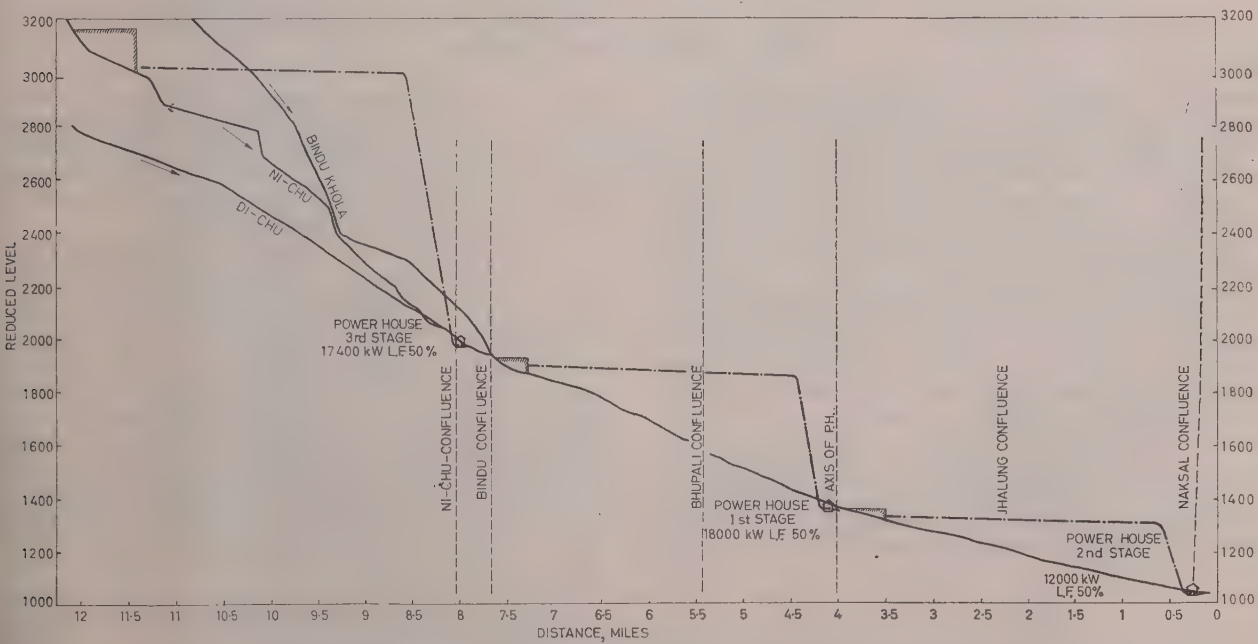

Fig. 3.—Longitudinal section of the River Jaldhaka, showing stagewise development.

The flow-duration curve suggests that the minimum discharge period extends over hardly three months (from February to April) while during the rest of the year the discharge is much in excess of that in the dry period. A unique feature of the project lies in the fact that availability of excess water synchronizes with the seasonal load demand of the tea industries of the Dooars and Darjeeling area—one of the major industries that earn the bulk of the foreign exchange for the country—for it is rain that brings out the tea leaves and at the same time swells the river. Normally, it is very difficult to find a market for such seasonal supply and excess water of the streams runs to waste, but Jaldhaka is fortunate enough to have a ready market for this secondary power in the tea industries of North Bengal. Considering all these factors, Jaldhaka Project, 1st stage, has been designed to give a firm output of 18 MW at 50% load factor, provision being made for the same quantity of seasonal power generation.

The Jaldhaka Project, 1st stage (Fig. 4), envisages the construction of a short diversion weir across the river below its confluence with Bindu Khola. The intake at the weir will divert water into a water-conductor system consisting of a reinforced-concrete duct and a two-and-a-half-mile tunnel terminating in a surge chamber, from which four steel-lined tunnels and afterwards steel penstocks run up to the power house located at an elevation of 1 364 ft on the river bed below Paren Hill. The tail-race water will be discharged into a channel.

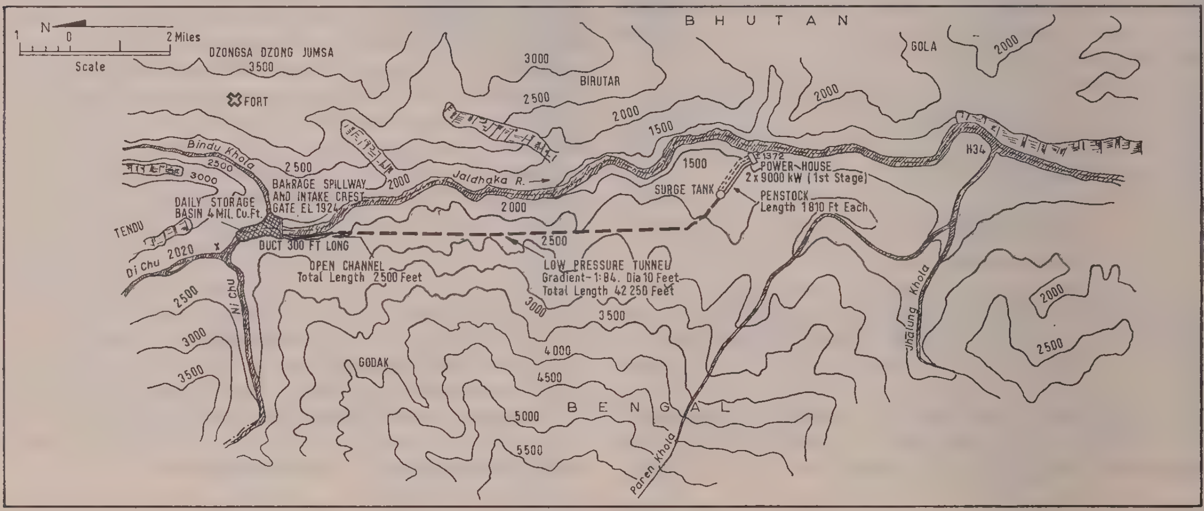

Fig. 4.—Jaldhaka Hydro-Electric Project.

The power house will accommodate four turbo-alternator sets each having a rated capacity of 9 MW, generated at 11 kV and stepped up to 66 kV in the adjoining outdoor switchyard. The transmission lines will run to Jalpaiguri in the south, Darjeeling in the west and Cooch-Behar in the east to supply electricity to the power-hungry areas of North Bengal.

Other Prospective Projects of North Bengal

In North Bengal, which has an area of about 2 500 sq. miles (6·5% of the entire area of West Bengal), the potential water-power resources have been estimated at 1 300 MW (600 MW on the border with Bhutan and Sikkim) at 50% load factor, the basin-wise analysis of which is as follows:

	MW
Balasun Mahananda Basin	43·4
Teesta (Great Rangit) Basin	570·4
Torsa Basin	400·0
Jaldhaka Basin	150·5
Raidhak Basin	162·1
	1 336·4

A hydro-electric survey of India currently carried out by the Central Water and Power Commission reveals that the technical potential of the country is of the order of 40 000 MW at 60% load factor.

Interconnected Operation of Power Supply Systems

As most of the schemes to be implemented in North Bengal are run-of-the-river type, the generation capacity of individual stations has been designed on a minimum-discharge basis. The discharge is well above the minimum for more than nine months in the year, and interconnection of some peaking station will increase the capacity considerably. Further, such interconnection will enable utilization of off-season potential (occasional run-off during winter) which cannot be used when all the stations are designed as run-of-the-river developments.

Particularly suitable for the regulation of greatly varying loads is the pumped-storage station. By the artificial creation of loads, this can absorb excess power produced during off-peak periods to pump water from a low-level reservoir into a high-level one, from which it can be taken during the peak-load period to drive water-wheel generators, which in turn supply additional energy to the system when needed during the day. This involves not only storage but also an improvement in the economic aspect of power generation. It is hoped that such pumped-storage installations will be developed to overcome the difficulties of having sufficient steam power to supplement the output of water-power stations during the periods of low flows.

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THE FUNDAMENTAL CHARACTERISTICS OF PILOT-WIRE DIFFERENTIAL PROTECTION SYSTEMS

By J. RUSHTON, Associate Member.

(*The paper was first received* 12*th October*, 1960, *and in revised form* 11*th April*, 1961.)

SUMMARY

In the design of pilot-wire protection systems the pilot circuit is usually considered to be either purely resistive or completely balanced and able to transmit information simultaneously from end to end. Characteristics are given for long practical circuits showing that these are invariably unsymmetrical owing to their heterogeneous composition, and it is shown that the change in pilot-circuit input admittance due to remote-end voltage variation gives a reliable indication of design parameters.

A locus diagram is given for pilot-circuit admittance upon which the relay operating characteristics can be superimposed, giving complete design information for the protection scheme. By a change of axis this diagram can be redrawn in terms of power-system input currents to the relay system, showing the performance of the relays at both ends of the line.

Consideration is given to the design of practical protection schemes using the longest pilot circuits, which are treated as completely unsymmetrical networks. The effect of the non-linearities in the voltage-limiting circuits is also considered.

A design technique is illustrated which enables the protective scheme characteristics to be predicted and verified by simple tests. Practical limits and limitations inherent in the use of rented telephone-type circuits are discussed briefly, since these exert considerable influence on practical scheme design.

LIST OF PRINCIPAL SYMBOLS

K_R = Relay constant proportional to coil ampere-turns.
N_O = Effective number of turns on relay operating coil.
N_R = Effective number of turns on relay restraining coil.
A, B, C, D = 4-terminal network parameters of pilot circuit.
I_A, I_B = Effective input currents to pilot-wire protection system.
ϕ = Angle between I_A and I_B.
V_S, V_R = Input voltages to pilot circuit.
K = Scalar ratio of input voltages to pilot circuit $= |V_S/V_R|$.
δ = Angle between V_S and V_R.

$$K'/\overline{\delta} = \frac{1}{K\angle\delta}$$

I_S, I_R = Input currents to pilot circuit.
γ = Angle between I_S and I_R.

(1) INTRODUCTION

The basic requirement of any unit protection scheme is that it must operate for all faults within the protected power circuit. Equally, it must remain inoperative both for healthy power-system conditions and for faults external to the protected circuit.

In the protection of 3-phase circuits by pilot-wire differential systems the function of the pilot circuit is usually considered to be the simultaneous transmission of information from end to end so that the relays can effect a continuous comparison of quantities derived at the two ends. Thus the pilot circuit has usually been represented by its resistance only. More recently, the distributed nature of long pilot circuits has become apparent, and the pilot circuit has been regarded as a correctly terminated infinitely long transmission line. Neither of these assumptions is found to be valid in practice.

Historically, the evolution of protective systems for pilot-wire protection followed closely after the conventional time-graded systems and preceded by many years the introduction of distance and carrier-pilot systems.

The original balanced-current and balanced-voltage arrangements of Merz and Price formed the essential basis of all pilot-wire differential systems and it soon became apparent that even the characteristics of the short pilot circuits then in use exerted considerable influence upon the performance of the protection. Particularly, it was found that the pilot capacitance currents which were present during through-fault conditions appeared in the relay-operating-coil circuits and seriously reduced the available discriminating margins.

Early solutions involved the use of special pilot cable designed to eliminate these capacitance currents from the relay coil, but such measures were expensive and it was found that discrimination could be obtained by providing the relay circuit with a compensating (or replica) impedance, or a bias feature.

The former comprised the provision of an impedance associated with the relay-operating-coil circuit connected to neutralize the relay-operating-coil current under through-fault conditions, whilst the latter ensured stability by providing the relay with a restraining force which increased (linearly or otherwise) as the system fault current increased.

With modern schemes operating over electrically long pilot circuits, the use of both compensating impedance and bias feature is essential, and the paper demonstrates the complementary nature of these measures.

Hamilton[1] gives an excellent survey of modern practice in pilot-wire protection and deals fully with the historical and practical aspects of pilot-wire systems, as well as discussing most of the basic arrangements commercially available. This paper attempts to present a comprehensive theory embracing all these different arrangements so that a comparative assessment can be made.

(2) BASIC PILOT-WIRE PROTECTION SCHEME

Fig. 1 is a single-phase diagram showing the essential components of all pilot-wire differential protection equipments. These comprise main current transformers, summation network, relay and pilot wire. Each individual item must be designed correctly if a sound overall performance is to be obtained from the scheme, and also the characteristics of the various components must be mutually compatible.

(2.1) Main Current Transformers

The analysis of current-transformer performance under steady-state and transient conditions is well known.[2] For pilot-wire

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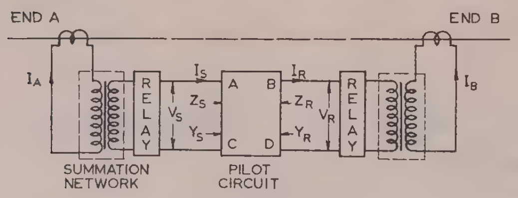

Fig. 1.—Basic arrangement of pilot-wire differential-protection scheme.
Conventions used in the analysis of overall performance are shown.

systems these requirements are self-evident. Clearly, if the pilot-wire protection is to remain stable under through-fault conditions the current output from the three current transformers at end A (only one is shown) must closely resemble the output from those at end B. Where the current transformers at both ends are of different design, or where two sets at one end are required to balance with one set at the other, it follows that excessive saturation must not occur. Where the current transformers at both ends are reasonably similar some degree of saturation is permissible. Usually, minimum current-transformer requirements for each type of scheme are determined by exhaustive testing on heavy-current test plant. In the following analysis current-transformer errors are ignored.

(2.2) Summation Network

The function of the summation network is to derive a single-phase voltage output from a 3-phase current input. This output must be available for all types of power-system fault condition and is usually provided by a summation transformer, although certain combinations of phase-sequence currents have been proved advantageous, particularly on resistance-earthed power systems.[3] The network may also provide a source-impedance conversion.

For short pilot circuits the use of a linear current/current (high-impedance) source may be permissible if the current-transformer secondary voltage under fault conditions (given by the product of current and pilot-circuit impedance) does not reach a high value. With long high-impedance pilot circuits a current/voltage (low-impedance) source is essential, particularly on rented telephone circuits where the maximum permissible voltage applied to the pilot circuit is restricted by the lessor.

Further, a typical protective scheme will be required to have a minimum setting current of approximately 20% current-transformer rating and must operate under fault conditions of up to 30 times the current-transformer rating, i.e. over a range of 150 : 1. If a linear voltage source were used with the maximum pilot voltage coinciding with the maximum fault condition, the voltage available for relay operation at minimum setting would be 1/150th of this, requiring a highly sensitive relay element. Such a relay would probably be unduly sensitive to spurious pick-up, particularly under system fault conditions.

The use of voltage-limiting devices permits a more robust relay element. Essentially, a voltage-limiting device comprises a non-linear resistor connected in parallel with the voltage source and the circuit is designed so that between 10 and 30% maximum voltage is available at setting. At high fault levels the action of the non-linear resistor holds the peak pilot voltage within limits. A typical arrangement is shown in Fig. 2.

Such devices clearly tend to introduce a non-sinusoidal voltage waveform to the pilot circuit and should be associated with a simple filter circuit (usually of high capacitance) to reduce the higher harmonic components in the voltage waveform. Thus, for protection schemes used on long pilot circuits the summation network will comprise a summation transformer (or phase-sequence network), a voltage limiting network and a filter

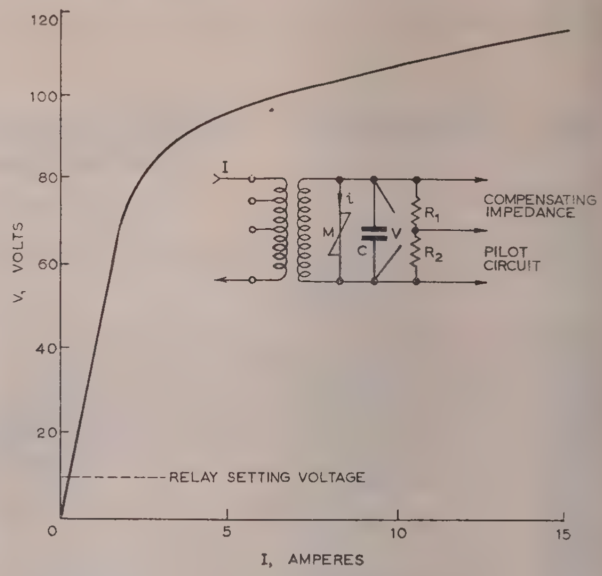

Fig. 2.—Characteristic of typical voltage/current network.
$R_1 = R_2 = 100\,\Omega$
$C = 40\,\mu F$
Non-linear resistor: $V = 100\,i^{0 \cdot 25}$.

circuit. The output voltages from the summation networks at the two ends may be connected in series or in opposition for the 'through-current' condition.

(2.3) Differential Relay

The electrical circuit of the relay essentially consists of operating coil(s) together with restraining coil(s) and a compensating or replica impedance, though in any particular case either of the last two features may be omitted.

The relay effects a comparison of the applied quantities and may be inherently responsive to either the amplitude or phase angle of the applied signals. Equivalence between amplitude and phase-angle comparator relays having two input quantities has been shown,[4] and the choice of relay element is thus arbitrary, provided that adequate sensitivity is obtainable. The following analysis assumes that the comparator characteristics are linear, though this is not always the case. In general, a linear characteristic will give a consistent and more predictable performance, and this is of great advantage where long pilot circuits are used. The use of a non-linear relay characteristic does not result in any improvement in overall performance.

(2.4) Pilot Circuit

For the general case, the pilot circuit must be assumed to have distributed constants. It must therefore be represented in calculation in hyperbolic form or, more conveniently, by A, B, C, D constants.

Rented telephone-type circuits over long routes invariably comprise various lengths of unloaded and loaded sections having different conductor cross-sectional areas. The electrical characteristics of such circuits are completely unsymmetrical and can be determined only by test. Rented circuits are also subject to limitations laid down by the lessor with regard to maximum applied voltages, insulation levels, security, etc.

(3) POWER-SYSTEM PERFORMANCE REQUIREMENTS

The power system must be investigated under extreme conditions of internal and external faults to determine the essential

basis of discrimination, so that a satisfactory overall characteristic can be embodied in the pilot-wire protective scheme.

(3.1) Load and Power-Swing Conditions

In considering the loci of relay currents for all possible healthy system conditions it is essential to include the shunt admittance of the protected line. An analysis by Donaldson[5] plots the power-swing and load conditions on a current/phase-angle diagram on which the protective-relay characteristic can also be drawn. The locus diagram obtained shows that a simple envelope can be derived to include the effect of all load and power-swing conditions upon the currents in the protected line. It is shown that minimum relay settings are determined by feeder-capacitance currents under light-load and through-fault conditions. For a pilot-wire protection scheme this locus diagram gives complete information of relay input quantities under healthy feeder conditions.

(3.2) Internal Fault Conditions

A single relaying quantity is required at each end of the protected feeder and the input currents for all three phases must be combined to achieve this, using either a summation transformer or symmetrical component networks. Adamson and Talkhan[3] have examined the outputs from these devices under various conditions of single-shunt faults on a simple power system, showing that the conventional summation transformer is generally satisfactory for solidly earthed systems and that a combination of positive- and negative-sequence currents is advantageous for resistance-earthed systems.

(3.3) Protective-Scheme Performance

From these investigations it is possible to list some of the desirable overall characteristics of a pilot-wire relay scheme:

(*a*) The scheme should have a small stable zone embracing the through-fault condition, and a large tripping zone. In general, a stability angle of ± 20–$40°$ gives an ideal phase-comparison characteristic.

(*b*) Amplitude comparison of currents is desirable at levels corresponding to the maximum circuit load, particularly on resistance-earthed systems.

(*c*) The fault setting of the protection should be sufficiently high to prevent incorrect comparison due to capacitance currents of the protected feeder.

(*d*) Summation transformers are generally satisfactory for solidly earthed systems, but the use of phase-sequence-current networks should be considered for resistance-earthed systems.

(4) METHODS OF PRESENTATION OF PILOT-WIRE PROTECTION CHARACTERISTICS

(4.1) Conventional Characteristics

In order to investigate the application of a pilot-wire protective system it is essential to examine the characteristics of the scheme in terms of power-system currents and phase angles during fault conditions. The characteristics usually presented include:

(*a*) Bias characteristic, taken with in-phase currents applied to the relays at each end, showing the magnitude of I_A with respect to I_B for relay A to trip.

(*b*) Phase-angle characteristic, showing the phase angle between I_A and I_B to cause tripping for different values of I_A equal to I_B.

(*c*) A combined characteristic of (*a*) and (*b*) in polar form showing the locus of $I_A\angle\phi$ to cause tripping with I_B fixed.[6] This requires a family of curves for various values of I_B to give a comprehensive characteristic.

Typical characteristics are shown in Fig. 3. In addition, these characteristics must be taken individually for different pilot conditions, although an ideal relay will incorporate adjustments

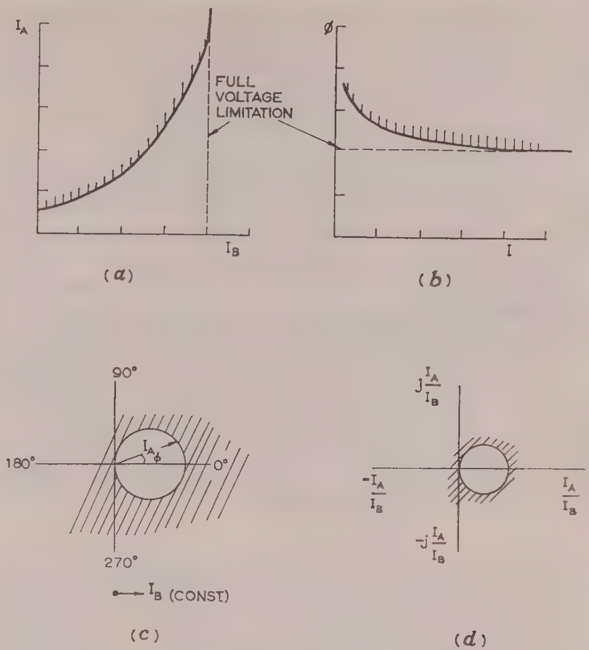

Fig. 3.—Presentation of pilot-wire relay characteristics.

(*a*) Bias characteristic ($\phi = 0°$).
(*b*) Phase-angle characteristic ($I_A = I_B$).
(*c*) Polar characteristic (I_B is constant).
(*d*) Complex-current-ratio characteristic.
Relay operates in the shaded area.

so that a reasonably consistent characteristic is obtainable for all types of pilot-wire circuit. The characteristics are, in general, obtained from measured values taken on a particular relay/pilot-wire combination and are useful for application purposes. They do not give any indication of fundamental design considerations.

(4.2) Complex-Current-Plane Presentation

The complex-current-plane presentation[7] is an extension of the polar-characteristic method, the protection characteristics being plotted in terms of I_A/I_B, the complex ratio of the currents to be compared. This method has advantages where the ratio of the two currents is linear, and for this case a single diagram would replace a family of polar diagrams drawn for various values of I_B. The non-linearities resulting from the use of voltage-limiting networks give rise to a non-linear ratio, I_A/I_B, so that for most practical purposes a family of curves is required for different values of I_B. This method of presentation then becomes identical with the polar method.

At high current levels the voltage-limiting feature of the current/voltage network will render the relay insensitive to changes in magnitude of input current and the overall scheme will be sensitive merely to phase displacement between currents at each end, i.e. the scheme will have a phase-comparison characteristic.

The performance diagram must include the effect of pilot-circuit parameters and the protection scheme must provide adjustment of relay characteristic so that the ideal characteristics can be achieved for all pilot-circuit constants within its capability range.

(4.3) Fundamental Requirements

The method of presentation of relay characteristics should fulfil the two functions of providing a design basis and of providing sufficient information to enable the relay to be applied

correctly. With pilot-wire systems using long pilot circuits these two requirements are essential if the user is required to make correct adjustments at commissioning so that the optimum design characteristics are obtained. Inevitably, therefore, he must be familiar with presentation of design and application characteristics and the relationship between them.

The overall characteristic in terms of primary-current values should be readily obtainable for any pilot circuit from a knowledge of pilot-circuit constants (open- and short-circuit impedances) and relay parameters (coil turns, resistances, etc.), i.e. the relay and pilot circuit must be considered as a unit, so that protective-gear characteristics can be correlated with power-system performance.

This is precisely attainable in any linear circuit, but since, with most high-resistance pilot-wire schemes, the current/voltage transformation is non-linear owing to the voltage-limiting network, it follows that an exhaustive overall presentation is impossible. The protection-scheme performance under both linear and full voltage-limiting conditions should be presented with reasonable accuracy. The non-linear transition between these conditions is of limited interest.

(5) DESIGN BASIS OF PILOT-WIRE PROTECTION SYSTEMS

Pilot-circuit constants must be considered in evaluating the performance of a pilot-wire protection scheme. The long pilot circuit must be represented by distributed-constant transmission-line theory. The concept of a homogeneous transmission line with correct terminations permits a simpler approach to the problem, but practical non-homogeneous circuits require a more generalized treatment.

(5.1) Infinite-Length Pilot Circuit

In an experimental system described by Neher[8] the pilot circuit is considered from the communication viewpoint, assuming a homogeneous line correctly terminated at each end in its characteristic impedance such that simultaneous end-to-end signal transmission may be carried out without interference; compensation is included in the relay circuit for signal attenuation and phase shift. Similar arrangements have been investigated in this country, though results have not been published. This approach permits a rigorous analysis by classical methods and has the advantage that the waveform of the applied pilot-circuit voltage need not be sinusoidal.

In practice, however, the 'infinite line' is difficult to obtain. A practical rented circuit comprises a complex 4-terminal *RLC* network, the resistance and capacitance being its normal properties and the inductance being due to audio-frequency loading coils fitted in all trunk circuits to facilitate speech transmission. Impedance matching between different sections is rarely included, and in any case such devices are designed to deal with low-level audio-frequency signals rather than the transmission of power frequencies at a higher level where they may introduce non-linear effects. Arrangements are usually made to remove these devices from the pilot circuit. A protective scheme designed on the basis of infinite-line theory would be of limited practical application.

(5.2) Unbalanced Pilot Circuit

Most practical pilot circuits fall into the unbalanced category and the only feasible approach to design calculation is by rigorous 4-terminal-network theory assuming sinusoidal applied voltages. Using this approach, the relay and pilot circuit can be investigated for all conditions of voltage input to the pilot circuit. The effect of the current/voltage characteristic of the summation network can then be included to derive the overall characteristics of the scheme in terms of input currents.

Most pilot-protection schemes quickly reach the constant-pilot-voltage condition where V_S/V_R equals unity over a wide range of I_A/I_B, so that the overall characteristic becomes independent of current magnitudes.

(5.3) Admittance-Plane Characteristics

In deriving the fundamental characteristics of the relay and pilot circuit the following procedure is proposed:

(*a*) A generalized locus diagram of input admittance to the pilot network is constructed for various values of the input-voltage ratio, V_S/V_R, using measured pilot-circuit constants (open- and short-circuit impedances).

(*b*) The relay equations are derived in terms of admittance measurement and are superimposed on the input admittance diagram. This combined diagram then indicates the correct values for relay parameters (replica impedance and bias resistance).

(*c*) Bias and phase-angle curves can readily be interpolated from this diagram, and by a simple change of reference axis the admittance diagram becomes a complex-current-plane diagram.

The relay performance over the actual pilot circuit can thus be adequately calculated and the calculation confirmed by a simple test. Overall characteristics for power-system application can then be predicted.

(5.4) Generalized Impedance and Admittance Diagrams for a 4-Terminal Network

In Fig. 1, keeping the current I_A fixed and assuming that an output voltage, V_S, is produced directly proportional to this current, the corresponding voltage, V_R, at the remote end can be considered to vary in phase and magnitude. As it does so, the input current, I_S, to the pilot circuit will vary, and it is required to investigate the locus of this current to determine the discriminating margin available between internal and external fault conditions. A relay characteristic must then be chosen such that the change in pilot-circuit current produced by the primary-circuit currents under internal and external fault conditions can be used to provide definite discrimination.

A locus diagram can be constructed (Section 12.1) showing the input impedance or admittance at one end of the pilot circuit for a voltage varying in phase and magnitude at the remote end. On this diagram can also be plotted the relay characteristic expressed in terms of impedance or admittance measurement. The relay characteristic can then be adjusted to embrace the actual phase and magnitude variations of the input from the power system under healthy conditions.

The input-admittance locus diagram of Fig. 4(*a*) is plotted for a typical practical pilot-wire circuit. The pilot-circuit constants were measured using an *RC* bridge and gave the following values:

	Resistance Ω	Capacitance μF
Z_{OCR}	4600	1·32
Z_{SCR}	2500	0·326
Z_{OCS}	4650	1·43
Z_{SCS}	2430	0·386

D.C. resistance, 2750 Ω.

The measured impedance for this bridge is with *R* and *C* connected in parallel.

These figures are typical of a 25-mile pilot circuit; this particular circuit has approximately 1½ miles of unloaded conductor at each end with the rest loaded for 7 miles at 88 mH per 2000 yd and for 15 miles at 44 mH per 2000 yd.

From these pilot-circuit constants the equivalent π-circuit is as shown in Fig. 4(*b*).

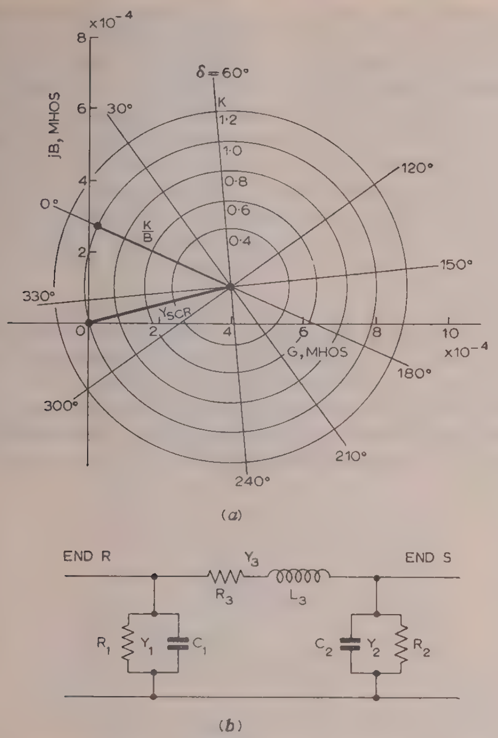

Fig. 4.—Characteristics of typical 25-mile-long practical pilot circuit.

(a) Input admittance diagram (end R) for all values of $V_s/V_R = K\angle\delta$.
(b) Equivalent π-network at 50 c/s.

R_1	22·5 kΩ	
C_1	0·864 μF	$Y_1 = (A - 1)/B$
R_2	30·3 kΩ	$Y_2 = (D - 1)/B$
C_2	0·804 μF	
R_3	2 330 Ω	$Y_3 = 1/B$
L_3	3·03 H	

Thus, $B = 2330 + j945$ and $1/B = (3{\cdot}69 - j1{\cdot}5) \times 10^{-4}$ mho.

$$Y_{SCR} = (4 + j1{\cdot}025) \times 10^{-4}\text{mho.}$$

$$Y_{SCS} = (4{\cdot}1 + j1{\cdot}21) \times 10^{-4}\text{mho.}$$

The admittance locus, Y_R, is given by $Y_R = (A - K\angle\delta)/B$ [eqn. (6)] and $Y_R = [-K\angle\delta\,(3{\cdot}69 - j1{\cdot}5) + (4 + j1{\cdot}025)] \times 10^{-4}$ from eqn. (9). This is plotted in Fig. 4(a).

(5.5) Voltage-Source Relay Characteristics on the Complex-Admittance Plane

Schemes of protection for long pilot circuits using current/voltage transformation (low source impedance) are considered in this Section.

A preliminary examination of the locus diagram of Fig. 4(a) reveals certain interesting features. Consider first the case of the conventional opposed-voltage-protection scheme where, for normal healthy conditions, $K = 1$ and $\delta = 0$ (i.e. voltages at both ends are in phase). The relay characteristic should embrace this point and, assuming that a stability angle of $\pm 30°$ for $K = 1$ is required, the ideal relay characteristic is that of a circle at centre $K = 1, \delta = 0$ passing through the points $K = 1, \delta = \pm 30°$ [i.e. radius $\simeq 1/(2B)$]. For this case the relay must be inoperative for all conditions within its characteristic circle (Fig. 5). To achieve this characteristic the relay must be provided with means to off-set the circle in a reactive direction, which entails the use of either a reactor or a capacitor for compensation, depending upon the type of circuit used. For the basic admittance relay

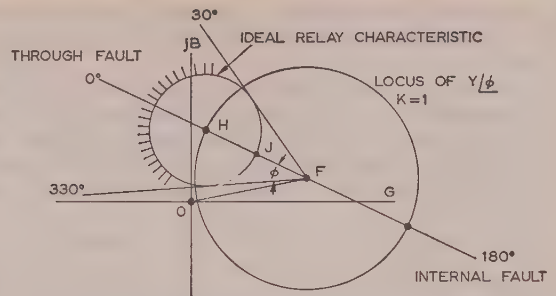

Fig. 5.—Opposed-voltage protection showing ideal relay characteristic superimposed on pilot input-admittance diagram.

Relay operates in the shaded area.

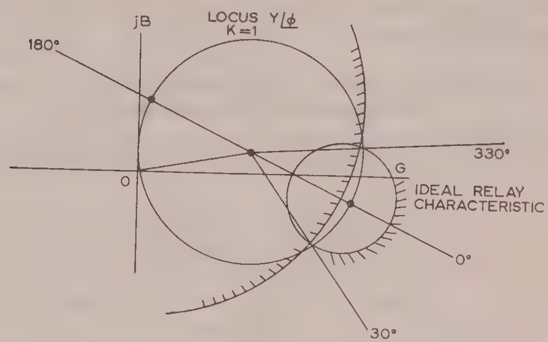

Fig. 6.—Series-voltage protection showing ideal relay characteristic obtainable using conventional voltage-bias relay.

The larger circle shows the characteristic obtainable from a current-bias relay. Relays operate in the shaded area.

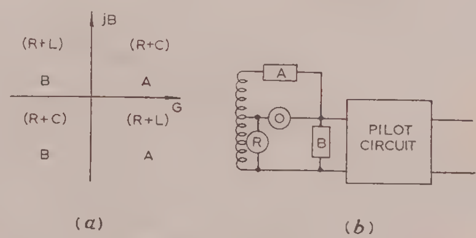

Fig. 7.—Available methods of pilot compensation.

(a) Type of compensating circuit required for all values of pilot-circuit input admittance. A and B refer to the method of connection of compensating admittance shown in (b).
(b) Fundamental methods of pilot compensation.
A Replica impedance.
B Tuning impedance.

the ideal compensation should provide a $G + jB$ displacement of the relay characteristic; this can easily be achieved using an RC compensation to nullify the relay operating force under healthy conditions, as shown in Fig. 7(b), method A. The conventional pilot tuning reactor [Fig. 7(b), method B] can provide only a $-G + jB$ compensation, which is not ideal as far as the local-end relay is concerned. It will be shown, however, that this method of compensation can approach the ideal for the remote relay and can therefore provide an identical overall characteristic. The method of compensation for admittances in different quadrants is shown in Fig. 7.

Modification of the connection to series voltage using the same relay connections merely changes the datum of δ by 180° and the relay can be recompensated using an RL circuit (in this case) to nullify the relay operating force and thus shift the relay characteristic to the ideal position (Fig. 6). Method B in Fig. 7

gives only partial compensation, and negative resistance would again be required to give complete compensation for the local-end relay.

The reversal of operating and bias coils gives the opposite relay characteristic, i.e. tripping inside the circle, which has obvious limitations if a $\pm 30°$ tripping angle is required over long pilot wires together with amplitude comparison of currents in the load-current region. This characteristic is given by the larger circle in Fig. 6.

It should be noted that the series-voltage arrangement using standard connections would require almost an in-phase compensation for the pilot circuit shown in Fig. 4. This illustrates the reason for the success of plain resistance compensation used on certain series-voltage schemes. Generally speaking, resistance compensation is usable where an unsymmetrical stability angle is permissible. There are undoubtedly cases, however, where such an approximate phase-angle compensation will give quite unsatisfactory results.

(5.6) Relationship between Complex-Current (Polar) and Admittance-Plane Characteristics

A typical admittance-plane characteristic is shown in Fig. 8(a) with the admittance circle drawn for $K = 1$ and the ideal relay

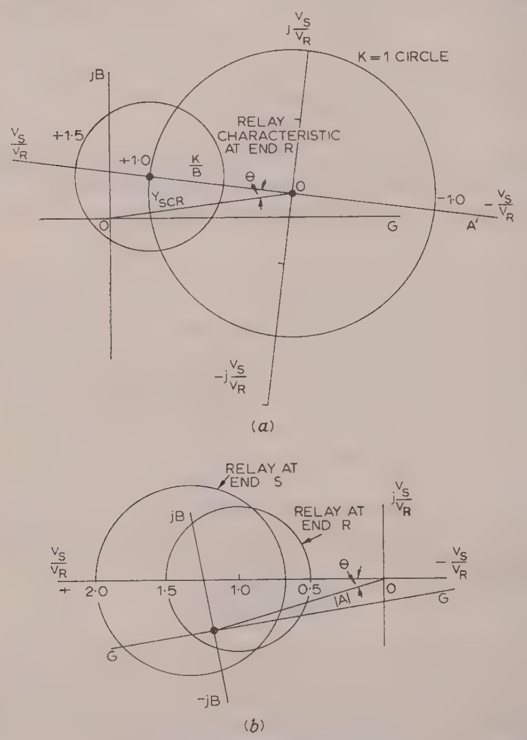

Fig. 8.—Derivation of complex-current-plane (polar) characteristics from admittance-plane diagram.

(a) Transition from Y-plane to V_S/V_R plane.
(b) Relay characteristics on the V_S/V_R diagram showing remote-end relay.
For linear I_A/V_S and I_B/V_R transformation these correspond to I_A/I_B characteristics.

characteristic circle drawn with centre at $K = 1$, $\delta = 0$ and radius $1/(2B)$, which approximates very closely to a $\pm 30°$ tripping angle for $K = 1$. The line OA′ is thus the datum of the voltage-ratio plane, and since, for a linear system, $V_S/V_R = I_A/I_B$, this is the in-phase axis of the complex-current-plane characteristic. Fig. 8(b) shows the characteristic redrawn using the V_S/V_R plane, from which it is seen, because of the manner of presentation, that the positive values of V_S/V_R occur to the left of the datum and the negative values to the right.

The relay characteristic at end R is plotted as before and also that at end S, and it is clear that, since (assuming ideal adjustment) the end S relay characteristic on the V_R/V_S plane is identical with the end R relay characteristic on the V_S/V_R plane, the end S relay characteristic on the V_S/V_R plane is the inverse of the end R relay characteristic. In both cases relay operation will occur outside the circle and the diagram thus shows the fault areas in which simultaneous tripping is obtained at both ends of the feeder.

Inevitably, with long pilot-protection schemes the major portion of the relay characteristic is influenced by the action of the voltage limiter. Thus, the relay characteristic operates about the $V_S/V_R = 1$ point and tripping occurs where the $K = 1$ circle cuts the relay circular characteristic. The relay then trips at constant phase angle (i.e. phase-comparison characteristic) over this region, which in this case is $\pm 30°$, and is completely insen-

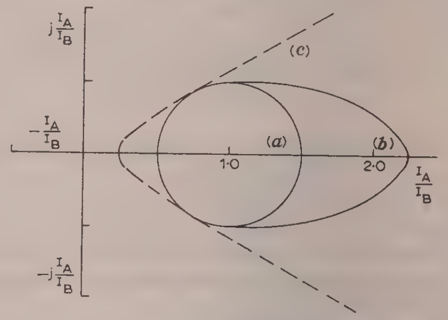

Fig. 9.—Effect of non-linearity on characteristic of end R relay of I_A/V_S transformation on I_A/I_B diagram.

(a) Linear characteristic corresponding to Fig. 8.
(b) Commencement of non-linearity where V_S is no longer proportional to I_A above $I_A/I_B = 1 \cdot 0$.
(c) Complete non-linearity of I_B as well as I_A, giving phase-comparison characteristic over most of working range.
Characteristics (a) and (c) are normally sufficient for application purposes.

sitive to current magnitude difference. Fig. 9 shows the transition from linear comparison to the phase-comparison relay characteristic.

(5.7) Conventional Pilot-Wire Relay Systems

Various relay characteristics will now be considered in detail. The relay basic equation will be expressed in terms of the pilot-circuit admittance required for operation, and the admittance characteristic will then be transferred to the complex-voltage plane, which coincides with the linear portion of the complex-current-plane diagram. In each case the influence exerted by the pilot circuit on the relay characteristic will be shown.

The majority of circuits described use an amplitude-comparator relay, i.e. a relay which is responsive to a difference in amplitude between the two compared currents and is insensitive to any phase difference between them. A highly sensitive relay element is required for protection over long pilot circuits and, in general, a moving-coil or transductor relay is used. Phase-comparator relays are equally applicable to pilot-wire systems so long as the basic comparator has a similar sensitivity. This tends to preclude the use of relatively insensitive induction-pattern phase-comparator relays, although electronic relays (using transistors) would appear satisfactory.

In the following analysis the voltage-source and relay-coil impedances are assumed to be negligible.

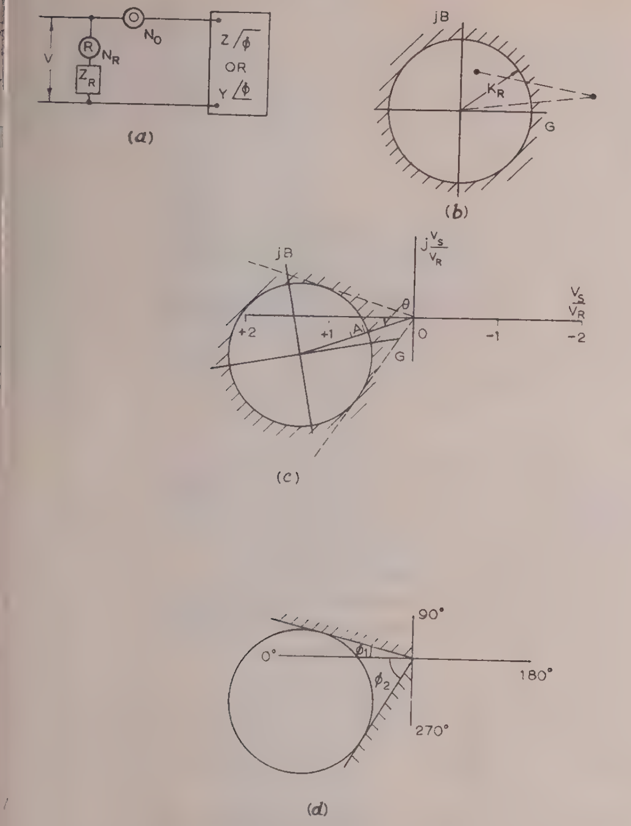

Fig. 10.—Basic impedance-type relay.

(*a*) For operation,
$|VY\angle\phi N_O| > |VN_R/Z_R|$, i.e. $|Y\angle\phi| > |N_R/(Z_R N_O)| = |K_R|$.
(*b*) Admittance characteristic. Relay operates in shaded area where $|Y\angle\phi| > |K_R|$.
(*c*) Polar characteristic showing end R relay. Characteristic of end S relay is inverse circle of end R relay.
(*d*) Phase-comparison characteristic under voltage-limiting conditions; note that $\phi_1 \neq \phi_2$.

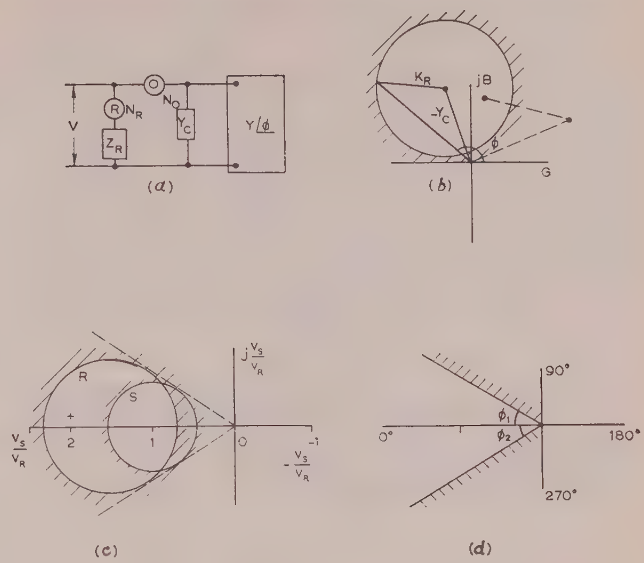

Fig. 11.—Compensated-impedance relay.

(*a*) For operation, $|V(Y_c + Y\angle\phi)N_O| > |VN_R/Z_R|$, i.e. $|Y\angle\phi + Y_O| > |K_R|$.
(*b*) Admittance characteristic; Y_O is shown inductive.
(*c*) Polar characteristic showing end S and end R relays. Both relays trip outside the circle.
(*d*) Phase-comparison characteristic under limiting conditions.

(5.7.1) Basic Impedance Relay.

The relay connection is shown in Fig. 10, which also gives the relay operating equation in terms of pilot admittance as $|Y\angle\phi| = |K_R|$. Thus the relay characteristic is a circle with its centre at the origin of the admittance axis. Fig. 10(*c*) shows the relay characteristic transferred to the polar plane. The centre of the circle is now displaced from the origin by a distance $|A|$ at an angle $\theta = \arg \angle(A-1)/B$. The relay characteristic shown is for relay R on the V_S/V_R diagram. The characteristic circle for the relay S on the same diagram is the inverse of the relay R circle, i.e. centre at distance $|A/(A^2-1)|$ at an angle of $-\theta$.

This characteristic is suitable only for a short pilot circuit where $(A-1)/B$ approaches zero and would be unsuitable for the pilot-circuit characteristics shown in Fig. 4 since it would give an asymmetrical tripping characteristic with insufficient discriminating margin.

(5.7.2.) Compensated Impedance Relay.

The relay connection is identical with the previous case, with the addition of the compensating admittance, Y_C, connected across the pilot input terminals (Fig. 11); Y_C is usually inductive. The effect of the shunt-reactance compensation is to displace the relay characteristic by $-Y_C$ in the $-G + jB$ section as shown.

Complete compensation for the local-end relay can never be achieved by this method since the point $K = 1$, $\delta = 0$ will invariably lie in the $G \pm jB$ sector. The polar diagram of Fig. 11(*c*) shows the local-end and remote-end relay characteristics, from which it is seen that the end S relay characteristic can, by suitable choice of adjustments, be arranged to provide the ideal characteristic surrounding the (1, 0) point of the polar characteristic diagram. Such a scheme can, therefore, in principle, be arranged to have ideal characteristics.

In practice, such ideal characteristics are difficult to attain owing to the non-linearity of tuning reactors, lack of fine adjustment on tuning-reactor tappings and inherent design difficulties using resonant *LC* circuits; nevertheless, the scheme is applicable over most practical pilot circuits up to 25–30 miles in length.

(5.7.3) Modified-Connection Impedance Relay.

The modified relay connections, equations and operating characteristic are shown in Fig. 12, from which it is seen that a characteristic generally similar to the previous case is obtainable with the added factor that the radius of the circle varies directly with the value of compensation. This is a disadvantage in that an additional adjustment of relay constants is required to obtain an independent adjustment for the radius of the circle. The scheme has not been applied practically to long pilot-wire circuits and although comparable with the previous method it is probably not so convenient.

(5.7.4) Bridge-Comparator Relay.

The bridge-comparator relay is illustrated in Fig. 13, from which it will be seen that an ideal characteristic is obtained for the R relay in the V_S/V_R plane (i.e. inverse to the compensated-impedance relay). Indeed, assuming sinusoidal voltages and a linear pilot-wire circuit, a scheme of this kind will operate successfully over pilots of any length, the only limitation being one of relay sensitivity to ensure that sufficient signal can be transmitted over the pilot circuit at a reasonable voltage level.

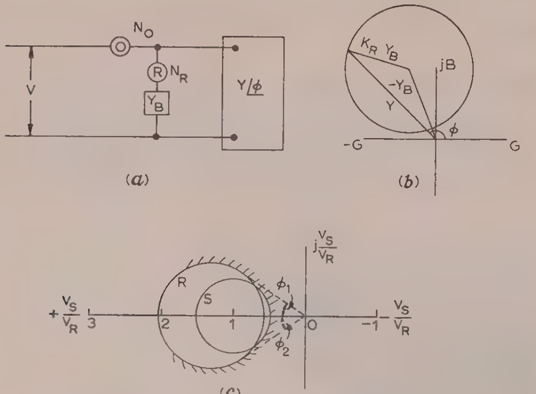

Fig. 12.—Modified impedance relay.

(a) For operation, $|V(Y_B + Y\angle\phi)N_O| > |VY_BN_R|$, i.e. $|Y\angle\phi + Y_B| > |Y_BK_R|$ where $K_R = N_R/N_O$.
(b) Admittance characteristic; Y_B is shown inductive.
(c) Polar characteristic. Both relays trip outside the circle.
Phase-comparison characteristic as for Fig. 11.

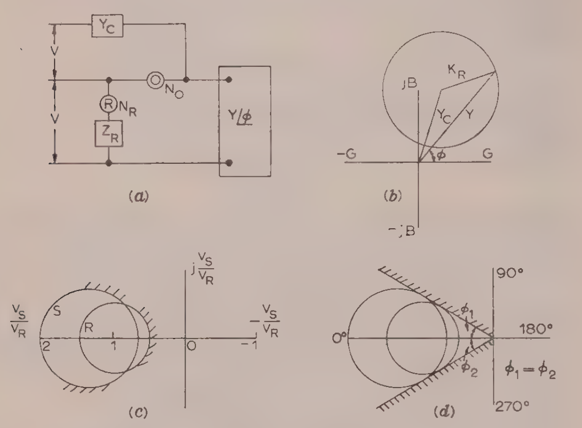

Fig. 13.—Bridge comparator relay.

(a) For operation, $|V(Y\angle\phi - Y_C)N_O| > |VN_R/Z_R|$
or $|Y\angle\phi - Y_C| > |K_R|$, where $K_R = N_R/(Z_RN_O)$.
(b) Admittance diagram; Y_C is shown capacitive.
(c) Polar diagram. Both relays trip outside the circle.
(d) Phase-comparison diagram; note that $\phi_1 = \phi_2$.

Schemes of this kind are not commercially available, but experimental equipments have given excellent results under all types of system fault condition.

(5.7.5) Inverted Impedance Relay.

The relay operating and restraining coils are transposed from the basic impedance relay connection, and a circle with centre at the origin is obtained (Fig. 14). Relay operation occurs inside the circle so that the relay compensation must be adjusted around the operating point rather than the stability point. Thus the relay is arranged to have a large operating zone. This has disadvantages where some degree of amplitude comparison of currents is required.

(5.7.6) Modified Inverted Impedance Relay.

The relay characteristic is inherently similar to the previous case with the circle displaced from the origin by $-Y_O$ (Fig. 15). The fundamental characteristic of this type of relay again does not provide a satisfactory discriminating basis for many system conditions where amplitude comparison is required at low currents.

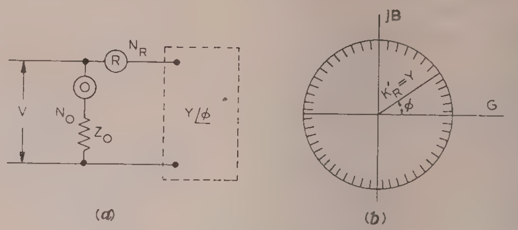

Fig. 14.—Inverted-impedance relay.

(a) For operation, $|VY_ON_O| > |VY\angle\phi N_R|$,
i.e. $|Y\angle\phi| < |N_O/(N_RZ_O)| = |K_R'|$.
(b) Admittance diagram. Relay operates inside the circle.

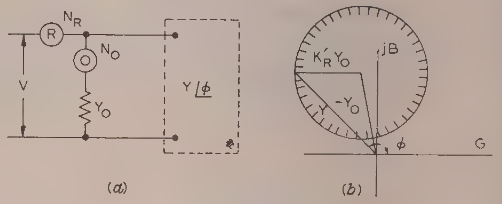

Fig. 15.—Modified inverted-impedance relay.

(a) For operation, $|VY_ON_O| > |V(Y_O + Y\angle\phi)N_R|$, i.e. $|Y\angle\phi + Y_O| < |K_R'Y_O|$.
(b) Admittance diagram. Relay operates inside the circle.

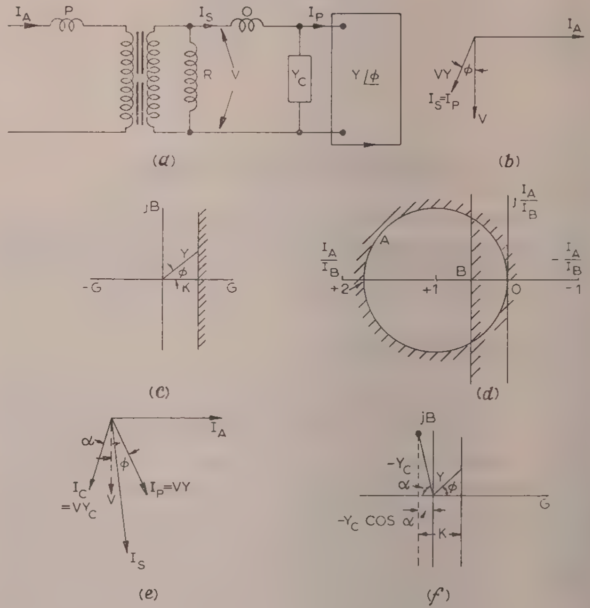

Fig. 16.—Translay-type relays, idealized characteristics.

(a) Basic circuit.
P Polarizing coil.
O Operating coil.
R Restraint coil.
(b) Vector diagram without compensation (Y_C not connected).
For operation, $K_1VYI_A \sin(90° + \phi) > K_2I_A^2$ and $V = K_3I_A$ (for linear system).
Thus, $Y \cos\phi = K_2/(K_1K_3) = K$.
(c) Admittance diagram for uncompensated relay.
(d) Polar diagram for uncompensated relay showing end A and end B relay characteristics.
(e) Vector diagram for compensated relay (Y_C connected).
Relay operating torque $= K_1I_A(VY\cos\phi + VY_C\cos\alpha)$
$= K_1K_3I_A^2(Y\cos\phi + Y_C\cos\alpha)$.

For operation, $$Y\cos\phi > \frac{K_2}{K_1K_3} - Y_C\cos\alpha$$
$$= K - Y_C\cos\alpha.$$

(f) Admittance diagram for compensated relay.

(5.7.7) Phase-Angle Comparator Relay (Translay).

The 'Translay' relay is inherently a phase-angle comparator relay of the admittance type. Fig. 16 shows the idealized characteristics of such a relay assuming linear measurement and high basic sensitivity. The basic relay measures a constant admittance (straight-line characteristic) for the B relay on the I_A/I_B plane which can be used to provide a reasonable discriminating factor, particularly on short pilot-wire circuits. Fig. 16(d) shows the polar-plane characteristics for a short pilot circuit. Modifications have been tried using a shunt-connected choke for pilot compensation, which reduces the effective reach of the relay; this has given improved characteristics over medium-length pilot circuits [Fig. 16(e) and (f)] though this improvement may be due largely to the improved power factor of the pilot current giving a more definite operating torque. The uncompensated pilot current is, of course, at a high leading power factor, which tends to overcome the ideal torque response of the relay.

(6) PRACTICAL APPLICATION OF DESIGN THEORY

A practical design of protective scheme will now be considered for the pilot-circuit characteristics shown in Fig. 5. Two schemes will be investigated; the compensated-impedance relay of Fig. 11 and the bridge comparator relay of Fig. 13. It has been shown that both types will provide equivalent overall characteristics, one being the inverse of the other.

The opposed-voltage connection is preferred since, for the bridge comparator relay, a capacitive compensation is then required. This is more flexible than the corresponding inductive compensation which may be required for the series-voltage arrangement. The tuned relay must, of necessity, use the opposed-voltage method since only this method will permit compensation.

The analyses in Section 5 assume sinusoidal applied voltages to the pilot circuit, negligible voltage-source impedance and negligible relay impedance. Thus, if the theories are to be valid these considerations must apply to the practical scheme design.

The use of a capacitive filter network to ensure a reasonably sinusoidal voltage waveform has been mentioned previously. The realization of a low voltage-source impedance presents little practical difficulty and the relay impedance can be made negligible by the choice of a sufficiently sensitive relay.

(6.1) Compensated Impedance Relay

For a practical design, the available adjustments are the reactance of the pilot tuning choke and the setting of the resistor in the relay restraint circuit. When these are set correctly the relay will provide the ideal overall characteristics shown in Fig. 11, though the practical difficulties of relay design listed in Section 5.7.2 have an increasing effect on longer pilot circuits. In calculating relay adjustments it follows from Fig. 11 that the ideal characteristic is attainable by relay S on the V_S/V_R diagram, so that the calculation of adjustments of the R relay characteristic on the V_S/V_R diagram must be based on the inverse circle.

The ideal circle on the current plane has its centre at (1, 0) and a radius of 0·5 (30° tripping angle) giving an inverse circle having its centre at (1·375, 0) and radius 0·625. When transformed to the admittance diagram this gives:

	Ideal circle	Inverse circle
Centre ..	$\dfrac{A-1}{B}$	$\dfrac{A-1\cdot375}{B}$
Radius ..	$\dfrac{0\cdot5}{B}$	$\dfrac{0\cdot625}{B}$

The admittance $(A-1)/B$ is, of course, the shunt-arm admittance of the nominal π-network.

For the compensated admittance relay, Fig. 11 gives the radius $K_R = |N_R/(N_O Z_R)|$ with the centre displaced $-Y_C$ from the origin.

Considering the practical pilot circuit of Fig. 4 and adjusting the end R relay for the inverse circle, the compensating admittance is $Y_C = (1\cdot375 - A)/B = (1\cdot06 - j3\cdot085) \times 10^{-4}$ mho, from which $Z_C = 1\,100 + j3\,210\,\Omega$, the circle radius $K_R = |N_R/(N_O Z_R)| = 0\cdot625/|B|$ and $|B| = 2\,520\,\Omega$. This gives the restraint circuit impedance $Z_R = |B|N_R/(0\cdot625 N_O) = 4030\, N_R/N_O$. Taking a practical case of $N_R/N_O = 0\cdot33$, Z_R equals $1\,343\,\Omega$. For other than $\pm 30°$ tripping angle the expression for the restraint resistance becomes $Z_R = |B|N_R/(1\cdot35 \sin\phi)N_O$, where ϕ is the required tripping angle. Thus, for $\pm 20°$ angle, $Z_R = 1\,820\,\Omega$.

(6.2) Bridge-Comparator Relay

A practical arrangement is shown in Fig. 17; the adjustments again are of replica impedance and restraint resistance.

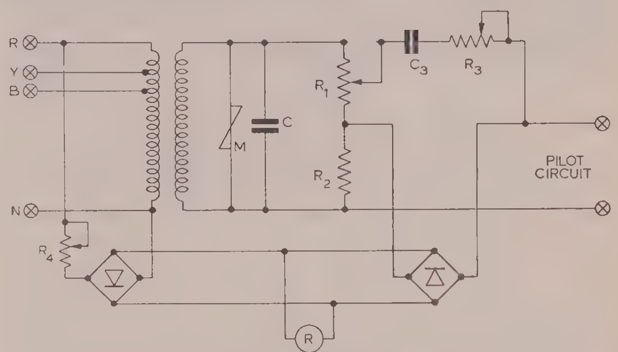

Fig. 17.—General arrangement of practical bridge comparator relay.

M, C Voltage-limiting network, as in Fig. 2.
R_1, R_2 Ratio arms of bridge network.
R_3, C_3 Pilot replica impedance network.
R_4 Restraint resistor.
R Relay coil.

Fig. 13 shows that the ideal circle is obtainable for the R relay on the V_S/V_R diagram and that the radius is $K_R = |N_O/(N_R Z_R)|$ with the centre displaced Y_C from the origin.

For the practical pilot circuit of Fig. 4 the relay adjustment values are: Replica admittance $Y_C = (A-1)/B = (0\cdot3 + j2\cdot53) \times 10^{-4}$ mhos. Replica impedance $Z_C = 426 - j3\,900\,\Omega$. The circle radius $K_R = |N_R/(N_O Z_R)| = 0\cdot5/|B|$ for $\pm 30°$ tripping angle, from which $Z_R = |B|N_R/(0\cdot5 N_O)\,\Omega$. If $N_R/N_O = 0\cdot33$, $Z_R = 1\,680\,\Omega$.

For tripping angles other than $\pm 30°$, the expression for Z_R becomes $|B|N_R/(\sin\phi)N_O$ so that, for $\phi = \pm 20°$, $Z_R = 2\,440\,\Omega$.

(6.3) Verification of Bridge-Comparator Relay Characteristic by Test

Normally the adjustment of replica impedance would be carried out by null balance at the relay under through-current conditions. The operating and restraining ampere-turns would then be measured separately for the single infeed condition.

The total operating ampere-turns correspond to the length of the $1/B$ vector on the admittance diagram (i.e. FH in Fig. 5) and the restraint ampere-turns correspond to the radius of the relay characteristic circle (JH in Fig. 5). Thus the relay circle can be adjusted and the overall tripping angle is given by

$$\phi = \sin^{-1}\frac{\text{Restraint-circuit ampere-turns}}{\text{Operating-circuit ampere-turns}}$$

to a close approximation.

The application limit of the scheme is reached when the relay-operating-coil current given by V_S/B approaches the basic sensitivity of the relay element.

(7) BALANCED-CURRENT SCHEMES

The balanced-current scheme operates on the circulating-current principle with the pilot circuit energized either directly or via a summation transformer from a current (high-impedance) source. To retain the high-source-impedance characteristic the summation-transformer output must remain linear. For this reason the principle is unsuitable for long pilot circuits where the pilot-circuit impedance constitutes a large burden and, in consequence, high pilot voltages are required for linear comparison. Where pilot voltages are limited owing to current-transformer saturation or other factors the scheme becomes in effect a series-voltage arrangement as described in the previous Sections, and under extreme limiting conditions the characteristics revert to phase comparison. A generalized input-impedance locus diagram exists for balanced-current schemes and this is derived in Section 12.2. A practical diagram for the pilot circuit of Fig. 4 is given in Fig. 18. Two typical balanced-

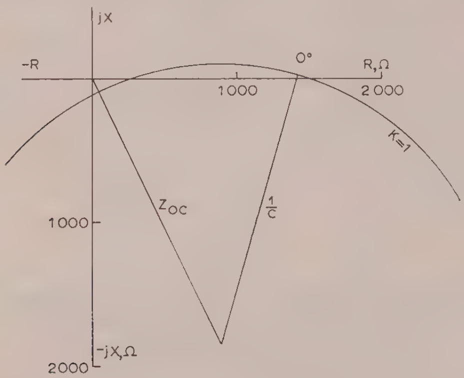

Fig. 18.—Impedance locus diagram for balance current system (high source impedance).

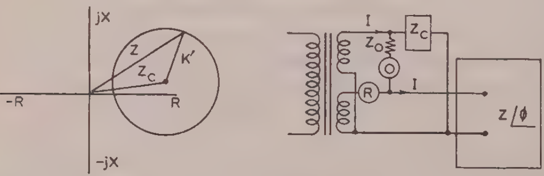

Fig. 19.—Bridge comparator relay for balanced current protection.

(a) For operation, $\frac{I}{Z_O}|(Z\angle\phi - Z_O)N_O| > IN_R$,

i.e. $|Z\angle\phi - Z_O| > \left|Z_O\frac{N_R}{N_O}\right| = |K'|$

(b) Admittance diagram; Z_O is shown inductive.

current relay arrangements are shown in Figs. 19 and 20, these being the circuit duals of the balanced-voltage arrangements of Figs. 13 and 11, respectively. The relay arrangement of Fig. 20 is clearly impractical since a large negative resistance is required for compensation. Fig. 19 shows an attractive arrangement since, for most cases, the compensating impedance can be purely resistive. Such a scheme is suitable for short privately-owned pilot circuits.

(8) ELECTRICAL CHARACTERISTICS OF PRACTICAL PILOT CIRCUITS

The essential information required for the construction of the pilot-circuit input-admittance diagrams can be obtained from

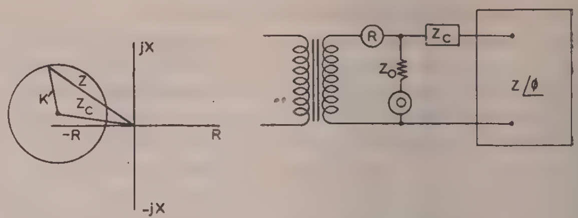

Fig. 20.—Compensated-impedance relay for balanced-current system.

(a) For operation, $\frac{I}{Z_O}|(Z\angle\phi + Z_O)N_O| > IN_R$,

i.e. $|Z\angle\phi + Z_O| > \left|Z_O\frac{N_R}{N_O}\right| = |K'|$

(b) Admittance diagram; Z_O is shown capacitive.

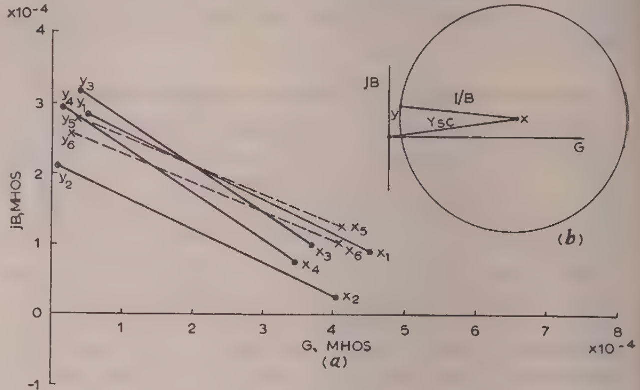

Fig. 21.—Practical telephone-type rented pilot-circuit constants on input admittance diagram.

(a) x_1y_1, x_2y_2 Pilot circuit No. 1, 2 [End 1, End 2]
x_3y_3, x_4y_4 Pilot circuit No. 3, 4 [End 3, End 4]
x_5y_5, x_6y_6 Pilot circuit No. 5, 6 [End 5, End 6]

(b) Construction of admittance diagram with circle centre at $x = Y_{SO}$, and radius $= 1/B$.

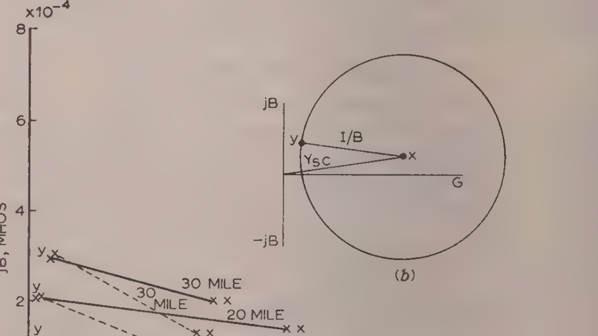

Fig. 22.—Calculated pilot-circuit constants for homogeneous telephone-type pilot circuits.

(a) ——— Unloaded 20 lb/mile circuits.
— — — Loaded 20 lb/mile circuits (loading 88 mH per 2000 yd).

(b) Construction of admittance diagram with circle centre at $x = Y_{SO}$ and radius $= 1/B$.

measurements of open- and short-circuit admittances. These measurements are best obtained at the normal operating voltage of the protection, using an isolated injection supply and conventional bridge methods. From them the required constants can be calculated by normal network theory since $1/\mathsf{B} = \sqrt{[Y_{SCR}(Y_{SCS} - Y_{OCS})]}$, $\mathsf{A}/\mathsf{B} = Y_{SCS}$ and $\mathsf{D}/\mathsf{B} = Y_{SCR}$. From these values the input-admittance locus diagram can be drawn.

Fig. 21 shows the position of the $1/\mathsf{B}$ vector for three practical pilot circuits and Fig. 22 shows calculated values for homogeneous 20 lb/mile conductor circuits both unloaded and with continuous 88 mH per 2000 yd loading. It should be noted that all the practical circuits shown are unsymmetrical (i.e. the input-admittance locus is different at the two ends). In making relay adjustments it clearly follows that different compensating or replica impedances will be required at the two ends, though the restraint resistor must have the same setting at each end since its value is governed by the $1/\mathsf{B}$ vector, which corresponds to the series arm of the equivalent π-network.

(9) CONCLUSIONS

The foregoing analysis of relay characteristics reviews most of the basic pilot-wire relay connections. Other connections are to be seen occasionally which include additional bias or operating coils, these being T-connected at the relaying point. Such connections give characteristics intermediate between the extremes shown, and do not, in general, give fundamental improvement.

Some relays employ a non-linear bias characteristic which, if designed carefully, will serve to increase the diameter of the relay operating circle (in the case of the impedance-type relay connection) and thereby improve stability. Whether or not this feature has any real value is doubtful, since the larger circle could probably have been provided in the first instance. Possibly on systems where, under low-current fault conditions, there is danger of 'masking' due to load currents, the feature has some marginal advantage, though this may be outweighed by the corresponding loss in calculability resulting from the use of additional non-linear circuit-elements.

The arrangements of Figs. 11 and 13 are amenable to linear circuit theory, and the equations developed for relay operation can be used as a basis for investigating overall design and adjustments.

It is shown how the characteristics obtained from these arrangements can be made to approach the ideal, in that a completely symmetrical circular characteristic round the through-fault operating point is obtainable. The diameter can be adjusted to give the required tripping angle in each case.

The design method has been applied to a practical pilot circuit by:

(a) Measurement of pilot-circuit open- and short-circuit admittances from both ends of the line.

(b) Computation of $1/\mathsf{B}$ and construction of the input-admittance diagram.

(c) Investigation of the fundamental relay characteristic on the admittance plane and evaluation of particular values required for compensating impedance and restraint-circuit resistor obtained from ideal values shown on the admittance plot.

(d) Transcription of the admittance-plane diagram to the current plane, involving a simple change of datum and scaling. Inversion of characteristic to obtain performance of remote end relay.

Using this procedure it is considered that any practical scheme can be investigated thoroughly and an accurate prediction made of scheme characteristics at the design stage. The procedure is also essential when commissioning a scheme using actual practical pilot circuits.

With pilot-wire protection systems the demarcation between prototype and commissioning testing is more clearly marked than usual. It is important, therefore, that the overall performance limits of any protection scheme shall be clearly established by exhaustive prototype testing and that commissioning tests should merely comprise the correct setting of all adjustments. The approach outlined in the paper attempts to set down a common philosophy for both purposes.

With the continuing improvements in telephone techniques it is probable that pilot conductor sizes will be further reduced, thereby exaggerating the problem of providing an effective protection system; again this approach will enable a fundamental evaluation to be made into the capability limit of any particular scheme.

(10) ACKNOWLEDGMENTS

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(12) APPENDICES

(12.1) Generalized Input-Admittance Diagram for a 4-Terminal Network Energized from Voltage Sources

The following analysis assumes sinusoidal voltages applied to the pilot circuits, and a relay of negligible impedance.

A 4-terminal network (Fig. 1) having constants A, B, C, D is considered, these being related by the well-known equations

$$V_S = \mathsf{A}V_R + \mathsf{B}I_R \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$I_S = \mathsf{C}V_R + \mathsf{D}I_R \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which

$$\mathsf{A} = 1 + Y_R Z$$

$$\mathsf{B} = Z$$

$$\mathsf{C} = Y_R + Y_S + Y_S Y_R Z$$

and

$$\mathsf{D} = 1 + Y_S Z$$

expressed in terms of the equivalent π-network. Taking the general case specified, of V_R fixed with V_S varying in phase and magnitude, $V_S = KV_R \angle\delta$, where K is the modulus of the ratio

V_S/V_R and δ is the phase angle between the two voltages. Then substituting in eqn. (1) for V_S, we have

$$KV_R\angle\delta - \mathsf{A}V_R = \mathsf{B}I_R$$

and for the local or receiving end,

$$\frac{V_R}{I_R} = Z_R = \frac{\mathsf{B}}{K\angle\delta - \mathsf{A}} \quad . \quad . \quad . \quad . \quad (3)$$

or in admittance form,

$$Y_R = \frac{K\angle\delta - \mathsf{A}}{\mathsf{B}} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Similarly for the remote end,

$$Z_S = \frac{\mathsf{B}}{\mathsf{D} - 1/K\angle\delta} \text{ and } Y_S = \frac{\mathsf{D} - 1/K\angle\delta}{\mathsf{B}}$$

Since the current at end R is assumed to be leaving the 4-terminal network, the expressions for Y_R and Z_R should be multiplied by -1, giving

$$Z_R = \frac{\mathsf{B}}{\mathsf{A} - K\angle\delta} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and

$$Y_R = \frac{\mathsf{A} - K\angle\delta}{\mathsf{B}} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Similarly for end S, if $1/K\angle\delta = V_R/V_S = K'/\overline{\delta}$, say, then

$$Z_S = \frac{\mathsf{B}}{\mathsf{D} - K'/\overline{\delta}} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and

$$Y_S = \frac{\mathsf{D} - K'/\overline{\delta}}{\mathsf{B}} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

These expressions give a locus diagram in terms of input impedance or admittance for a generalized 4-terminal network. They are, of course, vector quantities.

For the impedance equation, the locus of input impedance is a circle of radius $\mathsf{B}K/(\mathsf{A}^2 - K^2)$ at a distance $\mathsf{AB}/(\mathsf{A}^2 - K^2)$ from the origin (Fig. 23). This construction is by no means as convenient as the admittance form since both parameters vary with K.

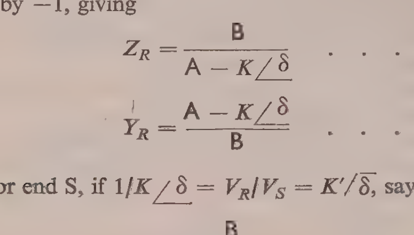

Fig. 23.—Locus diagram for input admittance of 4-terminal network showing inversion for input-impedance diagram.

$$\text{Oa} = \frac{|\mathsf{A}| + K}{|\mathsf{B}|} \qquad \text{Oa}' = \frac{|\mathsf{B}|}{|\mathsf{A}| + K}$$

$$\text{Ob} = \frac{|\mathsf{A}| - K}{|\mathsf{B}|} \qquad \text{Ob}' = \frac{|\mathsf{B}|}{|\mathsf{A}| - K}$$

$$\text{ab} = \frac{2K}{|\mathsf{B}|} \qquad \text{a}'\text{b}' = \frac{|\mathsf{B}|}{|\mathsf{A}| - K} - \frac{|\mathsf{B}|}{|\mathsf{A}| + K} = \frac{2|\mathsf{B}|K}{\mathsf{A}^2 + K^2}$$

$$\text{Oc} = \frac{|\mathsf{A}|}{|\mathsf{B}|} \qquad \text{Oc}' = \frac{|\mathsf{B}|}{2}\left(\frac{1}{|\mathsf{A}| - K} + \frac{1}{|\mathsf{A}| + K}\right) = \frac{|\mathsf{A}|\,|\mathsf{B}|}{\mathsf{A}^2 + K^2}$$

Radius and centre for impedance diagram both depend on A, B and K. Circles are not concentric.

By inspection of eqn. (6), it is noted that the locus of Y_R is a series of circles of radius K/B at a distance of A/B from the origin. These circles are concentric and vary linearly with K. The phase angle can be plotted using a protractor and has its datum 0° along the line—$1/\mathsf{B}$ drawn from the point A/B. The conventions and symbols adopted in this and subsequent derivations are shown in Fig. 1.

In producing a practical locus diagram, the constants A and B must be found. This will normally be carried out by measurement of the open- and short-circuit impedance of the pilot-wire circuit. From these measurements $\mathsf{A}/\mathsf{B} = Y_{SCR}$ (short-circuit admittance viewed from end R) and

$$\mathsf{B} = 1/\sqrt{[Y_{SCS}(Y_{SCR} - Y_{OCR})]}$$

where Y_{SCS} is the short-circuit admittance of the pilot circuit viewed from end S and Y_{OCR} is the open-circuit admittance viewed from end R. Thus from eqn. (6),

$$Y_R = Y_{SCR} - \frac{K\angle\delta}{\sqrt{[Y_{SCS}(Y_{SCR} - Y_{OCR})]}} \quad . \quad . \quad (9)$$

and similarly from eqn. (8),

$$Y_S = Y_{SCS} - \frac{K'/\overline{\delta}}{\sqrt{[Y_{SCSI}(Y_{SCR} - Y_{OCR})]}} \quad . \quad . \quad (10)$$

where

$$K'/\overline{\delta} = \frac{1}{K\angle\delta}$$

(12.2) General Locus Diagram for Balanced-Current Schemes

The following extends the approach to balanced-current (high source impedance) types of pilot-wire protection, assuming that sinusoidal quantities are applied to the pilot circuit.

From the basic equations (1) and (2) of a 4-terminal network, putting $I_R = KI_S\angle 180° - \gamma = -KI_S\angle\gamma$,

$$V_S = \mathsf{A}V_R - \mathsf{B}KI_S\angle\gamma \quad . \quad . \quad . \quad . \quad (11)$$

$$I_S = \mathsf{C}V_R - \mathsf{D}KI_S\angle\gamma \quad . \quad . \quad . \quad . \quad (12)$$

From eqn. (12),

$$V_R = \frac{I_S(1 + \mathsf{D}K\angle\gamma)}{\mathsf{C}}$$

Substituting in eqn. (11),

$$Z_S = \frac{V_S}{I_S} = \frac{\mathsf{A}(1 + \mathsf{D}K\angle\gamma)}{\mathsf{C}} - \mathsf{B}K\angle\gamma$$

$$= \frac{\mathsf{A} + K\angle\gamma}{\mathsf{C}} \quad (\text{since } \mathsf{AD} - \mathsf{BC} = 1)$$

$$= Z_{OCS} + \frac{K\angle\gamma}{\mathsf{C}}$$

This is plotted in Fig. 18 for the practical pilot circuit depicted in Fig. 4, for which

$$\frac{1}{\mathsf{C}} = \sqrt{[Z_{OCS}(Z_{OCR} - Z_{SCR})]} = +380 + j1\,890$$

$$Z_{OCR} = 960 - j1\,880$$

$$Z_{OCS} = 840 - j1\,860$$

$$Z_{SCR} = 2\,350 - j600$$

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A DYNAMIC MODEL FOR STUDYING THE BEHAVIOUR OF THE OVERHEAD EQUIPMENT USED IN ELECTRIC RAILWAY TRACTION

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SUMMARY

In electric traction systems in which power is supplied from an overhead contact wire via a pantograph, the mechanical design of the contact wire and its supporting equipment is clearly of importance in relation to the problem of current collection at high speed. Because of the great difficulty of making accurate measurements on the full-size system and the difficulty of making experimental modifications to it, a dynamically equivalent laboratory scale model has been designed and constructed. The limited information which was obtainable from the full-size system has been used to verify the performance of the model, which can now be used for a systematic study of the effects of design variables.

(1) INTRODUCTION

The use of models for studying various types of engineering projects is a well accepted practice. There are, broadly, two reasons for constructing a model for conducting systematic investigations:

(*a*) It may be practically impossible, or economically prohibitive, to carry out experimental tests on the full-scale version, apart from observation of its performance under normal working conditions in service.

(*b*) A complete theoretical study may not be practicable because of mathematical complexity or inadequate knowledge.

In the case of the system comprising an overhead conductor providing the motive power for electric railway traction and the current collector (pantograph) used in conjunction with it, both these reasons are valid. Experiments on a railway track in commercial service are fraught with considerable difficulties, and the construction and maintenance of a special test section of sufficient length for high-speed running is extremely costly; theoretical study, although in progress with some prospect of ultimate success, is only in its early stages. Therefore, the design and construction of a dynamically equivalent model, capable of representing high-speed running up to 100 m.p.h., was considered to be justified in order to investigate the effects of variations in design features.

Before constructing any model, the situation must be explored theoretically to ensure that the model will faithfully reproduce all relevant modes of behaviour of the full-scale prototype, and that any extraneous modes of behaviour are identifiable. It is not necessary that the theoretical background be complete, in the sense that rigorous equations of motion, equilibrium, etc., must be derived, but, on the other hand, a very thorough understanding of the physical properties involved is essential, and sound judgment must be applied in deciding which properties of the system must be taken into account. The principles of dimensional analysis can then be used to give a high degree of assurance that the behaviour of the model will be realistic.

(2) OVERHEAD SYSTEM TO BE MODELLED

Overhead electric traction equipment is far from being standardized, but all systems have certain features in common. In all cases, the object is to provide an overhead conductor from which an uninterrupted current may be taken to the traction motors via one or more pantographs mounted on the locomotive or multiple unit.

Ideally, the conductor should be continuous and at a constant height above the track, and its mechanical properties should be uniform along its length; the pantograph should exert a constant upward thrust, regardless of speed. In practice, these ideals cannot be realized. The conductor can be produced and installed only in lengths of about a mile; its height must change, for example, at over-bridges, tunnels and level crossings; it must be supported from structures often positioned at irregular intervals, and electrical insulation may need to be inserted in it. The pantograph, however good its control mechanism and design, has inertia and cannot respond instantaneously to changes in the rigidity of the conductor or to oscillatory movements of the locomotive caused by its spring suspension or by the track itself; also, the pantograph has aerodynamic resistance so that its characteristics are, to some extent, speed-dependent.

In consequence, the contact pressure between the conductor and the contact strips of the pantograph can vary considerably, and, for satisfactory performance, it must neither be so low that effective contact is lost nor so high that excessive movement, wear or mechanical damage occurs. The problem is not so difficult with low-voltage systems as it is with the high-voltage systems which are now being much more extensively used. This is because in a low-voltage system (e.g. 1·5 or 3 kV) heavy conductors are necessary and they exert some measure of control over the pantograph. In high-voltage systems (e.g. 25 kV), the overhead conductor is, in itself, lighter and more flexible, but it needs larger and heavier insulators; therefore, uniform rigidity is much more difficult to achieve.

Some typical arrangements of overhead equipment, as used in the British Railways 25 kV a.c. system, are briefly described.

(2.1) Simple Catenary Equipment

The simple catenary equipment is illustrated in Fig. 1(*a*). A solid cadmium-copper contact wire, of cross-sectional area $0 \cdot 166\,\text{in}^2$, is supported at intervals of 15–20 ft by solid wire 'telescopic' droppers from a stranded cadmium-copper cable (19/·083 in) which is suspended approximately in the form of a catenary between supporting structures. The droppers have a chain-link arrangement (Fig. 4), so that, however much the contact wire is raised by the pantograph, they are never in compression. The distance between adjacent structures is 240 ft maximum, but it is often less because of local requirements.

In 'fixed' simple catenary equipment, both the contact and catenary wires are rigidly fixed at the terminations, and the tension in them, and their sags, vary with temperature. Thus, this arrangement is not usually considered suitable for very-high-speed running.

For faster running, 'weight-tensioned' equipment is used. Both wires are rigidly anchored to a structure at the mid-point of their length, and, at the ends, at a distance of about half a

The authors are with British Insulated Callender's Cables, Ltd.

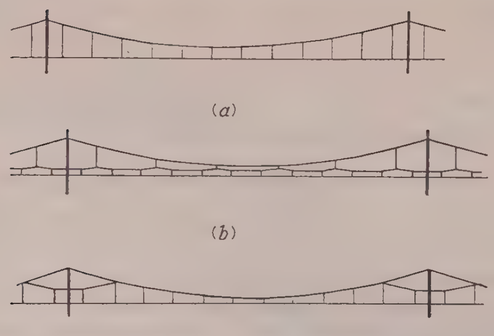


Fig. 1.—Schematic elevations of three principle systems of overhead equipment.

(a) Simple catenary.
(b) Compound catenary.
(c) Stitched catenary.

mile from the anchor point, constant tension is maintained by a suitable mechanism and dead weights. At the intermediate supporting structures, provision is made for the along-track movement of the wires which results from temperature changes.

(2.2) Compound Catenary Equipment

Weight-tensioned simple catenary equipment suffers from the disadvantage that the attachment of the contact wire to the supporting structures, even by hinged members free to move, causes variations in flexibility ('hard spots'). The situation is improved by using a compound catenary as shown in Fig. 1(b). The contact wire is suspended from a cadmium-copper auxiliary catenary by solid-wire loop droppers (Fig. 4) which are secured to the contact wire but are free to slide vertically with respect to the auxiliary catenary, which is itself suspended by rigid solid-wire droppers from the main catenary. With this arrangement, speeds in excess of 100 m.p.h. are possible.

(2.3) Stitched Catenary Equipment

The stitched catenary equipment is shown in Fig. 1(c) and has been used as an alternative to compound catenary equipment. The contact wire near the structure is supported by an additional wire which is attached to the catenary on each side of the structure, thus reducing the irregularity in flexibility. The droppers are of the same design as in simple catenary equipment.

(2.4) General

In all of the arrangements described, the contact wire is 'staggered' from side to side in a horizontal plane to distribute the wear on the rubbing strips of the pantographs. These are about 4 ft long to allow for the effects of contact-wire displacement by wind, vehicle sway and the offsetting of the contact wire at curves and junctions.

It is convenient to defer a description of the main supports for the overhead equipment and the pantograph until the modelling of these components is considered.

(3) GENERAL CONSIDERATIONS AFFECTING THE MODEL

(3.1) Requirements

It is desirable that the model should be geometrically similar to the prototype, and the scale must be such that the model can be conveniently housed, but not so small that the displacements and forces within it are too small for accurate measurement.

Dynamic similarity is essential; i.e. suitably scaled-down values of forces in the prototype, when applied to the model, must produce geometrically equivalent displacements, or be associated with velocities, accelerations, strains, etc., in the model, which are scaled-down values of the corresponding quantities in the prototype. Therefore, it must be decided what forces are operative in the prototype, what movements they produce, and which physical properties of the wires and other parts of the equipment determine the extent of the movement.

The forces on the wires are as follows:

(a) Gravitational, arising from the weight of the equipment.
(b) Upthrust from the pantograph, consisting of a vertical component to maintain contact. There are also horizontal frictional forces, which may be resolved into longitudinal and lateral components, but these are judged to be small.
(c) Longitudinal (static) tension in the contact wire and catenaries.
(d) Wind pressure, mainly lateral but perhaps with a vertical component, and also aerodynamic forces caused by the passage of a train.

These forces would be expected to produce a vertical movement of the contact and catenary wires, with an acceleration determined by the inertial mass of the system and, possibly, oscillations, travelling waves or standing waves. (On curved portions of the track, lateral movements may occur.) The physical properties of the overhead system which determine the nature and extent of these movements are as follows:

(a) The mass per unit length of the contact and catenary wires.
(b) The inertial masses of other component parts which are free to move.
(c) The extensibility, e, of each of the wires, i.e. the fractional extension per unit force, which, in a solid member of constant cross-sectional area, A, is related to Young's modulus, E, by $e = 1/EA$.
(d) The flexural rigidity, B, of each wire, defined as $B = MR$, where M is the bending moment required to produce a radius of curvature, R.

(3.2) Scaling

The scaling factors of the three fundamental concepts of length l, mass, m, and time, t, are defined as λ, μ, and τ, respectively; i.e. lengths in the model are in the ratio of $1/\lambda$ to lengths in the prototype, etc. These scaling factors are, in themselves, arbitrary and can be chosen to suit convenience or to permit compliance with other restrictions. However, the requirements of dynamic similarity demand that consideration be given to the physical properties of the system, and the materials used in the model must be so selected that similarity is produced.

As the prototype and the model operate in the same gravitational field, the same gravitational acceleration is present in both, so that the acceleration must be scaled by unity. Thus, using the subscript m to refer to the model:

$$\frac{[l]}{[t^2]} = \frac{[l_m]}{[t_m^2]} = \frac{[l/\lambda]}{[t^2/\tau^2]} = \frac{\tau^2}{\lambda}\frac{[l]}{[t^2]}$$

Whence $$\tau = \sqrt{\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The scaling of the relevant derived quantities then follows automatically:

Velocity $$\frac{[l]}{[t]} = \frac{[\lambda l_m]}{[\tau t_m]} = \frac{\lambda}{\tau}\frac{[l_m]}{[t_m]} = \sqrt{\lambda}\frac{[l_m]}{[t_m]} \quad . \quad . \quad . \quad (2)$$

Force $$\frac{[ml]}{[t^2]} = \frac{\mu\lambda[m_m l_m]}{\tau^2[t_m^2]} = \frac{\mu[m_m l_m]}{[t_m^2]}$$

Extensibility (fractional extension per unit force)

$$\frac{[1]}{[ml/t^2]} = \frac{1}{\mu}\frac{[1]}{[m_m l_m/t_m^2]} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Flexural Rigidity ($B = MR$)

$$\frac{[ml^3]}{[t^2]} = \frac{\mu\lambda^3}{\tau^2}\frac{[m_m l_m^3]}{[t_m^2]} = \mu\lambda^2\frac{[m_m l_m^3]}{[t_m^2]} \quad . \quad . \quad . \quad (4)$$

Thus the following scaling factors, by which quantities in the prototype must be divided in order to obtain the corresponding quantities in the model, are derived in terms of the two factors, and μ:

Length	λ
Mass	μ
Time	$\sqrt{\lambda}$
Velocity	$\sqrt{\lambda}$
Acceleration	1
Force	μ
Extensibility	$1/\mu$
Flexural Rigidity	$\mu\lambda^2$

The choice of λ and μ is governed not only by convenience but also by the practicability of finding materials or forms of construction which will give the appropriate relationships between the scaling factors for mass per unit length, extensibility and flexural rigidity.

(3.3) Choice of Scaling Factors

The physical properties of mass per unit length, extensibility and flexural rigidity depend on the materials employed on the type of construction adopted in the model. If all lengths were scaled by λ, the scaling factor for mass, μ, would necessarily equal λ^3 for any given material, but the choice in the model of a material of density different from that in the prototype would permit a different factor.

Moreover, if the legitimate assumption is made that radial movements or oscillations within the body of a wire are without sensible effect on the behaviour of the system as a whole, the scaling of lengths within the wires, perpendicular to their axes, may be different from λ. This removes one of the restrictions in choosing an effective wire diameter in the model and makes it easier to satisfy the required scaling relationships for mass per unit length, extensibility and flexural rigidity.

(3.4) Physical Properties to be Scaled

The physical properties of the contact wire, main catenary and auxiliary catenary are given in Table 1.

Table 1

Physical Properties of Wires in the Prototype

Physical property	Contact wire	Main catenary	Auxiliary catenary
Mass per unit length, lb/in	0·0541	0·0345	0·0124
Extensibility, per lb ..	$3{\cdot}61 \times 10^{-7}$	$7{\cdot}10 \times 10^{-7}$	$15{\cdot}50 \times 10^{-7}$
Flexural rigidity, lb-in²	$4{\cdot}58 \times 10^{4}$	$2{\cdot}78 \times 10^{3}$	$6{\cdot}73 \times 10^{2}$

Space considerations suggested a scaling factor for length, λ, of about 40, resulting in a model length of 25–30 ft representing six spans of full-scale equipment; it was considered that anything smaller than this would involve great constructional difficulties.

The choice of a suitable value for μ is restricted not only by the availability of suitable materials, but also because a wide range of forces must be simulated. The static tension in the prototype contact wire and catenary is about 2000 lb, but the pantograph upthrust is only about 20 lb. The scaled upthrust cannot be less than about 0·01 lb without incurring difficulty in maintaining precision, or without rendering unjustifiable the ignoring of frictional forces. Thus μ must be not greater than 2000, and a value tending towards 1000 would be convenient

(4) DESIGN OF MODEL

(4.1) Contact and Catenary Wires

The obvious first choice would be to use circular wires. Assuming tentative values for λ and μ of 40 and 1000, respectively, the two main wires in the model would require the properties given in Table 2.

Table 2

Physical Properties required of Circular Wires in the Model

Physical property	Formulae	Contact wire	Main catenary
Mass per unit length, lb/in	$\pi r^2 \rho$	$2{\cdot}16 \times 10^{-3}$	$1{\cdot}38 \times 10^{-3}$
Extensibility, per lb ..	$\frac{1}{\pi r^2 E}$	$3{\cdot}61 \times 10^{-4}$	$7{\cdot}10 \times 10^{-4}$
Flexural rigidity, lb-in²	$\frac{1}{4}\pi r^4 E$	$2{\cdot}86 \times 10^{-2}$	$1{\cdot}74 \times 10^{-3}$

$\lambda = 40$.
$\mu = 1000$.
r = Radius, in.
ρ = Density, lb/in³.
E = Young's modulus.

With a circular wire of radius r, density ρ and Young's modulus E, the product of mass per unit length multiplied by extensibility is ρ/E, and Table 2 shows that this must be $7{\cdot}8 \times 10^{-7}$ for the contact wire. Since E is at least 2×10^6 for any metal, this leads to the impossible value of approximately 1·5 lb/in³ (40 g/cm³) for density. The situation with the stranded catenary wire is even worse, so that the use of simple circular wires must be discarded.

The possibility of using non-metallic threads (e.g. nylon, polythene, rubber, etc.) was considered, but this approach was ruled out because of the difficulty of obtaining a suitable combination of physical properties and because of the dependence of the properties of such materials on temperature, humidity and mechanical loading. Much time was spent in studying the possibility of using helical springs, but it was found that, although values of about 40 for λ are possible, the corresponding values of μ are unavoidably far too high (i.e. greater than 10^6), A similar disadvantage was found with other possible constructions such as twisted strip, crimped wires or strip, or punched strip.

The problem is to provide a sufficiently large mass per unit length without decreasing the extensibility or increasing the flexural rigidity. What was seen to be necessary was a form of construction having the elastic properties of a thin wire but the mass of a larger one. This can be achieved by the use of metal beads threaded on fine wire.

As the pantograph must be able to slide freely along the simulated contact wire, the beads must be cylindrical and concentric and at the closest possible spacing. Smooth running should then be ensured if the contact strip on the pantograph is so designed that it bridges three or four beads simultaneously, The feasibility of such a design may be tested by assuming that the beads are rigid, i.e. the required elasticity is provided only by the short lengths of exposed wire between beads. Then, using the following notation,

b = Bead length, in.
s = Spacing, in.

D = Bead diameter, in.
d = Wire diameter, in.
ρ_b = Bead density, lb/in³.
ρ_w = Wire density, lb/in³.
E = Young's modulus of wire.

The physical properties of a simulated wire in the model may be expressed as follows:

Mass per unit length, lb/in

$$\frac{\pi}{4}d^2\rho_w + \frac{\pi}{4}(D^2 - d^2)\rho_b\frac{b}{b+s}$$

Extensibility, per lb

$$\frac{4}{\pi d^2 E}\frac{s}{b+s}$$

Flexural rigidity, lb-in²

$$\frac{1}{64}\pi d^4 E\frac{b+s}{s}$$

The product of extensibility times flexural rigidity is $d^2/16$, which must be $1/\lambda^2$ times the corresponding product for a wire in the prototype. Thus, for the modelled contact wire,

$$\frac{d^2}{16} = \frac{1{\cdot}66 \times 10^{-2}}{\lambda^2}$$

which, with $\lambda = 40$, gives $d = 0{\cdot}0129$ in. Assuming a bead length of about $\frac{1}{16}$ in, at a spacing of about $\frac{1}{32}$ in, the required value of E for the contact wire is approximately 1×10^7, which is reasonable for one of the common metals. With the same trial values and assuming that both wire and beads are made of brass, the bead diameter required is about 0·13 in, which is also a reasonable value.

So far only one wire has been considered, but the catenary wires must also be designed for identical values of λ and μ. Once λ has been chosen, the diameters of the three wires for the model are fixed. Retaining the value $\lambda = 40$, they become:

Contact wire	0·0129 in
Main catenary wire	0·004 46 in
Auxiliary catenary wire	0·003 24 in

The ratio of extensibilities is $1/\mu$, so that, with μ the same for all three wires, fixed ratios between the Young's moduli must be preserved, which, in the order contact wire: main catenary wire: auxiliary catenary wire, are 1 : 4·2 : 3·6. Considering the contact and main catenary wires, a suitable selection is brass (60% copper, 40% zinc) and tungsten, having values for Young's modulus of $1{\cdot}41 \times 10^7$ and $5{\cdot}92 \times 10^7$, respectively. Tungsten is also the metal which gives the nearest value to the required ratio for the auxiliary catenary wire, and, as will be shown later, discrepancies can be allowed for by adjustments in dimensions.

(4.2) Final Development of the Wires

In practice, the elastic properties of a beaded wire do not depend only on the short lengths of exposed wire. The beads themselves make some contribution, and also the overall properties are considerably influenced by the method by which they are attached. This effect can only be determined by measurements on samples, and, for this purpose, it is convenient to consider the extensibility and flexural rigidity of the wires alone. The actual effect of fixing the beads to the wire will be to increase the flexural rigidity, B_m, and to reduce the extensibility, e_m. For a given spacing between beads, the effect is a direct function of the bead length b, provided that the bead diameter is adjusted to conform with the required mass per unit length, w. Then, from the scale factors set down in Section 3.2, and using symbols without suffices for the prototype,

$$\frac{w}{w_m} = \frac{\mu}{\lambda}, \frac{e}{e_m} = \frac{1}{\mu} \text{ and } \frac{B}{B_m} = \mu\lambda^2$$

From these relationships and using the values of E in Section 4.1, a new tentative value for μ of 1 500 is deduced (instead of the 1 000 which was previously assumed to demonstrate feasibility).

If, for a given bead length, the actual values of extensibility and flexural rigidity in the model wires are taken as e_m' and B_m',

$$e_m' = \frac{1}{x}e_m$$

and

$$B_m' = yB_m$$

For a constant mass per unit length, it follows that the values of λ and μ will both be decreased by a factor of $1/x$. However, from the requirement for flexural rigidity, the scaling equations can now be satisfied only if

$$y = x^3$$

Therefore, a value for the bead length must be found with which the cube of the decrease of extensibility, in comparison with the theoretical value is equal to the increase in the flexural rigidity. Clearly, this can only be done empirically, and 15 in lengths of model contact wire were made with bead lengths of 0·055 in, 0·0625 in, 0·072 in and 0·125 in, preliminary trials having indicated a probable solution in the region of $\frac{1}{16}$ in. A constant spacing of $\frac{1}{32}$ in was used.

The results of measurements of e_m' and B_m' for these test lengths are shown graphically in Fig. 2, where y and x^3 are plotted against bead length. The curves cross at a bead length of 0·072 in, at which $x = 1{\cdot}23$. This correction, applied to the

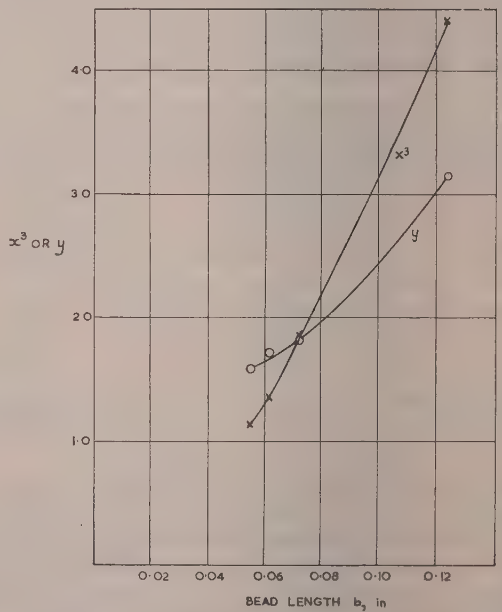


Fig. 2.—Effect of bead length upon extensibility and flexural rigidity of modelled contact wire.
x = Change in extensibility (decrease).
y = Change in flexural rigidity.

Table 3

FINAL DESCRIPTION AND PROPERTIES OF MODEL WIRES ($\lambda = 32 \cdot 5$, $\mu = 1\,220$)

Item	Wire		Beads			Mass per unit length	Extensibility	Flexural rigidity
	Material	Diameter	Material	Length	Diameter			
		in		in	in	lb/in $\times 10^{-3}$	per lb	lb-in^2 $\times 10^{-3}$
Contact	Brass	0·0129	Brass	0·072	0·0925	1·45	$4 \cdot 41 \times 10^{-4}$	35·5
Main catenary ..	Tungsten	0·0045	Brass	0·067	0·0738	0·92	$8 \cdot 67 \times 10^{-4}$	2·16
Auxiliary catenary	Tungsten	0·0032	Brass	0·053	0·0463	0·33	$18 \cdot 9 \times 10^{-4}$	0·52

previous values of 40 and 1 500 for λ and μ gives the final values of 32·5 and 1 220, respectively.

Using these scale factors, the required values of extensibility and flexural rigidity were then calculated for the two types of catenary wire. Retaining brass as the bead material, the bead sizes for each were determined empirically, as with the contact wire, and the bead length for the auxiliary contact wire was selected to accommodate the discrepancy in the ratio of Young's moduli, referred to earlier. With a bead spacing of $\frac{1}{32}$ in in all cases, the final description and properties of the model wires are summarized in Table 3.

(4.3) Manufacture of Model Wires

Because of the number of beads required (over 10 000 for a 30 ft model of compound catenary equipment), careful thought was given to the method of manufacture. After preliminary trials had revealed some of the difficulties, it was decided to use machined beads and to attach them to the wires by soft-soldering. To enable this to be done on the tungsten wires, they were coated by electro-deposition with a layer of copper, thick enough to permit soldering but too thin to affect the mechanical properties.

The wires were assembled in straight lengths of 6 ft in vertical jigs. Meticulous care had to be taken in the soldering operation; the holes in the beads were uniformly countersunk and an accurately controlled quantity of flux and solder was used to secure each bead, the soldering being done by resistance heating. Each finished length was tested for extensibility and flexural rigidity (see Section 10). Specimens of the finished wires are shown in Fig. 3. After acceptance testing, the individual lengths

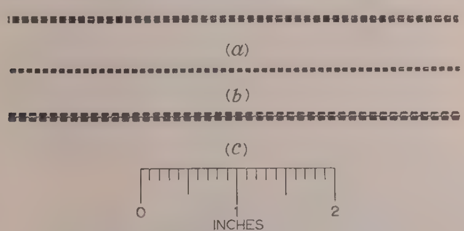


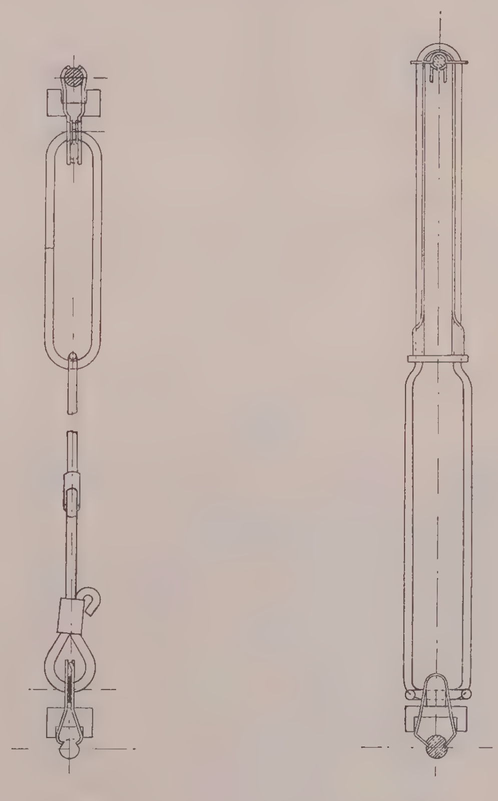
Fig. 3.—Completed model wires.

(a) Main catenary.
(b) Auxiliary catenary.
(c) Contact wire.

were connected by a mechanical joint in the wire, over which was positioned an additional soldered bead to preserve mechanical uniformity throughout.

(4.4) Droppers

In the prototype, the contact wire is supported from the catenary by droppers. In simple and stitched catenary equipment the telescopic droppers, made from $\frac{3}{16}$ in-diameter hard-drawn copper, contain chain links. In compound catenary equipment, the droppers between the main and auxiliary catenaries are rigid, but the contact wire is supported from the auxiliary catenary by loop droppers. Telescopic and loop droppers are shown in Fig. 4. Because these droppers are never



(a) (b)

Fig. 4.—Full-size droppers.

(a) Solid wire telescopic dropper.
(b) Solid wire loop dropper.

in compression, their elastic properties need not be reproduced in the model, but, as their mass constitutes a load on the system, this must be correctly scaled.

Brass-wire telescopic droppers were made in halves with a looped connection in the middle, the ends having hooks to grip the model wires in the space between the beads, where they were secured by a plastic cement. The correctly-scaled mass was achieved by affixing beads, or fractions of beads, to the dropper wires, as shown in Fig. 5.

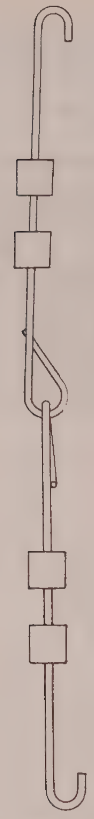


Fig. 5.—Model of simple catenary dropper.

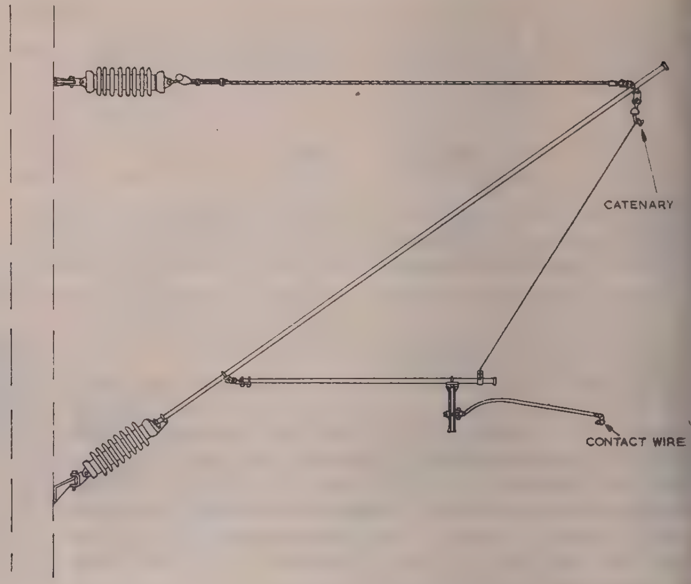


Fig. 6.—Full-size hinged cantilever.

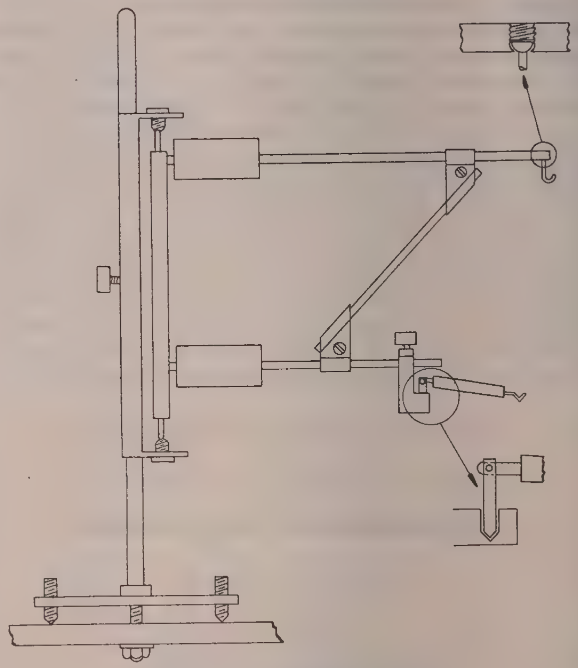


Fig. 7.—Model hinged cantilever.

(4.5) Supports

Depending on the number of parallel tracks to be served and other features of the system, the supports for the overhead wires in the full-scale system take numerous forms. Where weight tensioning is used, provision is made for the along-track movement of all wires, and the contact wire is allowed independent horizontal and vertical movement. All forms of support will eventually be reproduced in the model, but the first to be studied was the 'hinged cantilever' type, illustrated in Fig. 6. A diagonal strut tube is supported from porcelain insulators by a stranded tie wire and is hinged at the vertical mast. The catenary is fixed to the upper end of the tube, and the auxiliary catenary and contact wire are fixed to other hinged members, the 'registration arm' and 'steady arm', respectively, the latter being capable of movement independently of all other hinged members. The 'stagger' of the contact wire is achieved by reversing the steady arm at alternate supports (i.e. 'push-off' and 'pull-off'), so that the arm is always in tension.

The cantilever adds inertia to the system, so that its mass and the distribution of mass must be scaled correctly, using the already established value of $\mu = 1\,220$. The geometry of the hinges must be preserved, by scaling distances by the factor λ, but it is not necessary for the actual members to be reproduced. The design of the model hinged cantilever was based on the inclusion only of those hinges which affected the movement of the system as a whole, those which only affected the erection and adjustment of the support being ignored. As it is not possible to scale frictional effects, all the pivots in the model were made as free as possible. The model of a cantilever for the push-off side of the contact wire is shown in Fig. 7.

(4.6) Pantograph

The Stone–Faiveley pantograph used on the British Railways 25 kV a.c. system is shown in Fig. 8. The 'pan-head' is raised by means of two working springs acting on levers attached to a spindle at the base of the lower member; the upward pressure can be varied by the static tensioning of these springs. Since the tension in the springs varies with their extension, and as the upthrust is required to be constant, the point of attachment of each spring to the spindle is made through three flexible links; at four selected points, two on each side, abutment screws fixed

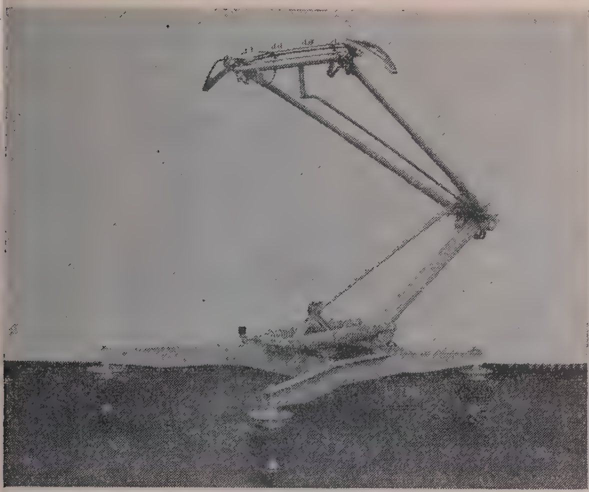


Fig. 8.—Full-size pantograph.

to the spindle thrust against these links, thus changing the operating length of the springs. The geometry of the system is so arranged that, at these four points (corresponding to four different heights of the pan-head above the roof of the locomotive), the upthrust is constant. The upthrust/height relationship is shown in Fig. 9, and the curve is, in fact, a series of discon-

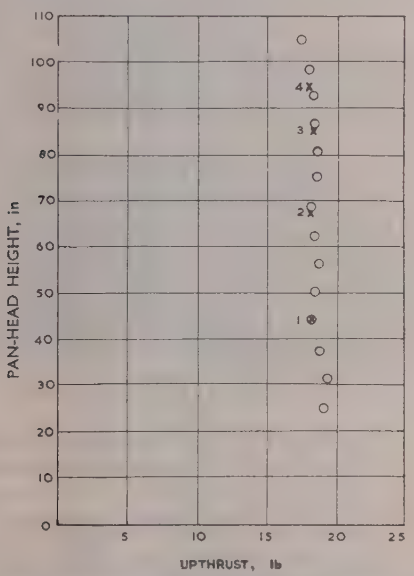


Fig. 9.—Characteristics of full-size pantograph.

Measurements were made at regular intervals of the vertical displacement of the panhead, those at the abutment positions being indicated by crosses.

tinuous curves, crossing at the abutment positions. The pan-head itself is sprung independently of the other members, springs and rubber bushes providing restoring forces should the pan-head be tilted in either an along-track or transverse direction.

The inertia of the pantograph must be scaled, and the first approach was to attempt to translate the inertia as a whole into a correctly scaled model system. Theoretical analysis showed that this method would be extremely complex, and so it was abandoned in favour of a simpler one.

In the absence of any reaction from the overhead wires, the only forces present in the pantograph are those due to spring tension and gravity. Ignoring friction, work is done against the inertia of the individual moving members, which experience both linear and angular accelerations. All forces are thus functions of mass, length and spring constants, while the accelerations of individual members are functions of length, mass and distribution of mass. Thus the inertia of the pantograph as a whole may be modelled by correctly scaling the length, mass and distribution of mass of each moving member; the contribution of the mass of the working springs to the inertia may be disregarded.

Consideration of the non-uniformity of the members showed that the theoretical design of an individual member for the model, to give the correct bearing distances, mass and distribution of mass, would be very difficult. Hence, each member was designed with the correct bearing length and approximately the correct position of the centre of gravity, but with a lower mass than required. The requisite additional mass was then added in two portions, movable along the main axis of the member, the spacing being adjusted to give the correct position of the centre of gravity and the correct period of oscillation. In some cases this led to the positioning of additional mass outside the bearing lengths. The pan-head springing was achieved with leaf springs. The characteristics of the model were confirmed by comparison with tests on a prototype pantograph.

(5) LAYOUT OF THE MODELS

It was decided that the first models would represent straight, level track with no complications such as gradients, electrical sectioning devices, cross-overs, etc.; these, and curved track, will be introduced later, as necessary.

(5.1) Track and Supporting Structure

The smallest single unit in weight-tensioned systems is a half-tension length, from the mid-point anchor to the tensioning weights, comprising half a mile of track and 11 spans of overhead wires, each nominally 240 ft in length. This was judged to be the minimum stretch which should be modelled, if the same interaction between spans were to occur in the model as in the full-scale system. It was also felt desirable that an additional lead-in span should be provided on the other side of the mid-point anchor. Thus, each model needed 12 spans, which, with a length scaling factor of $32 \cdot 5$, gave a length of $88 \cdot 6$ ft.

A steel-angle structure, 4 ft high and 2 ft wide, was erected in a closed loop with straight sides 144 ft in length and with 16 ft-diameter curves at each end. On top of this were mounted two continuous horizontal strips of chipboard, with a central space between. The chipboard was firmly secured with rubber cement and was well lacquered to prevent distortion through pick-up of moisture from the atmosphere. The running track, consisting of standard model-railway $\frac{3}{8}$ in aluminium flanged rail, was screwed to the chipboard. The complete structure and the rails were levelled by means of water-tube devices and spirit levels.

The space between the chipboard strips, carrying one rail on each side, permitted the underslinging of the driving motor of the trolley carrying the pantograph, to reduce the risk of derailment in negotiating the curves at speed. Even so, the curves imposed a speed limit around them of 12 m.p.h., even with check rails and banking. A full-scale speed of 100 m.p.h. corresponds to $17 \cdot 6$ m.p.h. in the model, and so provision had to be made for the rapid acceleration of the trolley as soon as it emerged from the curve, followed by the maintenance of a constant speed of the required value while actually in the test length, and then by rapid braking before entering the curve at

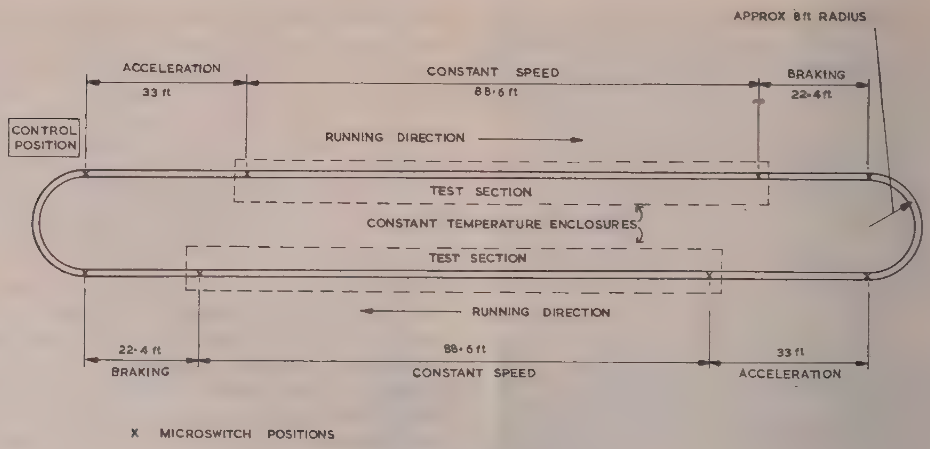


Fig. 10.—Schematic of track layout.

the other end. This was achieved by using a 50-volt d.c. motor, supplied from the rails, with microswitches placed at the necessary positions for operating control circuits to increase, reduce or reverse the voltage. (It was appreciated that such complications could have been reduced, or avoided, if proper transition curves and a larger diameter had been used, but, unfortunately, a much larger laboratory would have been needed and this was not available.)

Although it is possible to reproduce in the model changes due to changes in atmospheric temperature, the necessary complications were thought to be unjustified in the early stages of the researches which are being carried out, and constant-temperature enclosures (25° C) were built around the test lengths in each of the straight sides of the track. To control the height of the pan-head while travelling outside the test lengths, and thus prevent possible damage, a continuous length of nylon curtain-rail was fitted at the required height above the track. The whole arrangement is shown schematically in Fig. 10.

(6) MEASUREMENTS

The information extracted from the models is derived from two sources—the trolley carrying the pantograph and the overhead wires. The first measuring devices which have been installed are exploratory and will be modified or replaced if experience shows it to be necessary.

(6.1) Measurements from the Trolley and Pantograph

The position, velocity and acceleration of the pantograph trolley must be known at any instant. Signals proportional in magnitude to these three parameters are fed back to a receiver from a transmitter carried on the trolley. The transmitter aerial consists of two flat plates which span a wire constituting the receiver aerial, which is carried by the supporting structure.

An extra wheel on the trolley carries a number of equally spaced circumferential holes which allow a beam of light to be focused on to a photo-electric cell; movement of the trolley causes rotation of the wheel and interruption of the beam, and pulses proportional to the trolley velocity are fed from the photo-electric cell and transmitted to the receiver, where they trigger a multivibrator producing pulses of constant length, the mean level of which is read on a meter calibrated in feet per second. The meter output is taken through a differentiating circuit to another meter to measure acceleration.

On the other side of the trolley, a beam of light is directed downwards, and its reflection from small movable polished aluminium blocks on the structure is collected by a photo-electric cell to provide pulses indicating the position of the trolley. A view of one side of the trolley is shown in Fig. 11.

Two further devices are contemplated for the pantograph, one to measure loss of contact and the other to measure contact pressure, but the design of these is presenting considerable difficulty, because the inertia of the pantograph must not be affected.

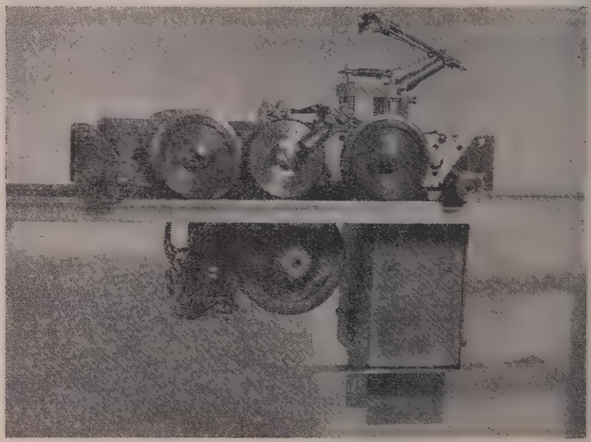


Fig. 11.—The trolley, showing device for measuring velocity and acceleration.

(6.2) Measurements from the Overhead Wires

Several methods, both electrical and optical, for measuring wire displacement are being explored.

In the first to be used, a 60 kc/s signal is supplied to the contact wire and a screened capacitive coupling to an amplifier provides a measure of the uplift of the wire; Fig. 12 shows the coupling plate in position above the contact wire. By placing three or more such couplings at desired positions, the amplitude and phase of any oscillations in the wire can be displayed on a multi-channel recorder, together with the signals of position, velocity and acceleration of the trolley. This method, although excellent for measuring displacement before the pantograph reaches the coupling plate and after it has left it, suffers from the defect that the capacitive effect of the pantograph and trolley influences the result when in immediate proximity. It is therefore not entirely suitable for all types of experiment.

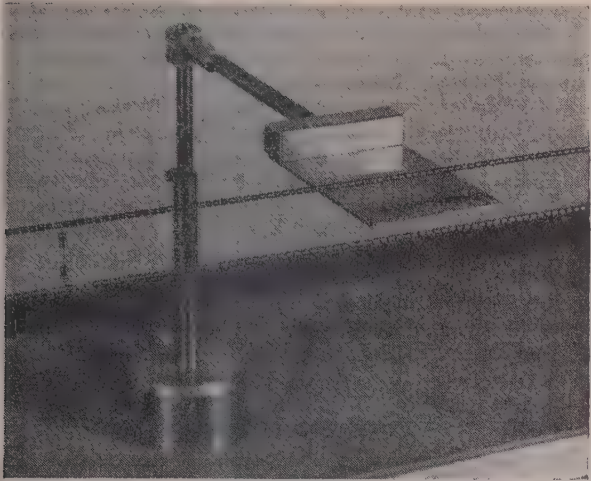


Fig. 12.—Screened capacitive coupling plate for measurement of displacement of contact wire.

An optical method, in which the shadow of the wire is cast on to a suitable photo-electric device is being developed, but, for the first experiments to verify the dynamic performance of the model, resort was had to high-speed cinematography which, although rather tedious, is quite certain and precise in its results. A scale was set up behind the model and a film was taken at 750 frames/sec. This was projected at 16 frames/sec for general observation, while accurate measurements were made on individual frames, as required.

(7) BEHAVIOUR OF THE MODEL

As stated in Section 1, the model was constructed because of the difficulty of making the necessary studies on the full-size system. Precisely this difficulty is met, of course, when it is desired to obtain assurance of the realistic behaviour of the model before proceeding with extensive experimental work. It was found that there nowhere existed full-size equipment exactly similar to the model in respect of straightness, flatness, uniformity of support spacing, etc., deviations from the ideal arrangement having always been necessitated by local requirements. The problem was therefore approached with the attitude that there was no reason to doubt the underlying theory of the model or the accuracy of its construction, so that, even if only limited confirmation were obtainable, it could be used with confidence at least to elucidate the general rules of behaviour of the system. This, alone, would be a considerable step forward; if agreement in detail could be obtained, it would be even better, but not essential at this stage.

First, tests under static conditions were made by applying forces equivalent to 10, 20 or 30 lb to the contact wire of the simple catenary model at various positions throughout a span, and measuring the displacement. The values obtained were compared with those deduced theoretically, the theoretical treatment (which will be reported separately) having already been proved accurate by tests on the only similar full-size equipment available for experiment, but which had, unfortunately, span lengths rather different from those in the model. The results are given in Table 4, where the positions to which the measurements or calculations apply are indicated by the 'dropper number'. The agreement between the two sets of figures is generally very good.

Next, experimental runs were made on a full-size system and compared with equivalent runs on the model. Full-scale

Table 4

Vertical Displacement of Simple Catenary Contact Wire Produced by Static Forces

Position (dropper number)	Displacement					
	10 lb		20 lb		30 lb	
	Model	Calculation	Model	Calculation	Model	Calculation
	in	in	in	in	in	in
1 (near support)	0·38	0·36	0·90	0·90	1·50	1·35
2	0·90	0·96	1·80	1·90	2·80	2·82
3	1·30	1·34	2·60	2·70	3·90	4·05
4	1·39	1·65	3·20	3·30	4·75	4·95
5	1·70	1·90	3·50	3·70	5·32	5·52
6 (near mid-span)	1·85	1·95	3·68	3·90	5·65	5·85

measurements of pantograph displacement were made with an instrumented test coach, and, from the records which were obtained, the displacement of the contact wire at mid-span relative to that at the support was deduced at two different train speeds. These were compared with the results obtained by cinematography in the model, again using simple catenary equipment, and the results are given in Table 5. The figures in brackets in Table 5 are those interpolated assuming a smooth transition between individual values. Displacements in the model can be measured to within 1% and reproducibility is of the same order. Such accuracy is not possible on the full-size equipment, and the four values given were obtained on four adjacent 240 ft spans. The wide variation under nominally identical conditions well illustrates the difficulty of measurement, but it is clear, nevertheless, that the model produces results of the correct order and that the agreement between it and the prototype can be judged satisfactory. It will be noted that, in the model, the relative displacement does not increase regularly with speed; the reason for this is not yet understood, but there is very little doubt that it is a real phenomenon which further work will clarify.

Table 5

Contact-Wire Displacement at Mid-Span, Relative to Displacement at Supports. Simple Catenary Equipment, 240 ft Span

Train speed	Contact wire relative displacement	
	Model	Full-size
m.p.h.	in	in
10	2·56	
20	2·69	
30	3·60	
40	2·90	
50	3·60	
55	(3·95)	3·4, 3·8, 3·2, 4·4
60	4·08	
64	(4·05)	2·8, 4·2, 4·0, 4·6
70	3·90	

(8) CONCLUSIONS

A scale model has been constructed to reproduce the dynamic characteristics of full-size overhead equipment for supplying power for electric railway traction.

The present model is that of an idealized system; it has a straight, level track, a contact wire at constant height and equal spans. There is no difficulty in installing curves and gradients, the contact-wire height can be varied to correspond to the usual arrangements at level crossings, under-bridges, etc., and differences in span length can easily be introduced. Variations, such as dropper spacing, dropper length, longitudinal tension, different types of support, multiple pantographs, etc., present no problems. On the other hand, other less obvious factors influence the behaviour of the prototype, the two most important probably being aerodynamic forces and movements of the pantograph initiated by the motion of the vehicle carrying it. The procedure will be to ignore such factors in the first place, regarding them as influences which modify the behaviour of any particular pantograph/overhead-line combination, but which do not change its basic laws.

However, even such factors can be introduced, provided only that certain information can be obtained from full-scale equipment. For example, the effect of speed on pantograph upthrust can be measured by wind-tunnel experiments, and, indeed, information on the Stone–Faiveley pantograph is already available; all that is then necessary is to modify the model pantograph so that its pressure is correct in relation to the speed at which the particular experiment is conducted. Movements of the wires caused by the slipstream of the train are probably small, but they can be measured without too much difficulty by running a train, using Diesel locomotion and holding the pantograph just out of contact; if the movements are significant, mechanical means can be used to move the model wires a corresponding amount. Similarly, the displacement of the wires caused by wind can be reproduced. Vehicle movements due to springing, track irregularity, wheel slip, etc., are rather more difficult, but, again, if full-scale behaviour is known, even in only general terms, it should be possible to simulate equivalent movements in the model pantograph.

All this is in the future and very many experiments need to be made before such complications are introduced. In the first place, the comparative performance of simple equipment, compound catenary equipment and stitched equipment (of various designs) will be examined over a sufficiently wide range of speeds, pantograph pressures and span lengths. The results, apart from their factual value, should enable the mathematical theory to be advanced, should point the directions in which the designs can be improved and should indicate the next most useful avenues to explore. It is hoped that a further paper will be presented in due course.

(9) ACKNOWLEDGMENTS

The authors' thanks are due to Dr. L. G. Brazier, Director of Research, British Insulated Callender's Cables, Ltd., for permission to publish the paper.

They also wish to record their appreciation of the assistance of a number of their colleagues, of the London Midland Region of British Railways for the use of the test coach, and of J. Stone and Co. for the loan of a pantograph.

(10) APPENDIX. MEASUREMENT OF ELASTIC CONSTANTS

(10.1) Extensibility

No difficulty arises in the measurement of the extensibility of solid wires, such as the full-scale contact wire, but stranded cables which have not previously been stressed are not elastic throughout the range of stress to which they are subsequently subjected in operation. This is a phenomenon associated with the tightening of the strands during stressing, and therefore the first measurements of extensibility for a stranded cable are likely to be high. For this reason, several extensions were carried out on the full-scale stranded cables until two consecutive measurements agreed, this usually occurring with the third and fourth extensions. In terms of Young's modulus, the values of 16·5, 16·5 and $13 \cdot 3 \times 10^6 \mathrm{lb/in}^2$ were obtained for contact wire, auxiliary catenary and main catenary, respectively, showing a marked increase in elasticity for the 19-wire cable as compared with the solid, but no measurable difference between the solid and the 7-wire. The values for the main catenary may be compared with figures given by Stickley* for similar measurements on stranded hard-drawn copper cables, when values of 13·7, 14·7 and $13 \cdot 2 \times 10^6 \mathrm{lb/in}^2$ were obtained for 12-, 19- and 37-wire cables, respectively. The figures given in Table 1 are the mean of measurements on three samples of each wire; the measurements on both the full-scale and the model wires were carried out by taking extensions over a range of loads.

(10.2) Flexural Rigidity

The measurement of flexural rigidity is not simple. In a stranded cable, the degree of flexibility is determined by the inter-wire pressure, which, in turn, depends on the tension in the cable. It is therefore important to measure flexural rigidity at the tension at which the cable will normally operate. If a cable of length L under a constant tension T is deflected a distance δ by a transverse force F, then, by taking moments, the equation

$$\frac{d^2y}{dx^2} - \frac{Ty}{B} = \frac{M}{B} - \frac{FL}{4B} + \frac{Fx}{2B} \quad . \quad . \quad . \quad . \quad (5)$$

may be derived,† where M is the bending moment at the point of application of force F. The solution of this equation is

$$\tanh \frac{kL}{4} = K\left(\frac{L}{4} - \frac{T\delta}{F}\right) \quad . \quad . \quad . \quad . \quad (6)$$

where $k = \sqrt{(T/B)}$

Hence, for constant values of T and L, the ratio δ/F may be measured and the flexural rigidity, B, determined.

Eqn. (6) admits only of a graphical solution, and, for the cables measured, the terms $L/4$ and $T\delta/P$ are very nearly equal. The setting up of the tensioned cable is therefore really important; the clamps holding the ends of the cable must be parallel, to satisfy the boundary conditions for eqn. (5), and the point of support from which bending takes place must be known accurately.

The measurements on the full-scale cables were made with the cables tensioned vertically in an hydraulic tensile testing machine, and a range of transverse loads was applied by screw-thread, and the deflections measured by a travelling microscope. The values given in Table 1 are the mean of measurements of the three samples.

The flexural rigidity of stranded cables bears no simple relationship to their geometry, so that accurate theoretical determination of flexural rigidity is not possible. In the case of the solid contact wire, however, such a determination is possible, using the expression from beam formulae

$$B = EI$$

The value of E having been derived as above, the value of I (the second moment of area) may be calculated about a line through the centre of gravity of the wire, by dividing the cross-section into relevant portions and summing moments. This gives a value of $0 \cdot 002\,83\,\mathrm{in}^4$, and using $E = 16 \cdot 5 \times 10^6 \mathrm{lb\text{-}in}^2$,

* Stickley, G.: 'Stress-strain Studies of Transmission Line Conductors', *Transactions of the American I.E.E.*, 1932, 51, p. 1052.

† Sturm, R.: 'An Analysis of Vibration of Cables and Dampers', *Electrical Engineering*, 1936, 55, pp. 455 and 673.

the theoretical flexural rigidity is $4\cdot66 \times 10^4$ lb-in^2, which agrees with the measured value to within 2%.

Although such a treatment is not possible with the stranded cables, two limiting values for the second moment of area may be arrived at, depending upon whether it is assumed that the cable bends as a whole about a common diameter, or whether the individual wires bend independently of each other. In the latter case, the second moment of area may be calculated from the formula

$$I = \frac{n\pi d^4}{64}$$

where n is the number of wires and d is the diameter of each wire

On the other hand, if the cable bends as a whole, the second moment of area of each layer of wires may be found from the formula

$$I = \frac{n\pi d^2}{64}[d^2 + 2(d_1 + d)^2]$$

where n is the number of wires per layer and d_1 is the inner diameter of the layer. Moments of individual layers may be summed to give the total value of second moment of area.

The two limits for the 19/083 in^2 cable are 750 lb-in^2 and 13 000 lb-in^2, and for the 7/083 in^2 cable, 270 lb-in^2 and 1 450 lb-in^2. The measured values for these cables, 2 780 and 673 lb-in^2, tend, as might be expected, towards the case where wires move individually, the tendency being more marked in the cable with the greater number of strands.

In these measurements, the deflection of the cable, δ, is dependent on two terms, $L/4T$ and $(F/TK)\tanh KL/4$, one of which is a function of flexural rigidity and the other not. In the case of the contact wire, the term dependent on B is 70% of that dependent on tension alone, but in the case of the main and auxiliary catenaries, the proportions are only 15% and 13%, respectively. This means that greater errors may be tolerated in the value of B_m for the model catenary wires than for the model contact wire.

The necessity for an alternative method of measuring B_m for the model wires is also indicated. For a length of model wire $1/\lambda$ times the test lengths on full-scale cables, the percentage deflections given above will still hold, but such a length would be much too small for carrying out such a test (about 0·75 in). As the test length, L, increases, the term involving B_m becomes very small in comparison with $FL/4T$, and for a minimum value of L of main catenary for such a test (say 4 in), it represents only 4% of $FL/4T$. If a variation about the required value of B_m of $\pm 10\%$ is allowed (the figure eventually adopted for both model catenaries), these limits would be too small to be detected by measurement. In any case, the method used for the full-scale cables would have been very tedious and laborious for checking the characteristics of the 6 ft lengths of beaded wires for the model.

Therefore, the method adopted was to treat the ends of each 6 ft length of model wire as a cantilever, and to suspend a load on the unsupported extremity. The deflection in such a case is given by

$$\delta = \frac{W_1 l^3}{3B} + \frac{W_2 l^3}{8B}$$

where W_1 is the applied load, W_2 is the distributed load due to the weight of the sample and l is the length deflected. This formula is true only for small deflections, and if the deflection is measured from the original position of the sample before application of the load, the second term may be ignored.

DISCUSSION BEFORE THE UTILIZATION SECTION, 16TH MARCH, 1961

Mr. J. A. Broughall: It has always been one of the misfortunes of the traction engineer that the people who design the overhead equipment do not build the pantographs or the locomotives, or vice versa. So there is a change of contract, and often a change in the responsible engineers, at the very point at which contact is essential—namely where hundreds of amperes have to be collected from a wire which is not fixed in space. As Monsieur Garreau has said, this is the most impossible piece of switchgear ever invented, but it works.

A number of us have been conscious of this difficulty for some time, and I think that the authors will remember a meeting which took place in November, 1957, in an attempt to bridge this gap and to get nearer the ideal of a perfect pantograph running on a perfect wire.

Dr. Williams then said that he thought he could make a model of it, and he even suggested the kind of model which he had in mind. One year later he was more definite and said, 'We have worked out the mathematics of it but we are not quite sure how to make the model. It can be made either of beads or of springs.' Now he has shown that it can be done.

The authors have done very valuable work, but a great deal has still to be checked. I am not yet quite convinced that the frictional forces in the pantograph and in the moving parts of the wire are correctly dimensioned, but time alone will show that.

This is one of those occasions when it can fairly be claimed that British engineering is in advance of the field in discovering why things behave in the way they do.

Mr. I. M. Holmes: It is stated in the paper that investigations into the effects of variations in temperature will be left until later. Do the authors anticipate any difficulty with conductors of different coefficients of linear expansion which differ from each other and in combination would be different from the full-scale wire? We shall all be most interested in learning the results of the various experiments which have still to be made, and whether the model has lived up to the authors' expectations.

Dr. F. T. Barwell: The paper commences with a description of three types of construction—simple, compound and stitched; and it is stated quite correctly that the uplift at the centre of the span is always greater than that near the supports, although a certain improvement can be made by stitched or compound construction.

There are only two points on any catenary which have the same reaction to an upward force. A long time ago a compound construction was used on the Gothard Railway, but it was different from those illustrated in that there were only two droppers per span from the main catenary and only four from the auxiliary to the contact wire. This produced an arrangement which spread the uplift out uniformly, apart of course from the deflection occurring between droppers.

This design was not prepared for the reason we have in mind, but in order to allow for the effects of temperature; the contact wire went up and down a great deal with changes in temperature but it remained substantially level. With the introduction of automatic tensioning, of course, that scheme became obsolete.

Clearly when the main supports are as much as 240 ft apart, the dropper spacing would be excessive. However, the addition of a fourth wire would provide eight droppers per span as opposed to the twelve illustrated in the paper. This could probably be compensated by adjustment of contact-wire section and tension. I have called this idea the 'double compound' construction, but recent information from Japan indicates that

the principle has been adopted there for speeds up to 150 m.p.h., and is given the term 'mesh construction'. An acceptable compromise appears to be a stitched compound construction, which is achieved by superimposing (*c*) of Fig. 1 on to (*b*). The argument in favour of this method is based on static considerations, and the paper makes it possible to introduce evaluation of the dynamic effects, which may be more important than static deflections.

Clearly, such a construction would be more expensive but might enable a more satisfactory electrical design to be achieved. For example, if one of the auxiliary wires were made of non-conducting or insulated material, a very adequate creepage path would result, and it might be possible to dispense with porcelain insulators for the support of the main catenary.

The dimensional analysis in Section 3 is particularly interesting and well expressed. A more conventional approach would, however, have been to apply the π theory to the problem to determine the maximum number of non-dimensional parameters governing the situation. It is clear from the treatment in the paper that one of these would be the Froude number, which is satisfied by the scaling factors used, but, in addition, something analogous to the Mach number, and possibly even a new number, would be required to take account of the elasticity and flexural effects.

It would be interesting to have the authors' comments on the relative importance of the factors they have scaled. At first sight, it does not appear that flexural rigidity would account for much of the energy involved in the response of the full-scale equipment, but, of course, the authors were very wise in going to so much trouble to ensure that, in the model, flexural effects did not become disproportionately large.

In the Appendix, a little more information might have been given about the method of measuring the flexural rigidity of the samples. In measuring this quantity with unconventional materials, I had tended to avoid the cantilever method; the reason for this is that the important independent variable in the determination, namely the bending moment, increases linearly along the specimen and reaches its maximum only at the point of clamping. Clamping effects, which may therefore distort the sample, will have an undue effect on the total deflection.

By means of a simple 4-point loading, it is possible to subject a considerable length of the specimen to a constant bending moment and to measure its radius of curvature. This has a useful averaging effect and may be expected to give more reproducible results. It also has the advantage that shearing forces are not applied to the test length, and, in this connection, eqns. (5) and (6) are incomplete in so far as they neglect the deflection due to shearing. In conventional structural engineering, these terms are small and can be conveniently neglected, but it would be reassuring if the authors could substantiate that this is also true for their rather unconventional construction.

Whilst the authors are to be congratulated on applying the principle of dynamic similitude to the problem, I should not like it to be thought that the design problems of overhead equipment can all be solved by these means. The value of the model is that it can enable variations to be made more readily than on the full-scale version, and, indeed, over a range which might be unrealistic, thereby enabling proper weight to be given to the various terms which must appear in a complete mathematical treatment of the subject. Nevertheless, there are many important quantities which cannot be reproduced on the model. First, the effect of side winds; secondly, the detailed lateral motion of a vehicle at speed; thirdly, the dynamic and electrical behaviour of detailed fittings such as section insulators; and fourthly, the damping characteristics of the overhead construction and its supports. This is important as it is quite likely that a further advance in design may arise from the deliberate introduction of damping to the overhead construction. Full-scale testing is therefore essential, and the value of the model lies in reducing the time and labour so required, but it will have limited practical utility in the absence thereof.

Mr. W. Holttum: In spite of the careful determination of dimensions for the model, I have some doubts as to the adequacy of reproduction of practical conditions. Track irregularities must be a vital factor, and the difficulty of copying these is scarcely disposed of by the remarks in Section 8.

The Introduction gives the reason for employing the model technique, but there are other considerations. The desirable features of a current-collection system are well known, and it is difficult to visualize how a model will throw further light on them or how to achieve them. A section insulator will always be a hard spot, and all that can be done is to make it as light as possible. Practical trial is the final criterion of success, whatever the difficulty of making observations. Generally, the beneficial nature of any change will be self-evident, and the only way of assessing its effectiveness is practical use. It is not apparent that there are critical values which a model will help to determine. However, perhaps I do not appreciate certain aspects. Can the authors give a few examples of specific problems for which it is considered that the model will be a more convenient method of investigation than observation on a railway?

Mr. E. E. Chapman (*communicated*): At present, the only comparative results between experiments on the model and the behaviour of full-size equipment under dynamic conditions are indicated in Table 5. It would be helpful if the authors would give some indication as to how the figures for relative contact-wire displacement at mid-span and at the supports were established from the test coach. Was any stationary measuring equipment used at the line side in addition to the equipment on the coach?

With regard to the next phase of experimental work which the authors propose to undertake, I would like to make a plea that the effect of section insulators should be explored at an early date. As indicated in the paper, the inclusion of section insulators in the light-weight 25 kV overhead system now being used in this country results in far greater variations in the rigidity of the line than apply to systems designed for lower voltages. Additionally, since it is already the practice in this country to run multiple-unit trains using more than one pantograph, the use of the model to examine the interaction of the line and the pantographs under these conditions would be very valuable. In particular, an investigation into the effect of more than one pantograph on a train traversing a number of section insulators in series which occurs in a neutral section could contribute greatly to a better understanding of the arduous conditions which can arise in actual operation.

Finally, while fully recognizing the authors' ingenuity in scaling the pantograph, there appears to be an important difference between the model and the full-size unit. As stated in the paper, the collector head is supported on rubber mountings, and it will presumably be difficult to simulate the effect of these, under dynamic conditions, by the leaf springs used in the model as mentioned in Section 4.6. In particular, the rubber suspension members in the full-size pantograph are arranged to ensure that the virtual centre of rotation of the collector head is at, or slightly above, the contact wire. Such differences between the model and the actual equipment do not, of course, invalidate the important results, which will no doubt be obtained from experiments with the model, but some caution may be necessary in the interpretation of these results because of such differences.

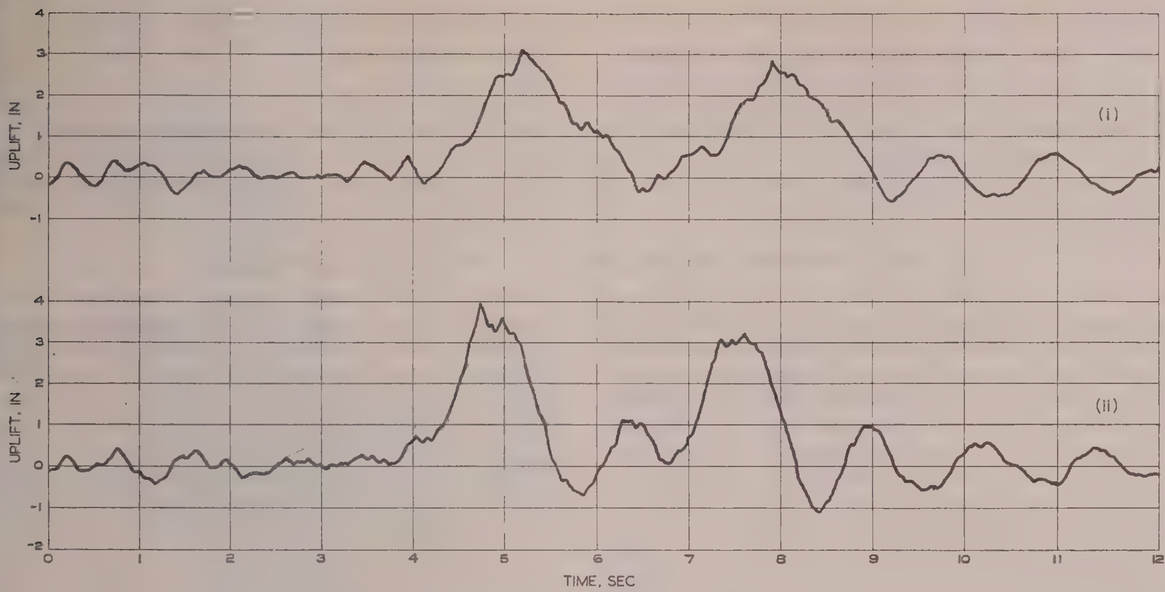


Fig. A.—Contact-wire uplift on passage of a pantograph.

Stitched catenary equipment, 200 ft span.
Uplift at mid-span: approximately 115/85 ft. Eight-car multiple-unit stock using two pantographs.
(i) Train speed: 63 m.p.h. approximately.
(ii) Train speed: 71 m.p.h. approximately.

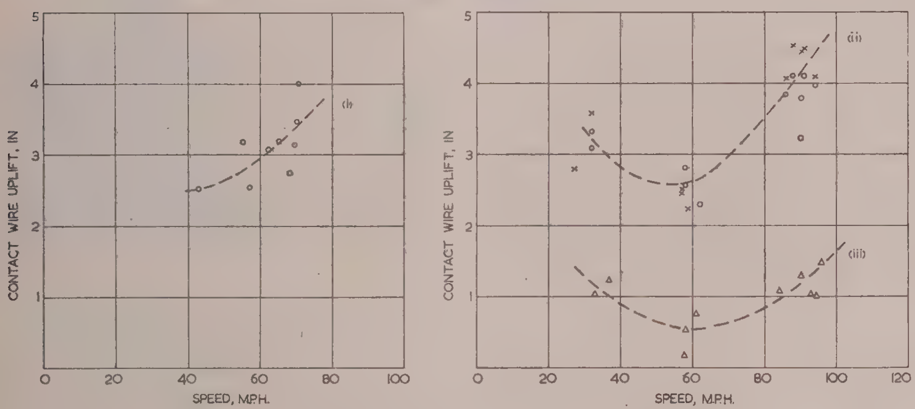


Fig. B.—Contact-wire uplift (maximum) on passage of pantograph.

(i) Stitched catenary equipment 200 ft span.
Uplift at mid-span: approximately 115/85 ft.
(ii) Stitched catenary equipment.
× Uplift at structures between spans of 200/190 ft.
○ Uplift at structures between spans of 190/180 ft.
(iii) Simple catenary equipment.
△ Uplift at structure between spans of 170/150 ft.

Messrs. R. G. Sell and D. R. Edwards (*communicated*): We ave been concerned with the development and application of st methods for measuring the performance of overhead ontact-wire equipment, and consequently we fully appreciate e authors' remarks about the difficulties involved when this as to be done on a line energized at 25 kV and with a public rvice running.

After some preliminary tests, apparatus was installed on the olchester–Clacton–Walton line just over a year ago. This as been briefly described,* and it is hoped that a more comete statement may be made at a later date.

The work has been directed mainly towards the study of particular features of the equipment, e.g. low bridge equipment, overlaps and neutral sections, and we think the authors may agree that, for such features, measurements will be easier on full-scale equipment and would present particular difficulties in a model. Although we had the advantage of having a railway ready made for these tests, we had to accept all the random variables that inevitably must occur, and the limitations imposed by normal operating conditions. For example, friction at supports is an unknown and not negligible factor on the full-size equipment, whereas it appears to have been virtually eliminated on the model.

However, there appear to be some interesting comparisons

* BARWELL, F. T.: 'Research for A.C. Traction', British Railways Electrification nference, 1960, Paper No. 11.

to be made with our results in respect of waveshape and frequencies, and maximum uplift/speed characteristics.

Fig. A illustrates a typical record of contact-wire uplift in a span 200 ft long of stitched catenary equipment at a point 115–85 ft from adjacent supports where a structure was available to carry the measuring transducer. The waves preceding the pantograph and subsequent oscillations of the contact wire will be noted, with their high- and low-frequency components, the latter being of the order of 0·8 c/s. More data are required to complete the uplift/speed curves shown in Fig. B, but there is evidence of an inflected curve, and the reduced resilience at supports without stitch wires is clearly seen by comparing curves (ii) and (iii).

The authors have devised a technique which is likely to provide a short cut in the study of design parameters, but the ultimate criterion is what happens to the full-size equipment. Would the authors agree that the model must be regarded as ancillary to, rather than a replacement for, full-scale testing?

THE AUTHORS' REPLY TO THE ABOVE DISCUSSION

Messrs. D. S. Farr and **H. C. Hall,** and **Dr. A. L. Williams** (*in reply*): We are grateful to the contributors for their valuable comments. First, to answer a general question posed by more than one of them, we certainly do not believe that models will obviate the need for full-scale testing. We are convinced, however, that such costly and difficult testing can be very much reduced and, further, that studies with idealized models will greatly facilitate the interpretation of full-scale results.

Mr. Broughall refers to the effects of friction (which we have tried to avoid in the models), and reference is also made to other complicating factors. We can only repeat that we regard all of these as influences which may modify the behaviour of a system, but which do not change its basic laws, which we hope to establish.

Mr. Holmes asks about the investigation of the effects of variations of temperature. If necessary, these effects will be studied by using electrical heating in each wire (catenaries or contact wire) separately, so as to produce such temperature rises in each as will give the same proportional extensions that arise with a given temperature increase in the prototype.

Dr. Barwell's comments on dimensional analysis are interesting, and it is true that the more conventional approach which he seems to favour might have been followed. However, we do not think that any material advantage would have been gained in this way. He asks us to comment upon the relative importance of the factors which we have scaled. The most important are mass and extensibility, and it is true that flexural rigidity is of considerably less importance. Morse* shows that the effect of flexural rigidity upon the frequency of transverse waves in a line of length l depends upon whether this quantity is negligible as compared with l^2T, where T is the tension in the line. For our model contact wire, the flexural rigidity is 0·035 5 lb-in^2, while, for the shortest unclamped length, the spacing between droppers, l, is approximately 7 in, and $l^2T \simeq 80$ lb-in^2.

We are indebted to Dr. Barwell for pointing out his method of measurement of flexural rigidity. There seems no doubt that this method would give rather more accurate results, but probably at the cost of increased testing time and complication of test equipment. With regard to the other points, it seems quite possible to scale wind effects and lateral motion of the vehicle (so long as it can be specified). The effect of the inclusion of section insulators or similar fittings, referred to also by Messrs. Holttum and Chapman, is an important item in our future programme. With damping, again, there seems no reason why this cannot be studied, and this is, in fact, currently under consideration.

The measurements and curves produced by Messrs. Sell and Edwards are very interesting indeed. We note with satisfaction that they find on the full-scale system a low-frequency component of oscillation of the order 0·8 c/s, which would correspond to 4·5 c/s in the model. We actually find such a component at a frequency of approximately 4 c/s.

In reply to Mr. Chapman, measurements in the test coach were made by recording the vertical movement of a light rod attached to the pantograph-head and passing through the roof of the coach. No track-side equipment was used.

* MORSE, P. M.: 'Vibration and Sound' (McGraw Hill), p. 170.

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THE PLACE OF FORMAL STUDY IN THE POST-GRADUATE TRAINING OF AN ELECTRICAL ENGINEER

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'he paper was first received 19*th November*, 1960, *and in revised form* 6*th January*, 1961. *It was published in February*, 1961, *and was read before* THE INSTITUTION 2*nd March, and the* NORTH-WESTERN CENTRE 14*th March*, 1961.)

SUMMARY

The paper examines the need for a period of formal post-graduate udy by electrical engineers who will be concerned with research and evelopment work in industry, as an integral part of their training. is concluded that the need is real, and that study should be deferred or a year after graduation. Study courses can with advantage be rganized co-operatively by an academic institution and local industry, nd should be held at the academic institution. The possibilities of full-time full-sessional course, a full-time course lasting about one rm, and a part-time course are considered. The advantages, both ducational and practical, are discussed and it is concluded that the ational and industrial need is best met by the full-time one-term ourse. The problems this involves for the academic institution are onsidered, particularly in the recruitment of part-time lecturers from dustry and the granting to them of some official status within the cademic institution. The conclusions are illustrated by reference to course with which the authors are concerned.

(1) INTRODUCTION

It is widely held that, under modern conditions, the normal ndergraduate course of three years' duration is inadequate, and at some further period of formal study is required in addition the practical training provided by an apprenticeship.[1–6] here is, however, by no means any equally accepted view of the rm which such study should take. It is perhaps for this reason at attempts to provide formal courses of post-graduate study, distinct from research, have been relatively few and have met ith varying success.

The paper is a review of some of the problems involved in e organization of post-graduate courses and is largely based the authors' experience with such a course. This course will briefly described, not necessarily as a pattern for universal nulation, but as an example. The paper is presented in the ope that it will provide a basis for discussion to the advantage all concerned.

It should be recognized at the outset that a man's need for ost-graduate study will depend very much on the particular pect of the wide field of electrical engineering which is to gage his attention. The number of different types of course hich could be conceived is great. To render the subject anageable, and to focus attention on a type of course for hich the need is great, consideration is restricted to the needs of en who will subsequently find employment in the research, velopment and design activities of industry. Such men will ve attained the standard of an honours degree, and most will, fact, be university graduates with first or second class degrees. ne needs of those who will be concerned with the production d commercial sections of industry are not considered. qually, the requirements of those proceeding to a higher gree, with the intention of entering industry later or of pur- ing an academic career, are outside the scope of the paper: these are a minority and the urgent need at the present time is to provide for the majority. Short *ad hoc* courses on special subjects of topical interest do not offer a solution, although they will continue to have their own place in the scheme of things. The paper is concerned with the increasing need for regular courses, available annually, which should come to form part of the general pattern of the education of no small number of professional electrical engineers.

(2) THE NEED FOR POST-GRADUATE STUDY COURSES

Even at this late stage it seems appropriate to begin with the query, 'Are 3-year undergraduate courses really inadequate?' The argument frequently employed—that engineering courses in some other countries last four or even five years—is irrelevant unless accompanied by a detailed discussion of the content and standard of such courses and the educational background against which they are set. Too often such discussion is lacking, and one suspects that much of the argument which arises when comparisons are drawn between practices in this country and oversea, particularly in a political context, is due to that lack. Moreover, it is insufficient to discuss only the educational background: the industrial climate for which the students are being prepared is equally important. It would seem to be more fundamental to consider the needs of the student himself. This should appeal to our preference for the pragmatical approach to the solution of problems such as this, and also to our belief that in the long run it is the man himself who is important rather than the system.

One feels much sympathy with the newly fledged graduate who is perhaps dismayed to find that three years of hard work are not enough, and that a further period of formal study is required of him. Why, he may well ask, cannot a 3-year engineering degree course be so arranged that it fits him to make an immediate contribution, even if a modest one, to the research, development or design activities of the firm he joins? To a certain extent, of course, it does, because it conveys to him a measure of the factual sort of information on which his firm's activities are based. But strictly speaking this is not the object or purpose of a university education. The first concern of a university must be to develop and train a man's mind, to inculcate habits of critical inquiry, of careful observation and logical deduction, and to develop in him the ability to learn for himself. It will convey facts in the course of such training, but the facts are of use principally as means to this end. They should not, at this stage, be considered as ends in themselves. This is not to say, of course, that any facts will do. Other things being equal, the information conveyed should be topical and pertinent to the world of engineering thought and practice as it currently exists. Every teacher and lecturer worth his salt tries constantly to keep the factual content of his course up-to-date, provided only that its logical structure is not thereby impaired. It is one of the fascinations of the teacher's work that he must constantly keep himself abreast of current developments, he must distil from them what is significant and fundamental, discard what is of only ephemeral interest and must carefully

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build the new material into the structure of his courses. In particular, he must constantly resist the outside pressures that are brought to bear on him to include this or that topic either if it is merely fashionable or if its sole purpose is to make a graduate more immediately useful to an employer. It may well be that, if there were more vigorous educational research into the matter, a 3-year course could be made to encompass more of the significant material than it does at present without any loss of its fundamental character. The fact is, however, that as things now exist, there is available a greater body of distilled knowledge than can be conveyed in a 3-year course.

Nevertheless, the graduate may well ask why, if a 3-year undergraduate course has properly trained his mind to learn, he may not be allowed to find out for himself what he subsequently needs to know? There is no doubt, judging from the number of graduates who have entered employment without having any opportunity for post-graduate study, that, up to a point, the method seems to work. But it has always been inefficient, and has become quite inadequate with the enormous growth of technical knowledge. It would seem reasonable to suppose that, if a coherent body of knowledge, i.e. theory, exists, it is more efficient to convey it by a period of formal instruction and study than to rely on unguided, perhaps misguided, private study. Of course, there exists the brilliant man who much prefers to work on his own and feels stifled by study to an imposed programme; but such men will come to the fore however much they have been educationally mishandled. In any case, they are very much in a minority, and when discussing formal programmes of education one must legislate for a reasonable majority. The problem of today is not so much to improve the education of the brilliant man as to provide better opportunities and facilities for those of average-to-good abilities, who will certainly benefit from assistance. This is not merely a question of trying to impose a tidy pattern on post-graduate education; it is a question of economics. The difficulty is to prove it, or rather to convince some people that some formalization of post-graduate study is economically desirable. The costs of the amount of wasted effort in unguided private study are hidden; the expenditure required to mount formal courses is plain for all to see. Since scientifically acceptable proof is thus hard to acquire, the institution of formal courses must to some extent be an act of faith. This is no new thing in education: the road of educational progress is lit by many acts of faith of far-sighted men.

Granted, then, that a period of formal post-graduate study is desirable, three main questions have to be answered. When in a student's life should it occur? Where and by whom should it be run? How long should it last?

(3) THE TIMING OF POST-GRADUATE STUDY

'When' in this connection means 'should post-graduate study follow immediately on graduation, or should it be deferred for a period of time; and if the latter, how long should be the deferment?'

It is sometimes argued that there are advantages in making the post-graduate study follow immediately after graduation. The educational pattern is somewhat tidier; there is no break in which the student's facility in learning might deteriorate, and formal study is complete before the start of the student's period of practical training. For the small employer it has the attraction that it absolves him from any financial responsibility during the period of study. There is the very real difficulty with the alternative of deferred study that the student has a great financial disincentive to return to study unless his employer continues his salary or at least supplements any other grant he may get. As seen by the student, there is a further disadvantage, real or fancied, in immediate post-graduate study, namely that he has little evidence that such study is to his ultimate financial advantage in that an employer will pay him more than if he does not so study. Furthermore, there is the suspicion that the only result of deferring his entry into employment will be to lose seniority compared with his contemporaries who enter employment directly after graduation. This may or may not be true, but in this case the reality is the impression in the man's mind. If the impression is widespread few men will choose to enter such post-graduate courses, in spite of the availability of grants from public funds, and correspondingly the courses will be poorly supported. For the men with whom this paper is concerned the only acceptable arrangement seems to be that a period of post-graduate study should form an accepted and integral part of a 2-year apprenticeship.

On the other hand, there are great educational advantages in deferring study for a period of about one year after graduation. For purely practical reasons, post-graduate courses must be restricted to those who really need them and have shown that they would benefit from them. If post-graduate courses immediately follow graduation, there is the danger that they will be overloaded by some men who for various reasons should not have been allowed to take them. Furthermore, electrical engineering is now so wide a subject that some specialization in post-graduate study is necessary, and it would be pure speculation for the student to embark on such specialized study before he has had, if not a specific offer of employment in a particular field, at least sufficient experience such as is offered by an apprenticeship to guide him in his choice. Admittedly many undergraduate courses now offer some choice of specializations in the third year, so that the student may have some idea of what he would like to do, but it is arguable that specialization at the undergraduate level is undesirable. Certainly it should not be introduced, directly or indirectly, solely as the result of the sort of outside pressure mentioned earlier.

The greatest advantage of deferment is to the student himself. Every year at that stage of his development brings a marked increase in maturity, as a result of which further study becomes more effective and valuable. It is not unknown that men who have achieved only ordinary degrees have, under the stimulus of exposure for the first time to practical engineering, found their feet and profited greatly from post-graduate study to which they have subsequently returned. For these men particularly, the break of one year provides a valuable opportunity to consolidate what they have learnt in their undergraduate years.

There is a further reason for advocating deferment. If the period of post-graduate study follows immediately on graduation and is held at the university, there is the possibility that it will come to be regarded by staff and students, consciously or unconsciously, merely as an extension in time of the undergraduate studies, whereas it should be an extension in attitude and mode of thought. It was suggested above that, in undergraduate study, facts are really less important than the ideas for which they form a vehicle. In post-graduate study they must be considered as important in themselves, even though they are presented in the coherent form of a theory. There is really no dichotomy between theory and practice, no clash between the classroom and the design office, if one subscribes to Boltzmann's remark that 'there is nothing more practical than a good theory'.

More subtle—and in the long run more important—is the difference in the modes of thought appropriate to undergraduate and post-graduate study. One object of the latter should be to help the student in his transition from undergraduate life to his life as a productive, creative engineer. The modes of thought

appropriate to the latter are in many respects different from, and additional to, the ones he has been accustomed to in his undergraduate career. The latter is largely inquisitive, deductive, analytic; creative thought is this, but is also much more. Analysis proceeds from specified beginnings by logical deduction to an unknown end. With creative design the end is specified, but the beginnings are not, except in a general manner as the totality of a designer's experience. In so far as it is possible to systematize the creative process, it would seem that the designer has to shuffle and reshuffle the facts of his experience into different patterns until there comes the inductive mental leap at the true moment of creation—the recognition of a pattern which fits his needs. Creative thought is both inductive and synthetic. The wider the designer's experience, the more complicated are the patterns of fact he can comprehend, the more acute are his inductive mental powers; then the better is his creative ability. Some flavour of this should surely be imparted in a post-graduate course; and if it is, the more necessary it is for the student to have some period after graduation in which to widen his experience, for otherwise he will not really be able to relish the flavour.

Now that a preference for deferment of further study has been expressed it will be apparent why no consideration has been given to the direct extension of undergraduate courses to four years.

(4) THE LOCATION OF POST-GRADUATE STUDY COURSES

Study courses may be held either at an academic institution or, where the organization is large enough, at the employer's works. It is difficult to think of any arguments in favour of the latter, and even a large organization would find it difficult to invest in all the facilities required and to make available the staff to run it. It would certainly be impossible for any but the largest organizations.

On the other hand, the basic facilities—lecture rooms, laboratories, library, etc., are available at an academic institution, together with the needful administrative services. The principal capital expenditure required would be restricted to laboratory equipment appropriate to a post-graduate course. The recurring annual expenditure is briefly referred to later. There is the advantage that much of the teaching will be done by academic staff. It is to be expected that academic staff, whose profession it is, will usually be better at organizing and presenting the material content of their lectures than men with less teaching experience. This applies particularly where a continuous logical exposition of theory is involved. From the point of view of the staff themselves there is the advantage that they are presented with the opportunity to master, and to develop methods of exposition of, topics which are more advanced than are necessary for undergraduate work. This should not only be satisfying in itself, but should also benefit—even if indirectly—their undergraduate work.

It may well be, however, that for reasons discussed later the academic staff will be unable to handle the whole teaching load and will have to be supported by men from industry. This should be an advantage to both. Lecturers from industry bring to the classroom a welcome contact with reality and an acute awareness of current problems, to the solution of which the students themselves should be making a contribution in a very short time. The contacts between academic staff and industrial lecturers outside the classroom should help greatly to foster fruitful relations. The effort required for the busy industrial lecturer to prepare his material is admittedly great, but those who have made the effort will surely agree that the mental discipline involved brings its own reward in a broader grasp of principles and a clearer understanding of detail.

From the national aspect there are overriding advantages in organizing post-graduate courses at academic institutions. The courses are then open to the employees of all firms alike, whereas otherwise they would naturally be restricted to a firm's own employees. The importance of this to the smaller firms and their employees is obvious. There are economic advantages: the number of men from one firm, even a large one, attending a course may scarcely justify the expense, particularly when this number is split between several specializations. If the courses can draw their students from a number of firms, the size of class becomes more economic and it also becomes possible to provide for the number of men who, from a national point of view, should be taking post-graduate courses. This is not to say that every university or college of advanced technology should provide such courses; they should probably be confined to a few strategically distributed to meet the needs of the country.

With courses organized at educational establishments it becomes possible to recruit the industrial lecturers from a wider field than with courses organized by an individual firm. Within a single firm, even the largest, it is difficult to find a sufficient number of men who combine all the qualities needed to make good lecturers at this level. Good men, as they must be, are invariably very busy.

It is suggested, then, that the proper home for post-graduate courses is the university, that they should form an essential part of industrial apprenticeship schemes, and therefore require co-operation between university and industry. This raises two important points. The first is that the co-operation must be not only close but also cordial, since, in the last analysis, success depends so much on the personalities of the people running the courses, however good the formal structure of any organizational arrangements created. To this end it is desirable that some representative or representatives of the industrial concerns co-operating in the organization of the courses should have some official status in the academic institution. The course to be described later runs with comparative smoothness, which the authors attribute in no small measure to the fact that the one of them representing an industrial concern also has an official academic appointment as a Special Lecturer. This is an active, not merely a nominal appointment; for example, he is also concerned with some undergraduate teaching. Such appointments are not usual in British universities, although commoner oversea, but their value cannot be doubted. Another contributing factor is that both authors have a combination of industrial and academic experience.

The second point is that of finance. When a course is organized by the employer, finance is obviously a private matter; but when a university organizes the course, the problem is more involved. It becomes a matter for discussion how the total cost of the course should equitably be shared by employer and university. The total cost includes the students' salaries while they attend the course, the running expenses such as the fees paid to external lecturers and a reasonable proportion of the salaries of the university staff engaged on the course and the university overhead charges, and a reasonable proportion of the cost of the capital laboratory equipment. It is also a matter for discussion as to the appropriate sources on which the university should draw for its share of the cost. The authors offer no views on these matters in the paper, in the belief that it is essential to make sure that the educational need has been properly assessed and the proper method of satisfying it has been found before it can become clear what the actual cost would be, let alone how it should be met.

(5) THE DURATION OF THE COURSES

How long should a course of formal post-graduate study last? The short answer is 'as long as possible': there is no shortage

of material to be included in such courses. The practical answer is 'for as long as industry is prepared to release the students'.

There seem to be three main possibilities: a full-time course of normal duration, i.e. one university session; a full-time course of shorter duration, say one term; and a part-time course of, say, one day per week throughout a session.

Full-time full-sessional post-graduate study courses have been available in one or two institutions for some time. They evidently meet a need because they have continued for so long; but just as evidently they do not meet a general demand from industry, because if they did they would be offered by a greater number of institutions. They do not meet a national need, because the number of men taking them is relatively small. The reason for this, at least in part, must be the lack until recently of grants in any significant numbers which would attract students to take such courses. The financial pressure on a man to enter industry directly after graduation has been great. The chance that he would return on his own initiative to post-graduate study after a short period in industry has been very small, while the number of men who have been encouraged by industry to return, and have been paid to do so, has also been small. The reason is not far to seek: it is quite unrealistic to suppose that at the present time the electrical engineering industry is prepared to release men in any significant numbers for a full academic session, while continuing to pay their salaries and possibly fees as well. When it has been done, it has been only in special individual cases. It would be an impossible financial burden if the practice were extended to include all the men for whom post-graduate study of some sort is desirable. In addition, industry has not in general been satisfied that the time available in a full session has been used to the best advantage. Usually the proportion of time occupied by actual lectures and laboratory classes is about the same as in undergraduate work, with the remainder occupied by private study and project work. Industry considers this to be inappropriate. If, the argument runs, a man has had proper mental training in his undergraduate days, there is not the same need for the periods of private study which are admittedly necessary at the undergraduate stage. At a post-graduate stage, and taking into account the change in outlook in post-graduate study which was discussed earlier, it should be possible for a course to be more intensive and last for a shorter period without any substantial educational disadvantage, particularly since the men can digest the information conveyed after they have returned to industrial work. The value of experimental project work is also doubted. Ostensibly its purpose is to give the student an introduction to creative, as opposed to analytical, work; but in this it would seem to usurp one of the avowed functions of an apprenticeship. (Design study work conducted on the tutorial system is a different matter.) Moreover, the conditions under which project work is undertaken—the student working individually, with very limited resources and workshop facilities, for a relatively short and disjointed period of time—are not representative of the best industrial practice.

The recent introduction of grants on a substantial scale for full-sessional post-graduate study merits consideration at this point, because the scale of the grants is intended to be such that (allowing for income tax) they are not incomparable with the salary of a graduate apprentice. Coupled with the fact that a year's post-graduate course may possibly result in the award of a degree or diploma by examination, the incentives may now be sufficient to induce some men to take these grants. It appears that some universities think that they will, since they now offer post-graduate study courses. Two questions arise. Are such courses educationally desirable? Do they meet the needs of the students of the type considered in the paper? As to the educational desirability, the arguments in favour of deferring post-graduate study for at least a year after graduation still stand; but one is afraid that the tendency with the new grants will be for the study to follow immediately on graduation. And if it does, one wonders whether the student will thereafter be prepared to undertake the two years' practical training provided by an apprenticeship which so far The Institution has regarded as necessary for the professional electrical engineer. The consequences of this we need not discuss further here.

The alternative is that a man should undertake an apprenticeship immediately after graduation and then return to the university for a year's study. This might be suitable for men who wish to make their careers in the academic world, but will hardly appeal to those who wish to stay in industry. Moreover, the arguments in favour of shorter, more intensive, post-graduate courses still stand, so one is bound to ask whether public expenditure on full-sessional courses is the most economical method of achieving the declared aim, namely the provision of greater numbers of men who have studied at post-graduate level. Certainly, if the scheme is to provide the numbers which are required from a national point of view, it is going to be very expensive. One is driven to the conclusion that the scheme will meet the needs of only the comparatively small number of men who wish to undertake a lot of post-graduate study (and one questions whether they will be able to keep up for a whole session the intensive study that this implies). It will not meet the needs of the men considered here. The danger is that, by focusing attention on the usual full-sessional pattern of courses, it will make universities less willing to accept the view of industry that at the present time post-graduate study should be relatively intensive for a period shorter than a full session.

It is natural that universities should question this view. The pattern of sessional arrangements has worked well for a long time past, at least at the undergraduate level. If some other pattern is to be superimposed upon it, there may arise great organizational difficulties, particularly in staffing. Precisely what these difficulties will be depends on which of the two remaining alternatives for the form of the post-graduate course is chosen.

Consider the possibility of part-time study, e.g. one full day per week. With such an arrangement it is possible, without too great hardship for the students, to make the day's study intensive, allowing the intervening week for reflection, homework and so on. It is easier to fit in the required lectures with the other commitments of the staff, and there is no peak load on them. In fact, the course to be described below was originally run in this form, but recently the logic of events has forced a change to the other possible form of a short full-time course, and experience now shows that the advantages of the part-time course are more apparent than real. For the student the interruption of his study by the intervening days spent in the works disturbs the thread of his thought, and time must be spent each week in recalling what has gone before. Equally, his industrial work is interrupted. Experience has indicated that the students prefer, and work better with, an uninterrupted period of study.

From the aspect of the academic staff there is the danger that part-time extra commitments will be regarded as additional to, and outside the scope of, normal university activities. Faced with the difficulty of obtaining extra staff, the natural reaction of university authorities would be to ask existing staff to shoulder the extra burden, possibly in consideration for some special fees paid for out of the revenues from the course. This approach is to be deprecated, for two reasons. In the first place the extra work must inevitably detract from the essential non-lecturing activities of the staff, such as research. In the second place it is

he purpose of the paper to inquire to what extent and in what orm some post-graduate study should become a normal part f the development of the professional electrical engineer. As uch, post-graduate courses should be recognized by both taff and university authorities as an integral part of university ctivity and a logical extension of undergraduate work. Such n attitude is difficult to engender with part-time courses, but he authors firmly believe that only if this attitude is adopted vill the quality of the courses be such as to meet the needs of he case. If a post-graduate course is undertaken, extra staff nust be recruited as necessary so that the annual load on any nan does not exceed that which is proper.

Even if extra staff are appointed so that the total individual oad remains the same there is not, with a part-time course, the dvantage that a full-time course offers. The peak load on the taff is much greater, but if the total annual load on any one nember is held constant, he will be correspondingly lightly oaded during the rest of the year and his other activities, such as research, will be less interrupted. The peak load on the staff nevertheless remains a difficulty, and it is still necessary that the load should be lightened by lecturers from industry. However, with a full-time course there is a greater flexibility in the arrangement of the time-table, and it may be more easily possible to arrange that the timing of an industrial lecturer's periods suits his wishes. It would seem that, on balance, such a man would prefer to give his quota of lectures within a short space of time rather than to have a regular commitment on one day per week extending possibly over a long period—an arrangement which does not always fit in easily with industrial commitments.

There are two important administrative reasons in favour of a full-time course. In a large industrial organization with factories in several different parts of the country it is necessary to post the apprentices for certain periods of their training away from the parent factory. If a substantial number of them have to be held at the parent factory for a long period, because of a one-day-per-week commitment, it becomes extremely difficult to operate the posting system. There is, moreover, the likelihood that the apprentice himself will receive a less wide and varied practical experience. This argument does not apply to the smaller single-factory firm which is also sending men on the course. If, however, co-operative apprenticeship schemes among small firms in a locality became commoner, it is conceivable that they would be easier to organize if the period of academic study were continuous rather than part-time. The second reason is that a part-time course restricts its catchment area to those students living within daily travelling distance. For economic reasons it may be that relatively few universities will be able to offer courses, and their catchment areas must be correspondingly wide. A full-time course is then the only possibility, the students coming to live locally for the duration of the course.

There remains the question of the time of year at which the course should be held, and this may depend on local conditions. For example, if a university serves a wide area so that an appreciable number of students come from a distance, it may be considered preferable to hold the course during a vacation, when rooms are more readily available in halls of residence. There is then also less pressure on the lecture room and laboratory accommodation. But with any vacation course, as with a part-time course, there is the possibility that it will not be regarded as an integral part of the university activity. For the reasons given above the authors therefore believe that the course should be held, if at all possible, within the period of a normal term.

(6) THE COURSE

The ideas advanced above are in part the outcome of, and in part have always been the inspiration of, the course with which the authors are associated. It has had a long development, stemming directly from the late Sir Arthur Fleming's interest in the education and training of the young electrical engineers serving their apprenticeships with his company. For many years the programme included lectures given by practising engineers within the company, which as time went on increased in numbers and scope. After the last war lecture courses, which spanned both years of the apprenticeship, grew to such a size as to impose an undue burden on the staff concerned with them. Six years ago, therefore, the company invited the Manchester College of Technology, as it then was, to undertake the second-year part of the course. It is not without significance that the college, in addition to being the Faculty of Technology of the University of Manchester, had a tradition of part-time teaching. This doubtless made easier the acceptance of the invitation. Initially, this second-year course occupied 30 half-days; it was later increased to 20 whole days, and was more recently changed to a full-time course lasting eight weeks. The first-year lecture courses continued to be organized by the company, and over the years have developed correspondingly.

The current form of the courses is shown in Table 1, in which are included only those courses of direct interest in connection with this paper; others, for example on company organization and products, are what one would normally expect to find in any apprenticeship scheme. One of these other courses, on

Table 1

Outline of the Course with which the Authors are concerned

FIRST YEAR	6 weeks Nov.–Dec.	Preliminary mathematics			
	16 weeks Jan.–May	Main mathematics	Engineering tutorials	(a) Electromagnetic fields (b) Applied physics (c) Nuclear power	
SECOND YEAR	8 weeks Oct.–Dec.	Electrical machines	Power systems	Communications and electronics	Automatic control
		Common course			

industrial administration, will however be mentioned briefly below.

The purposes of the first-year courses are as follows. First, they insure against the students' minds lying idle in the interval between graduation and the second-year courses. Second, they provide an equitable method of selection of the men who will be permitted to continue to the second year. Third, they prepare the men for the second year, in a way to be described. Fourth, in certain cases they permit men to take a second-year subject which they may not have had the opportunity of studying in their undergraduate course. All the courses are voluntary, although, in fact, the usual difficulty is to persuade men not to attempt too many. There is progressive selection, and in practice only about half the number of men who start on the preliminary mathematics course are admitted to the second-year courses. No distinction in opportunity is made between home and oversea men. This is not the place to discuss the detailed syllabuses of the various courses, so the description that follows is concerned with principles rather than actual content.

The preliminary mathematics course is a revision course, and is open to all men at the start of their apprenticeship. It lasts for six weeks, with each week a 2-hour lecture held in the men's own time and a 2-hour tutorial class held in the company's time. There are usually not more than 10 men in each tutorial class. Homework is set; it and the tutorial class-work is assessed. About two-thirds of the men complete this course successfully, enabling them to continue with the courses offered after Christmas, which run concurrently for 16 weeks.

The main mathematics course follows the pattern of the preliminary course, except that the number of men in each tutorial class falls to about six. The content of the course is intended to ensure that, so far as possible, the treatment of the second-year topics is not limited by lack of mathematical knowledge. To some extent, therefore, it is post-graduate work (by engineering standards, and not, of course, a mathematician's standards). Again, homework and tutorial class-work are carefully assessed.

The engineering tutorial classes were introduced comparatively recently, because it was found to be necessary to ensure that men enter the second year with a suitable common standard, not only of mathematical, but also of engineering, knowledge. The men come from many different universities, with correspondingly different syllabuses, and a process of levelling-up is necessary. The engineering tutorials also provide for the man who, having done some specialist work in one subject at the university, wishes to change to another subject. This situation had previously led to some waste of time at the beginning of the second-year courses while common standards were being established. Because of the diversity of background and the nature of the subjects, this situation is best remedied by tutorial classes rather than, as with mathematics, formal lectures. Each man therefore chooses which of the four second-year courses he wishes to take, and then the men are allotted, in groups of not more than four, to tutors who are engineers practising in that subject. It is the duty of each tutor to work with his class through a syllabus which is agreed jointly between the company and the college. The weekly tutorial classes last for 1½–2 hours in the men's own time. Again, homework is set and assessed. Only those men who have shown their fitness in both mathematics and engineering are selected to attend the second-year courses. A not inconsiderable advantage of the tutorial system is that each apprentice gets to know one or two members of the company's staff who take a lively and personal interest in his technical progress.

The fact that a man must choose his subject of specialization at the beginning of the engineering tutorial classes is not finally binding. It is realized that a man, for perfectly good personal reasons, may wish to change his choice. This is permitted, usually on condition that he undertakes some private study during the following summer, and he is allowed to proceed to a different second-year course if the organizers are satisfied that he is in a position to benefit from it fully.

The remaining first-year course shown in Table 1 is open to all who are attending the main mathematics course. It takes the form of a 2-hour lecture each week in the company's time, but since the course does not lead directly to the second year, it will not be discussed further.

A course not shown in Table 1 is one on industrial administration, which is an evening course offered by the college. It is intended primarily for mechanical engineers, to enable them to satisfy the regulations of The Institution of Mechanical Engineers. However, electrical engineers who do not wish to take the courses previously described, or think they may not complete them, are encouraged to attend. Those who have every prospect of succeeding in the technological courses are permitted to attend, but are not encouraged to do so because experience shows that the load on them would then be excessive.

As has been mentioned, there are four specialized second-year courses, each comprising lectures, tutorial classes and laboratory work. In addition there is a common course of lectures which is attended by all the students. This is held partly to give a community of interest to all the men and to give cohesion to the course, and partly for economy because the subjects included are of interest to all. Abbreviated syllabuses of the four courses are given in the Appendix, but will not be discussed here because an informed discussion could only take place on the basis of much fuller information than there is room to give in the paper; moreover, the authors consider that at this juncture there is a much greater need for discussion of the background and pattern of post-graduate courses than for discussion of the detailed content.

The proportions in which the college and industry provide teaching staff have varied from year to year to meet temporary factors concerning the availability of men, as was only to be expected. As a rough average over the years and over the five courses the contributions to lecturing time (as opposed to tutorial and laboratory class supervision) are 70% by the college and 30% by industry. For the supervision of tutorial and laboratory classes a greater number of staff is required, most of whom must of necessity come from industry, so that the proportions are approximately reversed. It has been found that men who have attended the course in previous years and have then had two years or so of work in industry are very suitable for these tasks.

The number of students has fluctuated somewhat from year to year, but on balance has grown so that, for example, in the courses held in the autumn of 1960 the total was 59. Of the total, the proportion of students attending from firms other than the one whose activities have been described above has also increased, and in this respect the transfer of the second-year courses to the college has served one of its purposes.

The final question which must be answered is whether the courses fulfil their main purpose in improving the knowledge and capabilities of the students. A sufficient answer is that it is unthinkable to those who are concerned with them that the courses should be curtailed in any way; as has been described above, the history of the courses has always been one of expansion and extension and, indeed, there are no indications that this process will not continue.

(7) CONCLUSION

The paper has been concerned with the educational needs of electrical engineers who are to engage in the research, develop-

ment and design activities of industry. The arguments given point to the suggestion that, for a by no means negligible proportion of such men, some form of organized post-graduate study is necessary, that it should be an integral part of a two-year apprenticeship and that it should comprise a full-time period of, at present, about one term, held at a university. Experience with a course run on these lines has been encouraging, which prompts the suggestion that such courses should be made more widely available in order to meet the national need; but before that can be done it is necessary to examine closely the arguments given so that it may be reasonably certain that courses of this form are sound in principle. It is the purpose of this paper to initiate such an examination. If the results of it are favourable, further questions will follow, but an attempt to ask and answer them at this stage would perhaps be premature.

(8) ACKNOWLEDGMENTS

The courses which have been described are the outcome of the labours of many men. There is not the space to mention them all; to name only some would be invidious. They are selfless men and will understand. To them all the authors give credit and offer their thanks.

The authors wish to thank Sir Willis Jackson and Professor E. Bradshaw for their interest and encouragement.

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(10) APPENDIX: ABBREVIATED SYLLABUSES OF THE COURSES

Common Course.
- Matrix algebra.
- Numerical analysis.
- Digital-computer techniques.
- Analysis of observations.

Electrical Machines.
- Synchronous, induction and d.c. machines.
- Matrix and tensor analysis.
- Computers in machine design.
- Field theory.
- Cooling.

Control Systems.
- Analytical methods, linear and non-linear.
- Restrictions on physically realizable systems.
- Performance criteria.
- Properties of component parts and system design.

Electronics and Communications.
- Electromagnetic theory.
- Linearized network theory.
- Non-linear active circuits.
- Communication systems.

Power Systems.
- Analysis of synchronous machines and systems.
- Per-unit and co-ordinate systems.
- High-voltage transmission.
- System protection; power-system stability.

There is appropriate laboratory work in each of the four specialized courses, which occupies approximately 30% of a student's time.

DISCUSSION BEFORE THE INSTITUTION, 2ND MARCH, AND BEFORE THE NORTH-WESTERN CENTRE AT MANCHESTER, 14TH MARCH, 1961

Dr. W. J. Gibbs: I admire greatly the work that the authors are doing and have done at Manchester, and I agree with much of what they say. Drawing comparisons takes up time and I shall therefore discuss the paper on the basis only of the courses described in it.

The authors start quite properly by recommending that the period of the 3-year undergraduate course should remain unchanged. I believe all university staffs in the United Kingdom will agree with their description of the objects of this course, i.e. to develop in the young man the ability to learn for himself. For the post-graduate courses the aim is quite different: it is to impart new knowledge.

There is a case—although it is not a strong one, for students should be capable of self-instruction—for holding lecture courses to spread some of what the authors call the 'body of distilled knowledge'. The authors devote the whole of Section 3 to the question of when this should be done, and they provide closely reasoned arguments intended to show that the courses should not be held immediately after graduation. These arguments, in fact, really point to a period much later than apprenticeship.

Their course is frankly called a second-year course. Not every industrial concern could provide a course equivalent to their first year, so in fact this course is ideal only for one firm. If a young man is sent to take it from outside Manchester, he has to go for eight weeks to strange rooms or lodgings and therefore is not in the same position as a young apprentice based on Manchester.

It is very encouraging to find that the duration of the courses has been reduced from an academic year to eight weeks, but this is still far too long, for two reasons. To take eight weeks out of the very precious two years of practical and professional training is too much; secondly, these courses have to be run at a fast pace, and at such a pace eight weeks is too long. Young men can be pushed hard for three weeks, but towards the end of this period their efforts begin to flag.

To be constructive, I should like to see a few universities running courses of this nature of six weeks' duration, divided into two periods of three weeks each. The young man would not take all six weeks at once; he would take three weeks in one year and three in a second. The courses would be in the long vacations and residential. They would be aimed at young men who had been on the engineering staff at least one year and preferably two, so that the age range would be about 25–27. This is the period in a man's life when he is most receptive, and when he is specializing to the highest degree. I know that the universities do not like these short, sharp courses asked for by

industry, but the authors' objection will probably be that such courses are for staff, whereas they are talking about courses for graduate apprentices. In that case I suggest that they consider very carefully whether, at that stage in a man's career, day courses are necessary or desirable. I do not think that they are either. However, if they decide otherwise they should look very carefully at the syllabus.

The most heartening thing about the paper is the proof it provides that one university college and one manufacturing firm have not only got together on this matter but have agreed.

Prof. E. Bradshaw: The paper describes a significant experiment in engineering education. Not the least important aspect is the extent to which full-time teachers have co-operated with their colleagues in industry. An important facet of this co-operation is the substantial help given, in the Electrical Engineering Department of the Manchester College of Science and Technology, by visiting lecturers holding formal university appointments; one of the authors holds such an appointment.

The time, duration and content of courses of this nature must long remain matters for debate. If all one had to do was to extend the existing degree course by eight weeks, that would mean increasing its effective length by a factor of only about 10%. The critical point is, of course, the value of such an eight-week course if given at the most appropriate time. Have graduates had, just after the end of the first year of graduate training, sufficient industrial experience to enable them, for example, to select the most appropriate of the four options mentioned? To what extent have other graduates, at a later stage in their professional career, made use of the course, and what opinions, if any, are available from industry about those who have taken the course at various times?

The authors have described a course, mainly analytical in character, with options in four particular branches. Would they comment on the significance of the four branches offered and suggest any other fields which they consider should be covered in the future? Their views on post-graduate courses to develop professional engineering skills other than those of the strictly analytical kind would also be welcome.

The paper raises certain wider issues. New ideas are abroad in engineering education. Not only are new treatments being considered for the professional engineering subjects, but experiments are under discussion such as the introduction of substantial amounts of other disciplines, including the social sciences or economics; the consequential limitation of the depth to which engineering subjects could be taken would increase the need for post-graduate engineering courses of various kinds. Finally, the very existence of the need which has occasioned the course described by the authors is a measure of the growing pressure on electrical engineering students. We must keep constantly under review, not only the desirability of including more and more fundamental matter in a professional engineering course, but also the ability and willingness of students to face an increasingly onerous task. Failure to give sufficient consideration to this aspect could result in the loss of recruits to the profession. A long-term solution may involve a conscious limitation of the undergraduate task coupled with a variety of post-graduate opportunities in which courses of various types are to some extent regarded as alternatives to part of what has hitherto been regarded as post-graduate training.

Mr. H. W. French: During the last two years the Ministry of Education has received many inquiries and suggestions about post-graduate courses in all fields of engineering. Some have been very similar to those proposed by the authors, while others have advocated as much as two post-graduate years of sandwich-course instruction of a specialized nature.

We can surely do little more in the time available for our university or Diploma in Technology courses than lay a sound framework of electrical engineering education; we cannot include much specialized training. The questions are, when and for how long can we bring these young men back for further training? I do not agree that such a course should be 'as long as possible' but rather 'as long as necessary'; that may mean just a short course, as Dr. Gibbs has said, or in other cases a complete academic year.

The duration and the timing depend on the job one is trying to do. If a man has been generally educated in electrical engineering, and then joins a firm which specializes, for example, on the electrical side of control engineering, he may well need a considerable extension of his academic knowledge; the proper time to acquire this may be at once, so that he can then benefit more fully from the work which he does subsequently. That is how the Russians train their control engineers. They have $5\frac{1}{2}$-year courses, which we could approach by adding a period of specialized training to our present three years. That may be right for this particular kind of engineer.

On the other hand, the point which Dr. Gibbs raised is valid. At a later age, a different kind of short course might be right. To provide the post-graduate training which is necessary requires not one pattern of course but many, which we should be prepared to provide. This calls for extreme flexibility. I therefore suggest that a major contribution could be made by those technical colleges suitably staffed and equipped for post-graduate work. The paper has been written in a university context, no doubt because the universities have done and are still doing very good courses of this kind. I suggest, however, that their efforts could be augmented by our more advanced technical colleges, which are well used to varied and flexible arrangements.

With reference to part-time lecturers, we could with advantage have an increase in our Colleges of Advanced Technology of the sort of appointment which Mr. Taylor holds.

We should remember, too, that we have almost 5 000 Diploma in Technology students, the majority on a sandwich-course basis. For them, an acceptable answer to the post-graduate problem might very well be one additional sandwich-course session.

Finally, if we make our courses long—and some of them would have to be long judging by the approaches which have been made—we have the problem of an award. Students selected and financed by their firms may well be content with the possible advancement which this might imply. Those not so fortunate may well look for an M.Sc. or something similar. The problem of an appropriate award for post-graduate study, as distinct from research, is one that calls for immediate attention.

Dr. G. S. Atkinson: I have naturally regarded the paper from the point of view of Colleges of Technology such as that at Rugby. In the first place the need for a formal post-graduate course has not yet been firmly demonstrated. I agree that it is necessary to have some kind or kinds of formal course but I think there should be a variety.

There should be a gap between the end of the university graduate course and the post-graduate course, but the gap need not be so long in the case of works-based Diploma of Technology students, who will have had a considerable amount of useful industrial experience. If the gap were made longer, as Dr. Gibbs suggests, would it not be necessary to put something else in between?

I think that the problems of the academic institution are over-emphasized in the paper. We, in the Colleges of Technology, are accustomed to many types of course. Naturally, other things being equal, we normally prefer a full-time to a part-time course. But if there are reasons—other than educational—why the course cannot be full-time, then the organization of a part-time course would fall naturally into our scheme of things.

I was surprised to find in the course described that the first year is still organized by industry. Is there a special reason for this? Surely this first year is broader and less specialized than the second year.

It appears that the authors are suggesting that similar courses should be established in different parts of the country. Do the firms want this? If you have a large firm, big enough to organize its own course, does it want a national pattern which would mean a broad course? Or does it prefer to organize its own course because of the opportunity afforded to refer to its own procedures?

Mr. M. R. Krick: Is it not possible to do something about this problem before the undergraduate leaves university? Mention has been made of the long vacation. Several colleges run schemes whereby students are taken into industry during this vacation and given some of their practical training at this time. I am surprised that the authors have not mentioned this. I know from my own experience that such schemes are very useful, and popular. They are easy to institute if industry can be persuaded to co-operate, and I am certain that students would have no hesitation in participating. Such schemes at the end of the first and second years of the university training would give a certain benefit to the second and third years.

Professor Bradshaw mentioned that eight weeks represents only a 10% increase in the amount put into the 3-year university course. I feel that the scheme I suggest might be a means by which that 10% could be absorbed without any noticeable change in the syllabus itself, and that as a result the graduate would be a little more ready for a life in industry. Then, if the scheme which has been outlined this evening were to be put into force, the benefits from it would be enhanced.

Prof. J. Greig: As Mr. French has implied, there are in fact two fairly well-defined types of post-graduate course. One is, in general terms, like that which the authors have described, i.e. closely associated in its content with the industrial work in design, development and research with which the graduate will be concerned later. It contains a substantial amount of technological detail and advanced technological theory.

There is, however, a second type of a more fundamental character. This is based on a more advanced study of mathematics, an introduction to modern physics and the study of some branch of fundamental technology. The main emphasis must be on the introduction to modern physics, for I believe that the most serious chink in our intellectual armour, in engineering, results from our engineering graduates being largely ignorant of this subject. They study modern physics in some measure in the undergraduate course, but necessarily rather sketchily and without a very firmly set foundation. For a man who is going on the research and development side nowadays, it is clear that modern physical theory, in particular the theory of the solid state, will be of paramount importance.

The more fundamental type of course should, in my opinion, be pursued immediately after graduation. One cannot learn one's mathematics too young, and if one is to make a more advanced and rigorous study of mathematics there is much to be said for doing it immediately after the end of the undergraduate course, when memorizing is still easy.

Doubt has been expressed as to the desirability of lengthening the engineering undergraduate course, making it four years. For the majority of men entering industry, a 3-year course is, I believe, probably adequate. It is inadequate for the few who are going on to more advanced theoretical work and who have the potentiality for pursuing research and doing advanced design. It appears to me that our best solution in this country is simply to provide 1-year courses of post-graduate study, which would lead to a post-graduate qualification, in Colleges of Advanced Technology and universities as has been suggested.

The effective development of post-graduate study is probably the most important single factor in the development of our technological education at the present time.

Mr. D. A. Picken: There are two aspects which have in the past been largely overlooked in the training of an electrical engineer, both of which could well be included in a course of formal study. One is a knowledge of law, particularly of electrical engineering, in which we are bound by the Electricity Supply Regulations and the Electricity (Factories Act) Special Regulations, 1908 and 1944. Almost everyone who practices as an electrical engineer should have a general knowledge of these requirements and their legal and practical interpretation. The second aspect which should be dealt with is that of training in electrical safety.

Already some organizations, e.g. the Manchester College of Science and Technology and the Burton Manor Adult Education College, have organized courses in both these subjects, and the enthusiasm and interest of students and their employers is an indication of the need for such training.

Mr. D. H. Tompsett: It seems as if the word 'firm' is being identified in the paper with a manufacturer of electrical equipment. Many other types of organization are employers of qualified electrical engineers. I have in mind the electricity supply industry, the various authorities in the communication field, the railways, the armed services, other manufacturing and operating industries, consulting engineers and so on. Although the authors state in their summary that they are referring to 'engineers who will be concerned with development and research work in industry', I believe that some cross-fertilization is essential between people on the manufacturing and the operating sides of industry. The authors aim at widening the designer's experience and believe that his creative ability is thereby increased. I entirely agree with this point of view and I think that appreciation of the ultimate use to which his product will be put is at least as important as a deeper understanding of the mathematics and physics underlying analysis and performance calculations.

What steps have the authors taken, in the establishment and planning of these courses, to take care of this aspect? Have they asked the operating side what they look for in their engineers? I cannot believe that it is only the firms interested in manufacture who are involved in the training of engineers.

Dr. J. H. Westcott: There was a time when it was possible to turn out a finished graduate who could manage for the rest of his life with what he had learnt. I am afraid that those days have gone, and that post-graduate study is here to stay.

I believe that electrical engineering is possibly the technology *par excellence* in the sense that the man who enters the profession is in for a life of study. The technical men who get to the top are unable to learn from anybody else, being at the top, but may still be young and under an urgent necessity to learn. Possibly the provision of post-graduate courses helps to provide time when these men can be relieved of distraction in order to do some studying, which they cannot be expected to fit in with their domestic responsibilities. I do not know how this will be done, but I think it must be made possible in working hours.

Mr. K. P. Grimshaw: For several months I have had the opportunity and experience, as a part-time lecturer, of helping in a full-time post-graduate course at Imperial College, and I feel that it is a wonderful opportunity for any post-graduate, who has spent a period in industry, to go back to a university and to come again within the sphere of its intellectual life and activities. Such a full-time course lasting an academic year is of great educational benefit to the young engineer and I am

sure that he will acquire new abilities which otherwise might never develop.

We should distinguish between what I would regard as practical reasons for education, such as the need of a firm or organization or industry generally to have men taught certain analytical skills, etc., to do their professional work, and the desire that men, as human individuals, should be better equipped to become possible technological leaders in the country, with a wider range of knowledge.

Many good engineers tend to be limited by their experience. This is probably a truism which applies more or less to everyone, but for the benefit of the country we must widen the range of some of our best young engineers so that they are not so limited. I believe that a full-time post-graduate university-type course may be the best way to do it. This is very different from the shorter-time course with which the authors are concerned. I am quite sure their course is excellent for its purpose, but I believe there is this other need to be recognized and met.

Dr. D. A. Bell (*communicated*): We are, in Birmingham, running two post-graduate courses occupying a full 12 months, and although the response has not been overwhelming, we have had a continuing small number of students supported by their employers, as well as the larger numbers supported by the Department of Scientific and Industrial Research, oversea governments and private funds. In an attempt to meet all the needs of industry, we have arranged for some of the lectures covering particular topics to run within five weeks, but there has been very little demand for attendance at these limited specialist sessions.

Most of our students have industrial experience, but their financial situation is not as favourable as the authors suggest. The D.S.I.R. maintenance grant for a fresh graduate is £340 per annum, and though I do not know the salaries paid by the firm with which the authors' scheme is mainly associated, I believe a usual figure is £650. Even a bachelor in Britain today would not pay in tax anything like the difference between these two figures. After two years' industrial experience the D.S.I.R. grant is £540, but a graduate engineer after his apprenticeship might earn anything from £800 to £1 200. Our experience confirms the authors' view that a man who achieved only a pass degree may have been sufficiently stimulated by industry to benefit from an advanced course, but he will not be eligible for a D.S.I.R Advanced Course Studentship.

I do not agree with the suggestion that there is not the same need for private study in post-graduate as in undergraduate work. Our courses are intensive because a large amount of ground is covered, and the function of lectures is to provide an introduction and a framework for each topic. The student must fill in the details for himself, and the advantage of a session at a university is that he can do the job thoroughly, if necessary consulting books and lecturers in other departments. In Section 5 the authors say that industry doubts the value of experimental project work, but in Section 10 laboratory work is stated to occupy 30% of the student's time. Experimental work associated with the lectures also is valuable if it is designed to illustrate key features of theory.

The experimental project in Birmingham occupies three additional months (July–September). It tests whether the student is an engineer in the sense of being able to apply his learning to a specific research problem in the university, where he has greater intellectual resources at his disposal than he might have in industry, though possibly less material resources.

Mr. M. St. J. Candy (*communicated*): I served my graduate apprenticeship with a large broadcasting organization from 1957–9. The apprenticeship began with a course, lasting about six weeks at their Engineering Training School; this may be summarized as being a general introduction to broadcasting engineering theory to as great an extent as was possible in the time available. It included laboratory work. In the circumstances some form of course was essential, broadcasting being an application of electrical engineering, whereas an apprenticeship in industry would not require a course in quite the same way. A suggestion was made that the course would have been more useful later in the apprenticeship.

Mr. W. Hill (*at Manchester*): I would like to pay tribute to the late L. H. A. Carr, who had the not inconsiderable task of organizing the beginning of the course which forms the background of the paper. Having been associated with this course from the beginning I have always marvelled that whatever its duration, place and scope it seemed always right and 'unthinkable that it should be curtailed in any way'. I believe that it achieves a dual purpose which no extension of university courses can achieve.

It synthesizes the graduate's theoretical approach with that of his colleague in industry, until they start to understand each other by means of a common, basic language. It is then that textbook learning can be applied to urgent topical problems, and in their turn, original solutions to these problems become acceptable to the older engineers because common ground exists on which to judge the theoretical reasoning. This benefit will last only as long as the course remains a true partnership between industry and the universities.

If judged against the above criterion it will be seen that it matters little, except from an expediency point of view, whether the course is whole-time for a few weeks, or one day a week for a large number of weeks.

Mr. B. Birtwistle (*at Manchester*): I have been closely concerned with the particular course described since its inception some ten years ago, and it represents the absolute minimum that is required. Indeed it is little more than a sort of ripple filter smoothing out deficiencies in earlier education. It does not significantly amplify knowledge or understanding.

A major difficulty is the almost random nature of the knowledge and understanding possessed by graduate trainees entering industry, necessitating a revision of work starting at a relatively low level and persisting even into the second year of the scheme.

The question of how best to provide *advanced* training for future industrial research and development engineers is not answered in the paper. Reconditioning the graduate trainee is an essential preliminary but is in itself not sufficient. This advanced training will clearly have to be provided after the graduate-traineeship period, and following the authors' arguments, further short full-time courses are indicated, although the part-time variation may be more acceptable at present. Otherwise, with full industrial patronage and co-operation I believe the 1-year full-time courses could be developed into a more effective solution, particularly if they were organized in a sandwich form over a period of two years.

What is urgently needed is a comprehensive review of the entire period of ten years or so between leaving school and achieving full professional status, and this review should include a critical comparison with the methods adopted in other leading industrial countries. In the meantime it is to be hoped that The Institution can assist in fostering a more dynamic association between industry and the technical institutes in order that at least the minimum stop-gap proposals outlined in the paper may be implemented on a wider scale as soon as possible.

Dr. E. R. Laithwaite (*at Manchester*): I should like to raise a question in connection with what I call the 'extra years'. In 1946 a large number of ex-servicemen, aged between 25 and 30, entered first-year courses in university honours schools, and

many obtained excellent results. My own experience as one of them was that I was more critical and yet more adaptable to the receipt of new information than when I left school at 18. For some time after graduation I was convinced that to come to university with some extra years on one's shoulders was a distinct advantage, and I recommended prospective students, where they had the choice, to elect to do National Service before starting their university course. However, the record of students who begin their courses at the age of 20 or 21 now appears to be no better than those who come directly from school, and I wonder whether the reason is that they still lack some extra years. Have the authors any views on the question of the age at which a man is best able to absorb new ideas in advanced technology?

The question has already been raised as to how much longer we shall be able to go on teaching all the basic theory together with all the additional technological information which is discovered each year. I would put the question in another form by asking how much longer can we go on teaching electrical engineering historically? There are a number of fundamental 'tools' which should be sufficient to enable each student to resolve a wide variety of problems, and it would appear adequate merely to teach these aspects. Unfortunately this is extremely difficult. The student of electrical engineering may well quote Sir Winston Churchill and say, 'Give us the tools and we will finish the job', but the unfortunate teacher replies, 'But what are the tools?'

Mr. R. F. Marshall (*at Manchester*): The courses discussed by the authors form part of a graduate apprenticeship which, in accordance with the recommendations of The Institution, provides training and experience in three stages—basic workshop experience, general electrical and mechanical experience and sponsored or objective training—yet little is said in the paper about the place these formal studies occupy within the apprenticeship and the time such studies require or deserve. It is suggested that the duration of the course is entirely a matter of finance. Finance is, of course, important but the period of eight weeks was in fact selected because it represented what was thought to be the appropriate period for this phase of graduate training in relation to the other commitments upon the trainee's time. The period also fitted conveniently into the timetable of graduate-training movements which operate within a particular organization.

An important reason for the introduction of advanced courses was that they can and do provide an intellectual challenge to graduates in training and are an excellent answer to those who say that graduate apprenticeships represent an undesirable intellectual 'let down' when compared with university conditions.

A large company will have wide engineering interests, and its recruitment and training arrangements must, if possible, satisfy the needs of widely differing engineering technologies. The preselection (in the first year) of the second-year optional subjects may mean that too many men study a subject which may not be most important to the company's interests. The authors have explained that a man can choose a second-year option differing from the work he did during the first-year tutorials provided that the tutors are satisfied that he has the necessary capacity and is prepared to do some extra 'homework' in the summer months. This would seem to be a weakness in the arrangements, which needs re-examination.

Mr. C. B. Cooper (*at Manchester*): Having had an interest in the power systems tutorial course described in the paper, some of my experiences may be of interest. In this course about 50% of the students come from oversea, and the standard of training they have already undertaken varies very widely. This is particularly evident in the field of mathematics, where experience of relatively new topics is essential for the man who in future will be doing research and development work. Many of the oversea students come to Britain for just the period of their training, and so if they are to benefit from this type of course it must come during the training period.

We find that, although most of our students have covered the work needed for entry to the second year, they lack the experience and confidence which enables them to apply the principles taught at a university. The tutorial scheme gives this experience, and in addition gives a man time to think about his work and discuss his ideas freely with an engineer having some years' experience in industry in his chosen subject.

Power systems cover a wide range of topics requiring some knowledge of electrical machines, switchgear, and transformers, in addition to some experience of control methods. We have in the past drawn our tutors from men working in both the power-system and the switchgear fields, and in this way try to make the course broader in scope than it might appear to be from the four topics quoted in the paper.

THE AUTHORS' REPLY TO THE ABOVE DISCUSSION

Messrs. N. N. Hancock and **P. L. Taylor** (*in reply*): As the Duke of Edinburgh has so rightly observed, if you start a discussion on education you are likely to provoke a riot. One of the purposes of our paper was to promote discussion on this most important topic. We did not intend to imply that the particular course we described is the only type that need be considered. In fact we shall be happy if our paper moves those who advocate other types to present their experiences in the pages of the *Proceedings*.

Papers written jointly by authors representing the academic and the industrial sides of the profession would be particularly valuable. It is commonly realized that post-graduate courses require co-operation between the two sides, and lack of this implies inadequate contact between them, so that the resulting courses will not meet a real need. Another purpose of the paper was to show that, given good will, the difficulties involved in this co-operation can be overcome, and thereby perhaps to encourage other academic institutions. Of course, the resulting courses will be compromises; as Mr. Marshall has said, the precise form and duration of the course we described was decided by non-educational factors, and to this extent the educational pattern may be considered to be distorted. But it is as well to realize that an insistence, in inappropriate cases, on a full-session course at the end of which there is the prospect of an award may result in even grosser distortions. We agree with Mr. French that this particular problem calls for attention.

We are aware of the excellent courses organized by Dr. Gibbs for another part of the same industrial body and which accord with his comments. They fulfil a rather different purpose, and while admitting the need for them, we do not concede that there is not an equal or greater need for short full-time courses earlier in a man's career. We have referred to the College course as second-year because of the first-year courses organized (for reasons indicated in the paper) by the firm concerned. The College would willingly consider putting on a corresponding first-year course for employees of other firms, but so far, though the need has been felt in certain cases, the numbers involved have not made this possible. The course at the moment is ideal for only one firm. Another purpose in publishing the paper was to arouse interest from other firms and organizations. We

could then embark on the more difficult experiment of modifying the course to make it more generally suitable. Perhaps Mr. Tompsett would discuss with us whether the power systems course is suitable for the men he has in mind. We would welcome suggestions of topics for inclusion, and if other firms and organizations feel the need for other types of course we would be glad to hear from them. Academic institutions cannot sense every need and make all the proposals. From this point of view, the dearth of contributions from industry to the discussion in London has disappointed us.

We agree with Dr. Bell that D.S.I.R. training awards are unattractive when compared with industrial apprenticeship rates. It is interesting to note that the number of such awards in electrical engineering in the four years of the scheme have been 8, 16, 16, 20. The last figure is out of a total of 315 for all subjects. The total of 60 electrical-engineering awards in all the universities of this country in four years has to be set against 59 attending our course during the present session alone, and we do not think that the standard or content of our course is so much less than that achieved in a full-session course. We wonder whether Dr. Bell is confusing project work, which takes up so much time, with set experimental work, which is what our course includes. We are sure that he does not intend to imply that intellectual resources are less common in industry.

We agree with Mr. Marshall that eight weeks is a big slice out of an apprenticeship, but he is aware of the compromise on which this figure was chosen and also why the time comes out of the apprenticeship rather than later. The dilemma is that the later a man chooses his specialization, the more time he may waste beforehand; the earlier he decides, the greater is the likelihood of his wanting to change.

While endorsing Mr. Hill's tribute to the late Mr. L. H. A. Carr, who, about 15 years ago, restarted the industrial precursor to this course, we must, in fairness to others, point out that it actually dates back about 40 years. We would like to hear about what has resulted elsewhere after 40 years of development.

DISCUSSION ON 'THERMISTORS, THEIR THEORY, MANUFACTURE AND APPLICATION'*

MERSEY AND NORTH WALES CENTRE, AT LIVERPOOL, 28TH NOVEMBER, 1960

Mr. F. J. Hyde: I should like to mention three examples of my own use of thermistors.

In many h.f. measurements a variable-resistance element is required. The directly heated thermistor may conveniently be used for this.[A] A d.c. bridge circuit comprises equal high-resistance ratio arms (say), a thermistor and a standard resistance box. To obtain any required value of r.f. resistance in the thermistor the standard resistance is set to this value and the power dissipation in the thermistor is adjusted by altering the power applied to the bridge until balance is achieved. Then, since the h.f. isothermal resistance of the thermistor is equal to its d.c. resistance at the operating point, it has an h.f. resistance equal to the d.c. resistance of the standard.

Some years ago I was measuring very-low-frequency electrical noise associated with a current-carrying germanium filament.[B] It was necessary to keep temperature fluctuations of the specimen small since these would give rise to an electrical noise component via a temperature/resistance modulation mechanism. The temperature fluctuations in the oil bath in which the filament was placed were monitored using a thermistor and they were less than $\pm 0\cdot 001^\circ$C in a period of 8 hours. This measurement was possible because the thermistor material had a much higher temperature coefficient of resistance than germanium and did not generate as much electronic noise.

I have also been interested in the small-signal equivalent circuits of thermistors and helped to establish that given in Fig. 14(*a*). This equivalent circuit applies to a thermistor having a negative temperature coefficient of resistance. We predicted that positive-temperature-coefficient materials would have a similar equivalent circuit, but with a capacitance replacing the inductance. At that time no materials with a large positive temperature coefficient were available and so we were unable to confirm this experimentally. It seemed clear, however, that such materials would come, as indeed they have. I suggested that a 'thermal oscillator' could be produced by connecting together positive and negative temperature coefficient thermistors.[C] All the ingredients for oscillation are there—inductance, capacitance and a negative resistance if bias is beyond turnover. I am interested to see that this has now been done.[D, E]

I take issue with the authors on nomenclature. At the beginning of their paper they state 'A thermistor is a temperature-sensitive resistor. It has a large negative resistance/temperature coefficient. . . .' I submit that 'thermistor', which is clearly a condensation of 'thermal' and 'resistor', should be used in a general sense for any thermally-sensitive resistor. Then we shall not need a new name for each positive-temperature-coefficient device as it is invented. A few years ago it was reported from Bell Laboratories that mixtures of doped barium and strontium titanates could be devised with a positive temperature coefficient of approximately 10% per deg C. Are there any developments in this area?

Mr. J. R. Cannon: A simple case of compensation for the effects of temperature arises in very-narrow-band filters such as are used for pilot pick-off (see, for example, Fig. 10). Very constant loss is desirable; it is generally found, however, that the loss does change as a function of temperature, even though the resonant frequency may remain constant by virtue of using components having differing temperature coefficients so that positive drift in one is counterbalanced by negative drift in another. This is due to the fact that the circuit Q-factor is also a function of temperature. This effect may be substantially eliminated by employing a voltage divider arrangement, containing a suitable thermistor in one arm, connected in the path at a convenient point.

Would the authors agree that their statement which implies that the time-constant of the thermistors in a.g.c. systems must be long compared with other time-constants is an over-simplification? In any event one does not have complete freedom of choice, particularly in the application to a carrier system working on open-wire lines. Here the line loss is immediately

* SCARR, R. W. A., and SETTERINGTON, R. A.: Paper No. 3176 M, January, 1960 (see 107 B, p. 395).

influenced by every change of weather and the control system must be fast enough to regulate effectively in a time which is of the order of seconds.

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(E) Aleksander, I.: 'A V.L.F. Thermal Relaxation Oscillator', *British Communications and Electronics*, 1960, **7**, p. 352.

Dr. R. W. A. Scarr and **Mr. R. A. Setterington** (*in reply*): In reply to Mr. Hyde, the name 'thermistor' has by association come to be almost synonymous with a negative-temperature-coefficient device. For example, B.S. 204: 1960 defines a thermistor as 'A resistor having a very large temperature coefficient of resistance which is usually negative'. However, we are inclined to agree with Mr. Hyde that we went too far in our paper—the name should not be confined to a device having a negative temperature coefficient.

Recent developments on doped barium titanate and barium-strontium titanate systems are described in a paper by Sauer and Fisher.*

We do not entirely agree with Mr. Cannon; we feel that the two conditions given in the paper, a long-time-constant network together with the characteristics of a simple *RC* network (i.e. a minimum phase-shift characteristic) are sufficient, even if they are not readily achievable in a practical device.

* Sauer, H. A., and Fisher, J. R.: 'Processing of Positive Temperature Coefficient Thermistors', *Journal of the American Ceramic Society*, 1960, 43, p. 297.

DISCUSSION ON 'WATER-TURBINE-DRIVEN INDUCTION GENERATORS'*

Before the Rugby Sub-Centre *at* Rugby 20*th October*, *and the* North-Eastern Centre *at* Newcastle upon Tyne 7*th November*, 1960.

Mr. L. D. Anscombe (*at Rugby*): The application of induction generators to the North of Scotland system has been economically favourable because excess leading kVAr is available. In small systems having net lagging loads there is difficulty both in providing and economically justifying the provision of the necessary magnetizing kVA, which at full load must be equal to about two-thirds of the rated power of the induction generator, in addition to any which is required by the load itself.

The switch-in tests described in the paper show results very similar to those obtained with the closing-in of an unexcited synchronous generator. Such 'motor paralleling' had been used in cases where extreme simplicity of operation was considered necessary, the only difference in the case of the synchronous generator being the subsequent closing of the field switch to enable the machine to pull into step.

In discussing the economics of the schemes in Section 11, the author refers to capital savings attributable to the use of induction generators by the omission of the turbine-governor actuator and the automatic voltage regulator. Could not these items also have been omitted if synchronous generators were used? Must we always assume that because large synchronous-generator sets have to be so equipped they are a necessity for smaller units also? I believe that there is no more objection to operating small alternators on a large system with a constant preset power input and fixed excitation than there is in having a number of synchronous motors on the system.

In small sizes the induction generator should be appreciably less expensive than its synchronous equivalent, but with increasing size special constructional features are necessary and the inherently lower efficiency becomes significant. Has the author made any investigations to determine the approximate size above which the induction generator ceases to be competitive?

Mr. D. D. Stephen (*at Rugby*): The low-voltage protection referred to in Section 6.4 does not appear to provide complete protection, since the conditions following the opening of the main circuit-breaker may vary over a wide range. If the load passing through the circuit-breaker before opening were small, then stable operating conditions could be reached by the generator varying its speed to suit the following conditions. The fixed hydraulic power must be consumed by the remaining connected electrical load, and the generator voltage and frequency must be such as to pass this power to this load. Simultaneously the generator voltage and frequency must be such that the requisite magnetizing current is obtained from the section of line still connected to the generator.

It is quite possible, therefore, that after the circuit-breaker has opened the generator will continue to supply the remaining connected load at a voltage close enough to normal to prevent either the under-voltage or over-voltage relays from operating. Should the circuit-breaker be reclosed under these conditions the generator could suffer serious damage if its generated voltage were not in phase with the incoming supply voltage. The effect of reclosing on the generator could be as severe as starting direct-on-line at twice the normal voltage. What protection is provided against the possibility of such sudden restoration of supply to the generators?

Mr. A. J. Gilbert (*at Rugby*): I wonder whether the author has considered the possibilities of the compensated induction generator for application up to 600 kW. This type of machine has considerable advantages over the normal induction generator.

The machines, which are compensated induction motors,† run as generators, will do all and more than has been claimed for the lower-powered induction generators. They are particularly suitable for applications where there is a varying surplus of power available, such as process steam in factories or water power in waterworks installations and similar hydro-electric schemes. They are simple to start and operate and there is no synchronizing problem likely to alienate the supply undertaking.

When the machine runs in parallel with an incoming supply, adjustment of the brushgear controls the kVAr fed into the

* Allan, C. L. C.: Paper No. 3140, December, 1959 (see **107 A**, p. 529).

† Adkins, B., and Gibbs, W. J.: 'Polyphase Commutator Machines' (Cambridge University Press), p. 193.

supply. For a given brush setting the kVAr fed back is almost independent of the load on the machine.

The machine can also operate independently and be used to supply essential services such as lighting and auxiliaries. Adjustment of the brushgear under these conditions varies the voltage output. The voltage regulation when operating independently is comparable with that of a synchronous machine.

The brushgear may be hand or motor operated, and when motor operated it can be used in conjunction with a regulator to give automatic control of power factor when running in parallel with another supply, or of voltage when operating independently.

Experience with these machines has shown them to give extremely satisfactory service. When operating in conjunction with one supply system the switching and protective arrangements are comparatively simple, and there is no complication about determining the phase rotation.

Although the author appears to be almost embarrassed by his surplus of capacitive kVA, I should be interested to know whether he considered the use of compensated machines for his lower-powered stations or indeed for any other applications, and to have his comments on the possibility of their increased use in hydro-electric applications, particularly in remote areas.

Mr. P. L. Olsen (*at Newcastle upon Tyne*): Reliability and robustness are often put foward as advantages of the induction generator over the synchronous type, but cage windings are not as reliable or as robust as one is led to imagine, and difficulties with them are not uncommon. Experience with this type of generator has been mainly confined to machines with restricted over-speeds. In the application to hydro-electric plant with appreciable over-speeds, great care must be taken in the design of the cage windings and the end-rings in particular. Trouble often arises as the result of too frequent starting or difficult starting conditions, and the practice of switching in at or near synchronous speed as adopted by the Board is advisable.

The induction machine may be useful in small pump-storage schemes, and a recent development of the pole-changing 2-speed machine with the single winding can be applied to give the different speeds sometimes required for optimum operation of the pump and turbine.

With regard to generator drying-out methods, a further alternative would be to provide a transformer to give a low-voltage supply to the motor. Hydro-electric machines usually require some form of internal heater to prevent condensation inside, and one method of warming the machine when it is standing for long periods is to provide a transformer to give just sufficient current to maintain warm conditions. This transformer could be adapted to provide sufficient current to dry out the windings in the first instance, although it would not be economic if very high temperatures in the windings are required for complete drying out, as might be the case for 11 kV machines.

Mr. N. Young (*at Newcastle upon Tyne*): In Section 3.2.2 the author states that the speed at switch-in has little influence on the initial current. I think that the explanation of this is that there is an initial transient current, of fairly short time-constant, during which the flux is built up in the machine. The current is not controlled by the switch-in speed. If we examine the second oscillogram of Fig. 8, however, it will be seen that there is a period after the initial transient has died away when a current of about half the initial value is maintained for a few cycles, before falling to the no-load value. If the switch-in speed had been lower, this current would have been maintained for a longer period, and this is confirmed by the current/slip curve of Fig. 17, where the current is substantially constant for about 90% of the run-up.

I can confirm that this is the case when switching large induction motors and that the currents and time taken for the initial transient fall in current is not influenced by the switch-in speed. However, the time during which the current is maintained at the nominal starting value is very significantly influenced by the speed at switch in.

Has the author taken any measurements of starting currents when switching the machine from rest?

In view of the fact that the voltage dip in the 132 kV system at switching is insignficant, has the author considered starting the rather difficult Kaplan machines by opening the guide vanes to the no-load position and then immediately closing the circuit-breaker? The machine would start and run up in the manner of an induction motor switched direct on line, and the magnitude of the starting current and the associated forces would be no greater than those occurring when the machine is switched at synchronous speed, although they would be maintained for a longer period.

The author mentions the difficulty in keeping the system voltage down at times of light load. Are the induction generators themselves run at light load, their effect on the system being that of asynchronous reactors?

In addition and for the same reasons, are the machines left connected, but idling, at times when it is not convenient to allow water to flow through the turbines, i.e. water shortage, or high level in the lower reservoir of an inter-reservoir plant?

Mr. W. Brittlebank (*at Newcastle upon Tyne*): The ratio of capacitive current to active current will be large on the small machines, which can give rise to self-exciting conditions. For an induction generator connected locally to a long transmission line which becomes tripped at the remote end of the line, the generator may become self-excited. Over-voltage protection at the machine connection trips the circuit-breaker provided that the frequency is sufficiently high to cause its operation.

A case in question was when the induction generator was supplying load to the common-services boards of two power stations. When tripping occurred only at the remote end owing to a fault, the frequency was not high enough to trip the machine circuit-breaker on over-voltage protection. Some load from the power stations was still connected to it and it became self-excited. Normally power-station auxiliary motors are connected to these boards, but they also feed the valves for power-line carrier equipment. To guard against this it would seem better to have a lower over-voltage setting, for carrier equipment is sensitive to high-voltage conditions.

Mr. J. Taylor (*at Newcastle upon Tyne*): It is stated in the paper that there are considerable financial advantages in the employment of induction generators, largely owing to the omission of governor gear and voltage-regulating equipment and the simplification of control equipment. This obviously makes them an attractive proposition for use on the North of Scotland Hydro-Electric Board system where, for various reasons, whether it be for the benefit of the fish or for reservoir regulations, a certain amount of compensation water which would otherwise be wasted can be harnessed relatively cheaply by the employment of low-capital-cost plant.

The use of induction generators in a system must of necessity be limited according to the amount of synchronous plant connected to that system. In fact, about 3% of the North of Scotland Hydro-Electric Board generation is at present provided by induction generators. Is this balance likely to be increased appreciably, and if so, by what amount?

Mr. C. L. C. Allan (*in reply*):

To Mr. Anscombe.—It is interesting to have Mr. Anscombe's confirmation that the switch-in tests on unexcited synchronous

generators have shown results which correspond with those referred to in the paper. We have made tests on 'motor paralleling' with synchronous water-turbine-driven machines but have found that the rotor polarity frequently 'comes up wrong', and there is subsequent pole slipping when excitation is being applied.

In some cases synchronous machines do not require a turbine governor actuator, and its omission can be worth while on smaller sets. The automatic voltage regulator would still be needed, however. We have, in fact, commissioned within the last year a 540 kW synchronous machine without governor to run in parallel with a system of about 10 times its capacity. In larger sizes of machine the cost of the governor actuator becomes small in relation to the total costs, and there seems little to be gained by omitting it.

To Mr. Stephen.—Mr. Stephen is quite correct in saying that low-voltage protection may not disconnect the machine in the circumstances which he describes. We have had cases where a small part of the network has become disconnected from the main part and induction generators have maintained supply at reduced voltage and frequency.

No harm will result from a self-exciting system as long as the voltage and frequency are not allowed to depart too far from normal. Indeed some supplies may be kept going. However, reconnection to the main network under such circumstances would not be satisfactory. We are considering the adoption of frequency-sensitive relays in addition to over-voltage and under-voltage protection.

To Mr. Gilbert.—We have not considered the application of the compensated induction generator to any of our installations. I think that the machine referred to requires a main and compensating winding on the rotor, a main winding on the stator, a commutator, movable brush gear and a starting resistor. The machine has the advantage of voltage and consequently reactive-power control, and can generate on its own. Where these features are required the machine might be useful, but it must be more expensive than a plain induction generator, and it would not have the simplicity and robustness which are very desirable features in the applications described in the paper.

To Mr. Olsen.—Mr. Olsen points out that induction generators coupled to water turbines must be designed to be safe at the over-speeds possible. Our experience of induction generators has been very good, and there has been no trouble at all from the rotors. It is considered that machines should continue to be designed for switching in after running up from the turbine. Apart from avoiding shocks, as Mr. Olsen points out, this allows machines to be designed for better efficiency.

We have not had occasion to try a transformer giving reduced voltage for drying out. Unless more than one voltage were available the system would be a little inflexible, and the motor/generator arrangement referred to in Section 8 of the paper gives a wider range of control.

To Mr. Young.—Mr. Young's views about the duration of the transient as affected by switching speed coincides with the views we have formed. Machines are designed for switching-in at or about synchronous speed for the reasons given to Messrs. Stephen and Olsen.

At light load times most of the induction generators are normally running in any case, and we have not made any arrangements to leave machines connected or idling if they are temporarily not generating.

To Mr. Brittlebank.—Mr. Brittlebank agrees with the need for overall protection in self-exciting conditions. Quite considerable voltage rise can be impressed on a system by a synchronous generating plant, but not for a rather long period as in the case he quotes. The setting of voltage and frequency protection is a matter of choice to some extent, and suitable arrangements can be made although more refinement naturally costs more.

To Mr. Taylor.—It does not seem likely that the percentage of induction generators will be increased. Their use is mainly confined to smaller sizes of machines, and as new main generation projects are developed, the compensation flow or inter-reservoir flow sections will continue to use induction generators. But these applications will always be a small proportion of the total.

DISCUSSION ON 'THE INFLUENCE OF CONSUMERS' LOAD/CONSUMPTION CHARACTERISTICS ON METERING PRACTICE'*

Mr. A. McCulloch (*Australia: communicated*): On page 571* Mr. G. F. L. Dixon says, '. . . an Electricity Board cannot, in the long term, lose money. When we make meters more accurate we are not saving losses so much as spreading electricity charges more equitably. I think that the whole problem will eventually have to be viewed against a wider economic background'. I do most strongly support this approach, and think it is long overdue. I feel that the influence of meter-room specialists is causing an economic waste in the industry. A measuring device for any service or goods is merely a means of apportioning charges between customers, and the accuracy of an electricity meter is inherently so much higher than that of other measuring devices that the search for higher accuracies has generally no economic justification.

Electricity accounts involve so many variables such as reading periods, seasonal variations in usage, etc., that the vast majority of consumers would never be aware that we had in fact achieved a higher degree of accuracy. I cannot see why we should take any other approach. Any electricity undertaking budgets for its revenue on the basis of current meter-reading figures, and fixes tariffs accordingly. If we, by a freak device, could speed up all meters by, say, 5% we would reduce tariffs by 5% and no consumer would be required to pay any more or any less. We are merely concerned with fixing prices to meet costs, and we base our considerations on current meter accuracies.

The author seems to lay great stress on the price per kilowatt-hour and the statutory significance of this unit. But the same applies to other measuring devices and our accuracy can be so easily maintained at a higher level that stress on this seems quite unjustified. I cannot see how the higher degree of accuracy has any effect in promoting development. In the overall picture, the undertaking seeks the same total income regardless of meter accuracy. All we are attempting to do is change the distribution of charges between consumers, and then only to an extent which is practically not discernible by the consumer.

It is suggested that lack of accuracy misleads the undertaking and does not allow the efficiency to be measured accurately.

* GOLDS, L. B. S.: Paper No. 2863 M, February, 1959 (see 106 A, pp. 342 and 571).

Surely this is highly theoretical. Losses in the system take many forms, and the undertaking are informed of the difference between, say, units generated and purchased and units sold. They can never be informed in detail as to how that difference is made up. But if at great cost they could be so informed, I fail to see what difference it would make if they knew that meter-reading errors contributed a definite percentage to that difference. They could, if they so desired, spend a considerable sum to reduce these errors, thus increasing revenue, and subsequently reducing prices accordingly, whereas they could have merely accepted the error and avoided spending the money.

The author, because any under-registration means a loss of revenue from a particular consumer, assumes that we must spend money to avoid the loss, but the benefit is passed on to other consumers. He overlooks the fact that revenue is merely required to meet costs, and any surplus is to the benefit of consumers. I knew of a private company which supplied without meters, and they operated at a profit. Of course, the apportionment between consumers was fantastically inaccurate.

I think it is time that we as an industry recognized the fact that this is the prime purpose of meters. A reasonable accuracy is necessary but sufficient. We should not spend money—consumer money—to obtain and retain extreme accuracy.

Mr. L. B. S. Golds (*in reply*): In a paper published in 1953* I pointed out that 'Any expenditure upon unnecessarily high accuracy at any particular load point would be a waste of national effort, and an unjustifiable imposition of cost on the consumer, who would receive from it little or no benefit'. I therefore share Mr. McCulloch's desire to avoid an uneconomic waste in the industry and agree that the whole problem should be examined against as wide an economic background as possible. I deprecate, however, Mr. McCulloch's statement regarding the influence of 'meter-room specialists'. In these days of specialization it is not easy for the specialist or the administrator to take a balanced view of the whole economic background. But, provided that both are well informed—as surely they must be if they are taking their proper place in any organization—the right balance should be obtained. If that is not the case, it simply points to one or the other not having the correct appreciation of the specialist function.

The statement that electricity meters have an accuracy 'inherently so much higher than that of other measuring devices' does not accord with the facts, particularly when viewed against a term of years and various conditions of service. The United Kingdom statutory tolerances for electricity meters, compared with some other devices covered by Weights and Measures, are as follows:

Electricity meters	$2\frac{1}{2}\%+$ and $3\frac{1}{2}\%-$
Beam scales	0·0005%
Weighbridges	0·04%
Measures of length ..	0·025%

* GOLDS, L. B. S., and SCHILLER, P.: 'Meter Problems and Consumers' Load Characteristics', *Proceedings I.E.E.*, Paper No. 1483 M, April, 1953 (100, Part II, p. 619).

More than 10% of some types of meter in present-day use register outside the statutory tolerances after a period of years, with a tendency generally to under-register.

Regarding the controversy as to whether under-registration involves a loss in revenue, a simple illustration may be of some help. A single consumer owning generating plant is concerned only with the amount of fuel used and requires no electricity meter. When two or more consumers take a public supply they agree with the undertaking for the purposes of apportioning their shares of the costs to adopt a certain quantity of electricity as a standard unit. The unit is then the basis of the business transaction. If more electricity is actually supplied than is registered by the meter, this results in loss to the undertaking. The consumers will know only that they are being charged a larger amount per unit for an unspecified reason if the undertaking increases its prices to obtain the necessary revenue.

Whether or not the amount of money paid by the consumer is the same, the price per unit is the yardstick. When the amounts of money and number of consumers involved are small, the accuracy of the assessment of units is of less consequence than when the income is of the order of £50 million per annum, and the number of consumers of the order of 2 million. An under-registration of 1% then amounts to £500000 per annum, but the undertaking will not adjust its tariffs year by year to compensate for these errors, because it will not know they exist unless steps are taken by testing samples of meters to find out. The Area Boards in England and Wales are nowadays operating on a very small working margin of profit per unit but are dealing in large quantities, and for that reason accuracy of metering is today of vital importance to the industry.

Of course, the undertaking budgets for its revenue on the basis of current meter readings but tends to assume that those meters will continue to register at a constant rate year by year. In fact, there is a gradual deterioration due, not to 'freak devices', but mainly to the effect of friction. I have never been aware of a deliberate increase in price to compensate for slow meters, but I know of cases where refunds to individual consumers have had to be made owing to over-registration. Present-day electricity supply-tariff building is a complicated operation, and to complicate it further by taking account of a variable unit of measurement seems a retrograde step.

Mr. McCulloch refers to 'units generated and purchased and units sold'. If the 'freak device' were put into effect thus making the units sold say 5% higher, the effect might be that more units would be sold than generated. I wonder what the consumers would say?

I conclude by agreeing with Mr. McCulloch that money should not be spent to obtain and retain extreme accuracy. His contribution would have been of much more value if he had defined 'reasonable accuracy' in quantitative terms of average error and standard deviation and suggested the stage at which money should be spent to achieve that 'reasonable accuracy'.

DISCUSSION ON
'ELECTRICITY IN THE MANUFACTURE OF HYDROGEN PEROXIDE'*

NORTH MIDLAND UTILIZATION GROUP, 15TH NOVEMBER, 1960

Mr. F. Clarke: The paper indicates that the installation is now 0 years old and I wonder what changes would be made today. Semiconductor-type rectifiers would give a wider choice of voltage and would enable a lower and safer operating voltage to be used.

We are told that liquid fuels are no longer satisfactory for rockets and missiles and that Blue Streak was abandoned largely on this account. What do the authors consider are the marketing prospects for hydrogen peroxide in the quantities now being produced?

Mr. G. Sheppard: The process described can produce about 470 tons of gas per annum, a conservative estimate being 1 ton, or 1 500 yd^3, per day. The gas, which is composed of approximately 95% hydrogen and 4% oxygen, is released directly into the atmosphere above the cells. In the Central Electricity Generating Board, generating sets use hydrogen gas as a means of cooling and we have always been concerned about the risk of explosion in confined spaces or inverted pockets where an escape of gas might collect.

I note that the design is such that the hydrogen is removed by a continuous copious amount of air being swept downward over the cells. Is there an interlocking arrangement which will close the plant down should a failure of the fans occur?

Mr. F. H. Merrill (*communicated*):† The figures quoted for the cost of electricity generated by the back-pressure turbines is indicative of the savings which can be obtained in a set-up of this kind. The marginal cost per kilowatt-hour, C_1, i.e. the actual cost of the additional coal which must be fired to the boiler to produce a unit of electricity, can be calculated from

$$C_1 = 860 \frac{1}{\eta_1} \frac{1}{\eta_2} \frac{1}{7000} \frac{2 \cdot 204}{2240} \times C_2$$

where 860 is the heat equivalent in kilogramme-calories, η_1 is the efficiency figure for the turbo-alternator, η_2 is the boiler efficiency and C_2 is the cost of coal in pence per ton entering the boiler.

The turbo-alternator efficiency figure is of course concerned only with losses which are external to the turbine, i.e. gland losses, radiation losses and alternator losses. The internal efficiency of the turbine has no effect since losses here reappear in the heat of the exhaust steam.

From this formula the cost of coal per kilowatt-hour generated, assuming a turbo-generator efficiency of 0·9, a boiler efficiency of 0·85 and a coal cost of 93s. a ton, is 0·176d.

The authors state quite rightly that higher steam pressures than are used in their plant would give greater financial returns. However, pressures in excess of 900 lb/in^2 demand special materials of construction which increase the capital cost, and at the same time the yield of energy for each increment of pressure decreases with increase in pressure. While it is difficult to generalize, it is likely that the percentage return on capital will pass through a maximum and then fall, and the optimum point may well lie in the region of 900 lb/in^2. Such a figure need not be feared for back-pressure operation since experience has shown that such boilers can be operated quite satisfactorily even with a very high proportion of chemically treated make-up water.

Regarding the stability of d.c. machines operating in parallel with rectifiers, Fig. A shows a system connecting a number of

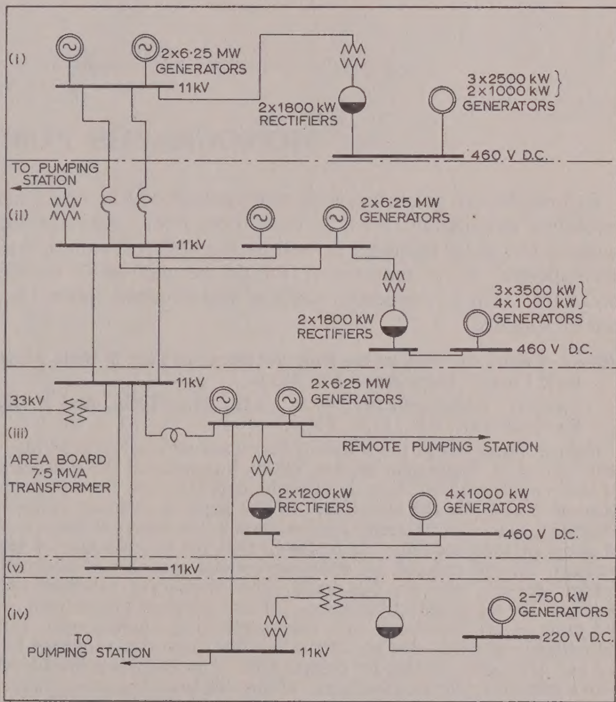

Fig. A.—Electrical system for group of five chemical works.

chemical plants, similar to that described by the authors in that d.c. machines are operating in parallel with rectifiers while the a.c. system operates in parallel with the public supply. It was found that, whenever there was a severe disturbance on the public supply system, the d.c. generators tended to overload and when the fault was cleared they usually lost load to the extent that they were being fed momentarily with power from the rectifier system. As the d.c. generators were fitted with reverse-current relays, primarily for protection of the armature windings, they almost invariably tripped out. The fitting of time delays to these relays has, however, enabled the machines to remain on the line during transient faults.

Two other forms of protection are considered necessary for back-pressure turbines shown in the system. First, when the steam turbine is tripped a switch is operated which trips the generator or alternator. This is essential, since if the turbine were motored with no steam flow, a dangerous temperature

* VIGERS, B. E. A., and FLETCHER, O.: Paper No. 3196 U, February, 1960 (see 107 A, p. 463).

† Mr. Merrill's contribution is based on his remarks at the District Meeting of the North-Western Centre at Northwich on the 26th October, 1960.

would quickly be reached on account of the churning of the steam inside the turbine casing.

It is now proposed that the protection should be extended slightly by a monitoring feature which will ensure that the generator or alternator can be tripped only when the steam flow through the turbine has ceased. This will be done by means of a wattmetric relay and is intended to guard against runaways should the governor system prove faulty or the steam valve stick when the turbine is tripped.

Messrs. B. E. A. Vigers and R. O. Fletcher (*in reply*): The question on semiconductor rectifiers and safe operating voltage has been dealt with in earlier discussions. If developments had continued on the electrolytic process these would very likely have led to the use of very much higher currents per battery at a lower overall voltage, in which case semiconductor rectifiers would be the obvious choice.

Regarding the use of liquid fuels for rockets and missiles mentioned by Mr. Clarke, it is a fact that the specific impulse available from hydrogen-peroxide/fuel systems is not high enough to achieve escape velocities. There are, however, valuable uses for hydrogen peroxide in connection with rockets, particularly those which may be used for manned-vehicle propulsion. The experience of our company is that the uses of hydrogen peroxide are continuing to expand.

In the plant described there are no confined spaces where hydrogen could collect to give a risk of explosion, and the very high rate of both natural as well as induced ventilation has made it unnecessary to provide additional safeguards.

Generator and alternator switches are arranged to trip from the steam end, as described by Mr. Merrill, and there is no advantage, therefore, in fitting reverse-current relays. It would be unusual for the steam valve to stick in the open position without the turbine driver noticing the condition when attempting to reduce load. Relays to protect against tripping the alternator under these conditions prevent faulty operation by the attendant, but may have disadvantages in other respects.

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Effects of Eddy Currents on the Rise and Decay of Flux in Solid Magnetic Cores. Monograph No. 463 U.

Prof. N. Kesavamurthy, M.A., B.E., M.Sc.Tech., and P. K. Rajagopalan, B.E., M.S., Ph.D.

Rigorous analysis of the transient behaviour of a solid-cored magnetic circuit of rectangular section, taking into account the resistance of the magnetizing winding, is difficult to obtain and has not been attempted so far in the literature. In this paper an attempt is made to obtain, to a close degree of approximation, the transient behaviour of cores of such sections. It is shown that the complexities of the problem can be reduced by choosing certain contours of constant current density, and on this basis, four alternative solutions are obtained for the transient behaviour. These contours are justified on the basis of (*a*) flux decay with the magnetizing winding open and (*b*) eddy-current loss due to alternating flux: rigorous solutions for the two cases are available for comparison. The analysis is illustrated with a specific application to the rise of flux due to an impact excitation on a solid core, and certain conclusions are drawn.

An interesting feature of the paper is that, in the majority of cases, the results are presented in a non-dimensional form.

Pole-Entry Flux Pulsations. Monograph No. 469 U.

K. J. Binns, B.Sc.

The rapid changes in flux distribution which occur as a slotted armature moves relative to a pole corner give rise to losses both in the copper and in the iron. These flux pulsations can have a considerable effect on the performance of both a.c. and d.c. machines.

The paper presents an accurate determination of the field of a slot passing under a rectangular pole corner, and the flux changes under relative motion are calculated over the range of dimensions of practical interest. The flux entering the armature slots is calculated and compared with values measured by search coils; the two sets of results are in close agreement. The effect of the slot width on the induced e.m.f. in the armature conductors is commonly ignored in design work. However, the effect is shown to be appreciable. Finally, the variation in the forces acting on the armature teeth is determined; this quantity is exceedingly difficult to obtain experimentally.

The Determination of the Transient Temperature Distribution in a Turbo-Alternator Rotor by means of an Analogue. Monograph No. 472 S.

B. M. Weedy, B.Sc.(Eng.), Ph.D.

Owing to the difficulty in the direct measurement of temperature in the winding and steel of turbo-alternator rotors, indirect methods, requiring a knowledge of the boundary conditions, can be used to advantage. In the present work the electrical-analogue approach is used, i.e. the replacement of thermal quantities by analogous electrical ones, giving both steady-state and transient solutions. The latter is of interest when considering overload capacity and unbalanced loading.

The rotor studied was that of a 60 MW 3000 r.p.m. 11 kV 50 c/s turbo-alternator with indirect cooling by hydrogen at a pressure of 4 lb/in^2 (gauge). Design and constructional features of a resistance-capacitance analogue representing two-dimensionally the thermal conditions in a 'slice' of rotor 1 in thick and containing the hottest slot are given. By simple adjustment any slice in the rotor length may be simulated. Incorporated in the analogue are electronic current-input units which vary the 'copper-loss' input currents to allow for the change in winding resistance with temperature.

Transient temperature rises at 20 points in and around the hottest slot in sections at the rotor end and at the axial centre are obtained for the application of step-functions of excitation currents of 500 A and 550 A and various values of surface loss. The mean winding temperature obtained is compared with that measured on the actual machine using the change-of-resistance method and good agreement is found. Variations in the coolant temperatures are seen to have small effect on the analogue temperature-rises.

PROCEEDINGS OF THE INSTITUTION OF ELECTRICAL ENGINEERS

Part A. POWER ENGINEERING, OCTOBER 1961

CONTENTS

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